Determination of the Coupling Strength of ρ Mesons and Nucleons†

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Consideration of pion-nucleon elastic scattering in the forward direction at high energies indicates a possible method of determining the coupling strength of the $I=1$, $J=1$ two-pion resonance (ρ meson) with nucleons for zero momentum transfer. As a specific example, we consider the case of π^-, p elastic scattering and show that $g_{\rho NN}^2/4\pi\hbar c\sim 0.2$.

CONSIDERATION of pion-nucleon elastic scattering in the forward direction at high energies indicates a possible method of determining the coupling strength of the $I=1$, $J=1$ two-pion resonance (ρ meson) with nucleons for zero momentum transfer. As a specific example, we consider the case of π^-, p elastic scattering and show that $g_{\rho NN}^2/4\pi\hbar c\sim 0.2$.

Since single pion exchange does not contribute,[1] one is obliged to consider processes involving the exchange of two or more π mesons. Of these, three states are of particular interest, since they seem to exhibit resonance properties.[2] They are the two-pion, $I=0$, $J=1$ state of Bowcock *et al.*,[3] the $I=1$, $J=1$ resonance of Frazer and Fulco,[4] and the B_0 meson of Sakurai and Gell-Mann. The first, tentatively identified with the ω^0 meson in Gell-Mann's unitary symmetry,[5] does not couple to the π-meson current. The second, the ρ meson, will contribute to the scattering process; however, the B_0 meson which couples to the total nucleon current is also absent. Moreover, since higher resonances would be expected to correspond to particles of greater mass than the ρ meson, one is justified in considering the cross section to be due primarily to the exchange of a single ρ vector meson (provided that we arrange the kinematics in such a way that the pole dominates). Finally, in order to exclude Coulomb scattering, it is necessary to examine the cross section for scattering angles greater than 5° in the center-of-mass system (in the Bev range).

The pole contribution to the matrix element caused by the coupling of the two currents, assuming the validity of unitary symmetry, is easily shown to be

$$R=\frac{2i(k_\mu+k_\mu')g_{\rho\pi\pi}g_{\rho pp}\bar{u}_{p'}\gamma_\mu u_p}{(k_\nu-k_\nu')^2-\mu_\rho^2}, \tag{1}$$

where k, k', p, and p' are the initial and final four-momenta of the pion and proton and μ_ρ is the mass of the vector meson. The center-of-mass cross section for the process, neglecting the mass of the pion, is readily derived and is equal to

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}\simeq 16\left(\frac{g_{\rho\pi\pi}^2}{4\pi\hbar c}\right)\left(\frac{g_{\rho pp}^2}{4\pi\hbar c}\right)\left(\frac{\hbar}{\mu_\rho c}\right)^2\left\{\frac{(E_pE_\pi+\mathbf{k}^2\cos\theta)(E_\pi E_p+\mathbf{k}^2)-\frac{1}{2}\mathbf{k}^2(1-\cos\theta)(E_\pi^2-\mathbf{k}^2)}{\mu_\rho^2E_{\text{tot}}^2[E_\pi^2(1-\cos\theta)/\mu_\rho^2+1]^2}\right\}, \tag{2}$$

with E_p, E_π, and E_{tot} the center-of-mass energies of the proton, pion, and the total energy, and $\mathbf{k}$ the three-momentum of the pion. θ is the center-of-mass scattering angle. The best experimental values are $g_{\rho\pi\pi}^2/4\pi\hbar c\simeq 1$ and $\mu_\rho\sim 750$ Mev, respectively.[6,7] With these, and the experimental cross section at 5 Bev ($6°\leqslant\theta\leqslant 10°$),[8] one finds

$$g_{\rho pp}^2/4\pi\hbar c\simeq 0.2. \tag{3}$$

This value of the coupling strength is somewhat smaller than that expected on the basis of Gell-Mann's scheme.[9] However, Gell-Mann's scheme only suggests that the coupling constants be equal (and of order unity) in the limit of no mass splitting (i.e., infinitely high energy) and does not indicate what the actual physical values might be, in the absence of unitary symmetry. However, it is in substantial agreement with a calculation by Watson[1] obtained by considering the proton-neutron charge-exchange cross section at high energies. Alternatively, one may use the present value of the coupling strength to calculate the total charge-exchange cross

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[1] W. K. R. Watson, Nuovo cimento **21**, 182 (1961).
[2] It has been pointed out by S. C. Frautschi and M. Gell-Mann (private communication) that the $I=0$, $J=0$ resonant state might also play a substantial role. However, the elastic cross section arising from this intermediate state vanishes in the limit of infinite energy whereas the elastic cross section due to the $I=1$, $J=1$ state approaches a constant in this limit and might, therefore, be expected to dominate in the 5-Bev region.
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[7] C. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961).
[8] R. G. Thomas, Jr., Phys. Rev. **120**, 1015 (1960).
[9] Hofstadter has shown that this value should be multiplied by $(1.2)^2\sim 1.5$ in order to obtain the actual coupling constant. See, e.g., R. Hofstadter and R. Herman, Phys. Rev. Letters **6**, 293 (1960).

section for $p-n$ scattering. At high energies one obtains

$$\sigma_{\text{exchange}} \simeq 8\pi\left(\frac{g_{\rho NN}{}^2}{4\pi\hbar c}\right)^2\left(\frac{\hbar}{\mu_\rho c}\right)^2 \simeq 0.6 \text{ mb}. \quad (4)$$

Experimentally this cross section is observed to be less than 1.5 mb,[10] but its determination is obviously rather difficult. One could thus conclude that the small ρ-nucleon coupling strength at $\mathbf{k}^2=0$ is responsible for the extremely small high-energy nucleon-nucleon charge-exchange scattering (due primarily to the exchange of a virtual charged ρ meson).

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Pion Production in the Low-Energy Limit*

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The low-energy production reaction $\pi+N\rightarrow\pi+\pi+N$ is discussed in the framework of a simplified kinematics. The discussion is based on the Khuri-Treiman representation for processes with a three-particle final state. Under the assumption that the pion-pion rescattering term is dominant, branching ratios are derived for various processes. Under the same assumption, a quantitative analysis is given for the effect of the final-state pion-pion interaction on the three-particle distribution functions.

I. INTRODUCTION

Many attempts have been made to treat the process of single pion production in pion-nucleon collisions. However, the difficulty in studying kinematics and analytic properties has kept us from going any further than using crude approximations. Among many such attempts, the Chew-Low static model has been used by several authors in calculating the total cross section[1] and, recently, in estimating strength of the pion-pion interaction.[2] But all these authors end up with the unique conclusion that a more satisfactory theory is desired.

In this note, we review the kinematical situation and propose a reasonable approximation scheme for low energies. For a fixed total energy, the distribution function for the three-particle final state is studied. We first replace the transition amplitude by that of the process in which the composite particle with its mass equal to the total energy in the center-of-mass system decays into the final state of our interest. For the decay amplitude of that particle, a Khuri-Treiman type spectral representation is adopted.[3] In this mathematical model, we shall study the effect of the final-particle interactions on the statistical distribution of the three-body final state.

In Sec. II, the kinematical situation is investigated. It is shown that, in the low-energy limit, the incoming beam interacts only through the $p_{\frac{1}{2}}$ channel. It is shown further that the spin structure of the transition amplitude is identical to that of the decay amplitude for a particle with spin $\frac{1}{2}$, even parity, and mass equal to the total center-of-mass energy of the process. In Sec. III, we adopt the view that the production amplitude for the present problem can be replaced by the decay amplitude just introduced. For the decay amplitude, a Khuri-Treiman type spectral representation is adopted. In the approximation of retaining only the lowest-mass intermediate states and under the assumption that the pion-pion interaction is much stronger than that of pion-nucleon, it is shown that the spectral representation corresponds to a rescattering amplitude in which the two final-state pions interact with each other. In Sec. IV, a system of soluble integral equations is derived and solved. Branching ratios are derived for various processes. In Sec. V, scattering-length approximation is made and the decay amplitudes are written in closed forms. In Sec. VI, statistical sum is made over the phase spaces of the three final-state particles. Both the purely statistical distribution and the dynamical deviation are discussed. In Sec. VII, our quantitative results are compared with the low energy data of Batusov *et al.*[4]

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