Magnetoacoustic Effects in Nondegenerate Semiconductors*

HAROLD N. SPECTOR†‡
Institute for the Study of Metals, University of Chicago, Chicago, Illinois
(Received October 11, 1961)

A self-consistent, semiclassical treatment is given for the attenuation of a sound wave in a nondegenerate impurity semiconductor with spherical energy bands, using the formalism developed by Cohen, Harrison, and Harrison. A treatment is given for propagation both parallel and perpendicular to the magnetic field and the behavior of the attenuation is examined for a wide range of frequencies, magnetic fields, and mean free paths. The phenomena studied include saturation with frequency in longitudinal magnetic fields, and cyclotron resonance and magnetoplasma effects in transverse magnetic fields. A magnetoplasma resonance for the transverse polarized waves in a longitudinal magnetic field has been discovered and has been identified with a resonance calculated by Cohen, Harrison, and Harrison. A qualitative physical interpretation of the various effects found in the detailed calculations is also presented.

I. INTRODUCTION

In the past few years, much work, both theoretical[1–8] and experimental,[9–12] has been done on the electronic contribution to the ultrasonic attenuation in metals. In contrast, little has been done in semiconductors, and the work that has been done is mainly concerned with the acoustoelectric effect.[13–15] Weinreich[16,17] has treated the acoustoelectric effect in semiconductors using a phenomenological approach and has derived[18] a relation between the acoustoelectric effect and the ultrasonic absorption which is valid in the presence of carriers of only one sign and in the absence of a magnetic field. Mikoshiba has rederived the same relations,[19,20] using a Boltzmann equation treatment and has shown that they also hold outside the region of validity of the phenomenological approach. Mikoshiba has also treated the caes of the ultrasonic attenuation in a degenerate semiconductor in zero magnetic field[21] and in the case of propagation parallel to the magnetic field, our results agree with his whenever the calculations overlap. Elsewhere, we find that Pokatilov[22] has calculated the acoustic absorption in transverse magnetic fields for nondegenerate semiconductors, but his treatment is incomplete in that he has neglected both screening effects due to the self-consistent electromagnetic fields and the nonuniform distribution of carriers due to the presence of the sound wave.

We present here a self-consistent semiclassical treatment of a free-electron gas, obeying nondegenerate or Boltzmann statistics, in a partially-ionized background supporting a sound wave. This model is a simple approximation to an n-type, impurity semiconductor. We follow the general approach of Cohen, Harrison, and Harrison[5] apart from the kind of statistics our free-electron gas obeys.

In Sec. II we present the general formulation of the theory of the attenuation. Starting from the Boltzmann equation, we develop a constitutive equation giving the response of the electron gas to the electric field, the collision drag, and the electron density gradient that accompany the sound wave. Using the constitutive equation together with Maxwell's equations, we derive a general formula for the attenuation. The conductivity tensor, which plays an important role in our formulation, is evaluated in Sec. III. In Sec. IV, we specialize to the case of propagation parallel to the magnetic field and we treat the long- and short-wavelength limits of the attenuation. In Sec. V we treat the case where the magnetic field is transverse to the direction of propagation and we examine in detail the high- and low-field limits as well as the cyclotron resonances found theoretically. The concluding section, Sec. VI, contains a qualitative physical interpretation of the various effects found in our detailed calculations and an estimate of which of the examined situations are physically realizable in the foreseeable future.

II. THEORY OF THE ATTENUATION

As our model of an impurity semiconductor we consider a gas of N_0 electrons per unit volume moving

* This work was supported in part by the Office of Naval Research.
† Shell Oil Predoctoral Fellow 1960–61.
‡ Presently National Science Foundation Postdoctoral Fellow at the Hebrew University, Jerusalem, Israel.
1 A. B. Pippard, Phil. Mag. **46**, 1104 (1955).
2 E. I. Blount, Phys. Rev. **114**, 418 (1959).
3 M. S. Steinberg, Phys. Rev. **109**, 1486 (1958); **110**, 772 (1958).
4 T. Kjeldaas and T. Holstein, Phys. Rev. Letters **2**, 346 (1959).
5 M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).
6 A. B. Pippard, Proc. Roy. Soc. (London) **A257**, 165 (1960).
7 M. J. Harrison, Phys. Rev. **119**, 1260 (1960).
8 V. L. Gurevich, J. Exptl. Theoret. Phys. (USSR) **37**, 71 (1959) [translation: Soviet Phys.—JETP **10**, 51 (1960)].
9 R. W. Morse, H. V. Bohm, and J. D. Gavenda, Phys. Rev. **109**, 1394 (1958).
10 T. Olson, Phys. Rev. **118**, 1007 (1960).
11 R. W. Morse, A. Myers, and C. T. Walker, Phys. Rev. Letters **4**, 605 (1960).
12 D. H. Reneker, Phys. Rev. **115**, 303 (1959).
13 R. H. Parmenter, Phys. Rev. **89**, 990 (1953).
14 R. H. Parmenter, Phys. Rev. **113**, 102 (1959).
15 A. Van den Beukel, Appl. Sci. Research **B5**, 459 (1956).
16 G. Weinreich, Phys. Rev. **104**, 321 (1956).
17 G. Weinreich, T. M. Sanders, and H. G. White, Phys. Rev. **114**, 33 (1959).
18 G. Weinreich, Phys. Rev. **107**, 317 (1957).
19 N. Mikoshiba, J. Phys. Soc. Japan **14**, 22 (1959).
20 N. Mikoshiba, J. Phys. Soc. Japan **14**, 1691 (1959).
21 N. Mikoshiba, J. Phys. Soc. Japan **15**, 982 (1960).
22 E. P. Pokatilov, J. Exptl. Theoret. Phys. (USSR) **36**, 1461 (1959) [translation: Soviet Phys.—JETP **9**, 1037 (1959)].

through a uniform background of positive charge of the same density. A sound wave of wave vector $\mathbf{q}$ and frequency ω shows itself as a velocity field $\mathbf{u}(\mathbf{r},t) \propto \exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)]$ in the background. The interactions between the particles are replaced in part by the interactions of the individual particles with a self-consistent electromagnetic field and in part by the modulation of the energies of the conduction electrons via the deformation potential. The relation between the self-consistent electromagnetic field, $\boldsymbol{\mathcal{E}}$, as derived from Maxwell's equations and the total current density $\mathbf{j}$ accompanying the sound wave can be written as

$$\mathbf{j}=-\sigma_0\mathbf{B}\cdot\boldsymbol{\mathcal{E}}. \tag{2.1}$$

In (2.1), $\mathbf{B}$ is a diagonal matrix with elements $B_{11}=-i\gamma$ and $B_{22}=B_{33}=i\beta$ relative to the axes defined by $\mathbf{q}$ and two transverse directions and

$$\gamma=\omega/\omega_p{}^2\tau,\quad \beta=(\gamma/\epsilon)(c/v_s)^2, \tag{2.2}$$

where $\omega_p=(4\pi N_0e^2/m\epsilon)^{\frac{1}{2}}$ is the plasma frequency of the electrons, m is their mass, ϵ is the dielectric constant of the background, and v_s is the sound velocity. The energy of a conduction electron in the presence of the sound wave becomes, according to Harrison,[23]

$$E_e=E_e{}^0-\mathbf{q}\cdot(\mathbf{V}/\omega)\cdot\mathbf{u}, \tag{2.3}$$

where $E_e{}^0$ is the energy of the conduction electron in the absence of the sound wave, and $\mathbf{V}$ is the deformation potential tensor.

The problem of finding the electronic current can now be solved by using the formalism originally developed by Cohen, Harrison, and Harrison[5] for the case of a free-electron gas. The Boltzmann equation is solved by using the path integral method of Chambers,[24] with the force being given as the sum of the Lorentz force and the deformation forces,

$$\mathbf{F}=-e(\boldsymbol{\mathcal{E}}+\mathbf{v}/c\times\mathbf{H})-\mathbf{q}(\mathbf{q}\cdot\mathbf{V}/i\omega)\cdot\mathbf{u}. \tag{2.4}$$

The unperturbed distribution function, for a nondegenerate semiconductor, is the Maxwell-Boltzmann distribution,

$$f_0(\mathbf{v})=\frac{N_0}{\pi^{\frac{3}{2}}v_0{}^3}\exp[-(v/v_0)^2], \tag{2.5}$$

where $v_0=(2kT/m)^{\frac{1}{2}}$ is the most probable speed of the conduction electrons, T is their mean temperature, and k is Boltzmann's constant. The number of conduction electrons N_0 is also a function of temperature in the extrinsic range except at temperatures of the exhaustion range.

The constitutive equation now can be written in the form

$$\mathbf{j}_e=\sigma_0\boldsymbol{\sigma}'\cdot\left(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\frac{\mathbf{V}}{ei\omega}\cdot\mathbf{u}-m\mathbf{u}/e\tau\right), \tag{2.6}$$

where $\boldsymbol{\sigma}'$ is an effective conductivity tensor defined by the relation

$$\boldsymbol{\sigma}'=[\mathbf{I}-\mathbf{R}]^{-1}\cdot\boldsymbol{\sigma}/\sigma_0, \tag{2.7}$$

and the tensor $\mathbf{R}$ has components

$$R_{ij}=R_i\delta_{1j}. \tag{2.8}$$

Using a general relation derived in reference 5, we find the following relation between $\boldsymbol{\sigma}$ and $\mathbf{R}$:

$$i\boldsymbol{\sigma}\cdot\mathbf{q}=-(2\sigma_0v_s/\tau v_0{}^2)(1-i\omega\tau)\mathbf{R}, \tag{2.9}$$

where

$$\boldsymbol{\sigma}=\frac{-2e}{mv_0{}^2}\int d^3v\,\mathbf{v}\mathbf{J}(\mathbf{v})f_0, \tag{2.10a}$$

$$\mathbf{R}=\frac{1}{N_0v_s\tau}\int d^3v\,\mathbf{v}K(\mathbf{v})f_0, \tag{2.10b}$$

and the function $\mathbf{J}(\mathbf{v})$ and $K(\mathbf{v})$ have been previously defined.[5] Using (2.9) we can write the nonvanishing components of $\mathbf{R}$ as

$$R_{i1}=\frac{-iql}{2\sigma_0(1-i\omega\tau)}\frac{v_0}{v_s}\sigma_{i1}=-\Delta\sigma_{i1}. \tag{2.11}$$

In this equation $\sigma_0=Ne^2\tau/m$ is the dc conductivity of the electrons and $l=v_0\tau$ is their mean free path.

It is possible to show[5,7] that the net power dissipated per unit volume is

$$Q=\tfrac{1}{2}\operatorname{Re}\left[\mathbf{j}_e{}^*\cdot\left(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\frac{\mathbf{V}}{ei\omega}\cdot\mathbf{u}\right)-\mathbf{u}^*\frac{N_0m}{\tau}(\langle\mathbf{v}\rangle-\mathbf{u})\right], \tag{2.12}$$

where the second term is related to the collision drag effect discussed by Holstein.[25] We can obtain an expression for the self-consistent electric field in terms of the velocity $\mathbf{u}$ of the background by combining (2.1), (2.6), and the expression for the total current and we get as the result

$$\boldsymbol{\mathcal{E}}=-[\boldsymbol{\sigma}'+\mathbf{B}]^{-1}\cdot\left[\boldsymbol{\sigma}'\cdot\mathbf{qq}\cdot\frac{\mathbf{V}}{im\omega}+(\mathbf{I}-\boldsymbol{\sigma}')\right]\cdot\frac{m\mathbf{u}}{e\tau}. \tag{2.13}$$

Using (2.1) and (2.13) we can transform expression (2.12) for Q to

$$Q=\tfrac{1}{2}(N_0m/\tau)|\mathbf{u}|^2\hat{u}\cdot\mathbf{S}\cdot\hat{u} \tag{2.14a}$$

where

$$\mathbf{S}=\operatorname{Re}\left[\mathbf{I}+\left(\mathbf{I}-\frac{\mathbf{V}}{im\omega}\cdot\mathbf{qq}\tau\right)\cdot\mathbf{B}\right]\cdot\left\{[\boldsymbol{\sigma}'+\mathbf{B}]^{-1}\right.$$
$$\left.\cdot\left[\mathbf{I}+\mathbf{B}\cdot\left(\mathbf{I}-\tau\mathbf{qq}\cdot\frac{\mathbf{V}}{im\omega}\right)\right]-\left(\mathbf{I}-\tau\mathbf{qq}\cdot\frac{\mathbf{V}}{im\omega}\right)\right\}, \tag{2.14b}$$

and $\hat{u}$ is a unit vector in the direction of polarization. We choose $\hat{u}_1$ to lie in the direction of $\mathbf{q}$ so that the three

[23] M. J. Harrison (private communication).

[24] R. G. Chambers, Proc. Phys. Soc. (London) **A65**, 458 (1952); **A238**, 344 (1957).

[25] T. Holstein, Phys. Rev. **113**, 479 (1959).

independent directions of polarization form the coordinate system in which **B** has been expressed.

The quantity of direct experimental interest is the absorption or attenuation coefficient α, which gives the exponential decay of the sound wave with distance. The absorption coefficient α is defined as the power dissipated per unit volume per unit incident energy flux, or

$$\alpha=Q/(\tfrac{1}{2}\rho|\mathbf{u}|^2v_s), \tag{2.15}$$

where ρ is the mass density of the semiconductor being represented by our simple model. For a particular polarization we have for α

$$\alpha_i=(N_0mv_0/\rho v_s)(S_{ii}/l). \tag{2.16}$$

Since α, by its nature, is the reciprocal of the mean free path L of the sound wave, we have

$$L=(\rho v_s/N_0mv_0)(l/S_{ii}). \tag{2.17}$$

For ease of measurement, we would need L to be smaller than approximately one centimeter. In germanium, for example, we have at temperatures of 10°K, a τ of 10^{-10} sec from cyclotron resonance experiments,[26] and an N of 10^{13} cm^{-3} is obtainable.[27] Therefore, with $\rho\approx 3$ g/cm^3, $v_0=10^6$ cm/sec, and $v_s=10^5$ cm/sec, we find that $L\approx 10^8/S_{ii}$. This implies that we must have $S_{ii}\gtrsim 10^8$ for the attenuation to be readily observable in a typical nondegenerate semiconductor.

III. CONDUCTIVITY TENSOR

The magnetic-field dependence of the conductivity tensor is implicit in the integral expressions (2.10a) and (2.10b). The present task is to obtain this dependence explicitly by evaluation of the integrals. We choose a coordinate system having the z axis in the direction of **H** and the x direction in the plane containing **H** and **q**. In this coordinate system, the relation between $(\mathbf{r},\mathbf{v})$ and $(\mathbf{r}',\mathbf{v}')$ is

$$\begin{aligned} v_x'&=v_0w\cos[\omega_c(t'-t)+\theta], & x'&=x+(v_0w/\omega)\{\sin[\omega_c(t'-t)+\theta]-\sin\theta\},\\ v_y'&=v_0w\sin[\omega_c(t'-t)+\theta], & y'&=y-(v_0w/\omega)\{\cos[\omega_c(t'-t)+\theta]-\cos\theta\},\\ v_z'&=v_0u, & z'&=z+v_0u(t'-t), \end{aligned} \tag{3.1}$$

where $\omega_c=eH/mc$ is the cyclotron frequency, θ is the polar angle, and w and u are the velocities in units of v_0 in the plane perpendicular to **H** and parallel to **H**, respectively.

The expression for $\mathbf{J}(\mathbf{v})$ has been evaluated previously[5] in this coordinate system and is

$$\mathbf{J}(\mathbf{v})=-el\exp[-iXw\sin\theta]\sum_{n=-\infty}^{\infty}\begin{pmatrix} n/X\\ -id/dX\\ u\end{pmatrix}\frac{J_n(Xw)\exp(in\theta)}{1+i(n\omega_c-\omega+q_zv_0u)\tau}, \tag{3.2}$$

where $X=q_xv_0/\omega_c$. Using (3.2) for $\mathbf{J}(\mathbf{v})$, expression (2.14a) for $\boldsymbol{\sigma}$ becomes

$$\boldsymbol{\sigma}=\frac{2\sigma_0}{\pi^{\frac{1}{2}}}\sum_{n=-\infty}^{\infty}\int_0^\infty dw\,w\exp(-w^2)\int_{-\infty}^{\infty}du\exp(-u^2)\int_0^{2\pi}d\theta \times\left[\begin{pmatrix}(i/X)d/d\theta\\ id/dX\\ u\end{pmatrix}\exp[-iXw\sin\theta]\right]\left[\begin{pmatrix} n/X\\ -id/dX\\ u\end{pmatrix}\frac{J_n(Xw)\exp(in\theta)}{1+i(n\omega_c-\omega+q_zv_0u)\tau}\right]. \tag{3.3}$$

The integral over the variable θ is easily performed using the periodicity of the limits of integration and the familiar relation[28]

$$\exp(iz\sin\theta)=\sum_{m=-\infty}^{\infty}J_m(z)e^{im\theta}. \tag{3.4}$$

Performing this evaluation, we find that

$$\boldsymbol{\sigma}=\frac{4\sigma_0}{\pi^{\frac{1}{2}}}\sum_{n=-\infty}^{\infty}\int_0^\infty dw\,w\exp(-w^2)\int_{-\infty}^{\infty}\frac{du\exp(-u^2)}{1+i(n\omega_c-\omega+q_zv_0u)\tau}\left[\begin{pmatrix} n/X\\ -id/dX\\ u\end{pmatrix}J_n(Xw)\right]\left[\begin{pmatrix} n/X\\ id/dX\\ u\end{pmatrix}J_n(Xw)\right]. \tag{3.5}$$

[26] B. Lax and J. G. Mavroides, *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1961), Vol. 11, p. 261.
[27] H. Fritzsche, J. Phys. Chem. Solids 6, 69 (1958).
[28] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. I, p. 620.

The kinds of integrals that appear in the integrations over the variables u and w are evaluated in the Appendix. Using the results of the Appendix, we find the following expressions for the components of the conductivity tensor:

$$\begin{aligned}
\sigma_{xx}&=\frac{2\pi^{\frac{1}{2}}\sigma_0}{q_z l}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\left(\frac{n}{X}\right)^2 I_n(\tfrac{1}{2}X^2)F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_z l}\right),\\
\sigma_{yy}&=\frac{\pi^{\frac{1}{2}}\sigma_0}{2q_z l}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\left(X^2 I_n(\tfrac{1}{2}X^2)+2(1-X^2)\frac{dI_n(\frac{1}{2}X^2)}{d(\frac{1}{2}X^2)}+X^2\frac{d^2I_n(\frac{1}{2}X^2)}{d(\frac{1}{2}X^2)}\right)F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_z l}\right),\\
\sigma_{zz}&=-\frac{2\pi^{\frac{1}{2}}\sigma_0}{(q_z l)^3}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}[1+i(n\omega_c-\omega)\tau]I_n(\tfrac{1}{2}X^2)\left[[1+i(n\omega_c-\omega)\tau]F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_z l}\right)-\frac{q_z l}{\pi^{\frac{1}{2}}}\right],\\
\sigma_{xy}&=-\sigma_{yx}=\frac{\pi^{\frac{1}{2}}\sigma_0}{q_z l}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}in\left(\frac{dI_n(\frac{1}{2}X^2)}{d(\frac{1}{2}X^2)}-I_n(\tfrac{1}{2}X^2)\right)F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_z l}\right),\\
\sigma_{xz}&=\sigma_{zx}=-\frac{2i\sigma_0\pi^{\frac{1}{2}}}{(q_z l)^2X}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}nI_n(\tfrac{1}{2}X^2)\left[F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_z l}\right)-\frac{q_z l}{\pi^{\frac{1}{2}}}\right],\\
\sigma_{yz}&=-\sigma_{zy}=\frac{i\sigma_0\pi^{\frac{1}{2}}}{(q_z l)^2}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\left(\frac{dI_n(\frac{1}{2}X^2)}{d(\frac{1}{2}X^2)}-I_n(\tfrac{1}{2}X^2)\right)\left[F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_z l}\right)-\frac{q_z l}{\pi^{\frac{1}{2}}}\right].
\end{aligned}\tag{3.6}$$

Here $I_n(z)$ is the hyperbolic Bessel function of order n, and[29]

$$F(x)\equiv\exp(x^2)\,\mathrm{erfc}(x);\quad \mathrm{erfc}(x)\equiv\frac{2}{\pi^{\frac{1}{2}}}\int_x^{\infty}\exp(-t^2)dt.\tag{3.7}$$

We shall now treat separately the cases of propagation parallel and transverse to the external magnetic field.

IV. LONGITUDINAL MAGNETIC FIELD

In the case where the direction of propagation is along the direction of the magnetic field, X vanishes and the components of the conductivity tensor reduce to

$$\begin{aligned}
\sigma_{11}&=-2\pi^{\frac{1}{2}}\sigma_0\frac{(1-i\omega\tau)}{(ql)^3}\left[(1-i\omega\tau)F\left(\frac{1-i\omega\tau}{ql}\right)-\frac{ql}{\pi^{\frac{1}{2}}}\right],\\
\sigma_{\pm}&=\sigma_{22}\pm i\sigma_{23}=\frac{\pi^{\frac{1}{2}}\sigma_0}{ql}F\left(\frac{1-i(\omega\pm\omega_c)\tau}{ql}\right).
\end{aligned}\tag{4.1}$$

In this case we let the 1 direction coincide with the z axis, and we let the transverse currents and fields be circularly polarized in the x-y plane. The components of the conductivity tensor, other than the ones appearing in (4.1), are equal to zero.

We may now write the relevant components of the tensor $\mathbf{S}$ (2.14b) (in forms correct for all the cases of interest discussed below) in terms of σ_{11}' and $\sigma_{\pm}'$, the nonvanishing components of the effective conductivity tensor $\boldsymbol{\sigma}'$,

$$S_{11}=\mathrm{Re}\left\{\frac{[1-iql(v_0/v_s)V_{11}\sigma_{11}'/mv_0^2][1+(\omega/\omega_p)^2(v_0/v_s)^2V_{11}/mv_0^2]}{\sigma_{11}'-i\gamma}\right\}-1\tag{4.2a}$$

$$S_{\pm}=\mathrm{Re}\left\{\frac{(1+i\beta)^2}{\sigma_{\pm}'+i\beta}+\frac{(\omega/\omega_p)^4(v_0/v_s)^4(V_{\pm}/mv_0^2)^2}{\sigma_{11}'-i\gamma}\right\}.\tag{4.2b}$$

[29] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge University Press, New York, 1950), 4th ed., p. 372 for definition of the hyperbolic Bessel function; p. 341 for definition of error function erfc(x).

We need now only calculate the tensor and substitute into the above expression. Using (2.7) and (4.1), we find

$$\sigma_{11}'=\frac{-\dfrac{2\pi^{\frac{1}{2}}}{(ql)^3}(1-i\omega\tau)\left[(1-i\omega\tau)F\left(\dfrac{1-i\omega\tau}{ql}\right)-\dfrac{ql}{\pi^{\frac{1}{2}}}\right]}{1-\dfrac{i\pi^{\frac{1}{2}}}{2}\dfrac{v_0}{v_s(ql)^2}\left[(1-i\omega\tau)F\left(\dfrac{1-i\omega\tau}{ql}\right)-\dfrac{ql}{\pi^{\frac{1}{2}}}\right]},$$
$$\sigma_{\pm}'=\frac{\pi^{\frac{1}{2}}}{ql}F\left(\frac{1-i(\omega\pm\omega_c)\tau}{ql}\right). \tag{4.3}$$

Since at the present time, acoustic frequencies up to 10^4 Mc/sec are available experimentally,[30] we must consider both the short-wavelength case $ql\gg1$, and the long-wavelength case of $ql\ll1$.

A. Short-Wavelength Case

When $ql\gg1$, the conductivity component σ_{11}' has the limiting behavior:

$$\sigma_{11}'=-\frac{2iv_s}{v_0ql}\left[1-\frac{\pi^{\frac{1}{2}}}{ql}+i\pi^{\frac{1}{2}}\frac{v_s}{v_0}\right] \tag{4.4}$$

for all values of the magnetic field. This can readily be seen from the behavior of the error function $\operatorname{erfc}(x)$ for small values of x:

$$\operatorname{erfc}(x)\approx1,\quad x\ll1. \tag{4.5}$$

Substitution of (4.4) into (4.2a) yields the result

$$S_{11}=\tfrac{1}{2}\pi^{\frac{1}{2}}ql\left[\frac{1+(\omega/\omega_p)^2(v_0/v_s)^2V_{11}/mv_0^2}{1+\frac{1}{2}(\omega/\omega_p)^2(v_0/v_s)^2}\right]^2. \tag{4.6}$$

For relevant values of the parameters, (4.6) yields three regions of interest. The first region is defined by frequencies less than the frequency ω_0, where

$$\omega_0/\omega_p=(v_s/v_0)(mv_0^2/V_{11})^{\frac{1}{2}}.$$

In this region S_{11} becomes

$$S_{11}=\tfrac{1}{2}\pi^{\frac{1}{2}}ql. \tag{4.7}$$

The attenuation is unobservable in this region, depends linearly on the frequency, and is independent of the mean free path. The second frequency range is defined by frequencies such that $\omega_0<\omega<\omega_1$, where $\omega_1/\omega_p=v_s/v_0$. Here

$$S_{11}=\tfrac{1}{2}\pi^{\frac{1}{2}}ql(v_0/v_s)^4(\omega/\omega_p)^4(V_{11}/mv_0^2)^2. \tag{4.8}$$

In this region there is a dramatic rise in the absorption with frequency. Over a narrow range of frequency, the absorption rises from an unobservable value to a fairly substantial value. The absorption varies as ω^5 in this frequency region.

Finally, in the third frequency region where $\omega>\omega_1$, we have

$$S_{11}=2\pi^{\frac{1}{2}}ql(V_{11}/mv_0^2)^2, \tag{4.9}$$

and the absorption again increases linearly with the frequency.

Our results (4.8) and (4.9) agree with those of Mikoshiba[21] for a longitudinally polarized wave in zero magnetic field within a numerical factor, which arises from our using different statistics. For $ql\gg1$, we find that the attenuation α becomes independent of the relaxation time. Mikoshiba did not calculate the absorption on the first frequency range $\omega<\omega_0$.

For an estimate of the order of magnitude of S_{11}, we take note that in germanium the deformation potential V_{11} is of the order of 20 ev,[31] while the mean energy of the conduction electrons is $mv_0^2\approx10^{-3}$ ev at $T=10°$K. Therefore the parameter V_{11}/mv_0^2 is of the order of magnitude of 10^4. Also v_0/v_s is of the order 10, while in the third frequency range S_{11} is of order 10^9 when $ql\gg1$. Thus the absorption increases by a factor of 10^8 or greater between $\omega/\omega_p=10^{-3}$ and $\omega/\omega_p=10^{-1}$. This behavior of the electronic contribution to the absorption as a function of frequency is shown in Fig. 1.

For the circularly-polarized transverse waves, we need the condition $\omega_c<qv_0$ in addition to $ql\gg1$ to use the approximation for $\operatorname{erfc}(x)$ [Eq. (4.5)]. In this short-wavelength, low-field limit we have

$$\sigma_{\pm}=\pi^{\frac{1}{2}}/2ql. \tag{4.10}$$

For all frequencies beyond a few cps, $\beta\gg1$ because of the low concentration of carriers that occur in semiconductors. Therefore when we substitute (4.4) and (4.10) into (4.2b), we drop terms of order $1/\beta$ compared to terms of order unity. Thus we obtain

$$S_{\pm}=1-\frac{\pi^{\frac{1}{2}}}{ql}+\tfrac{1}{2}\pi^{\frac{1}{2}}ql\left[\frac{(\omega/\omega_p)^2(v_0/v_s)^2(V_{\pm}/mv_0^2)^2}{1+\frac{1}{2}(\omega/\omega_p)^2(v_0/v_s)^2}\right]^2. \tag{4.11}$$

When the values of the deformation potential for transverse circularly-polarized waves are of the same order of magnitude as the deformation potential for

[30] K. N. Baranskii, Doklady Akad. Nauk S.S.S.R. **114**, 517 (1957) [translation: Soviet Phys.—Doklady **2**, 237 (1957)]. H. E. Bommel and K. Dransfeld, Phys. Rev. Letters **1**, 234 (1958); **2**, 298 (1959); **3**, 83 (1959). E. H. Jacobsen, *ibid.* **2**, 249 (1959). N. S. Shiren, *ibid.* **6**, 168 (1961).

[31] H. Fritzsche, Phys. Rev. **115**, 336 (1959).

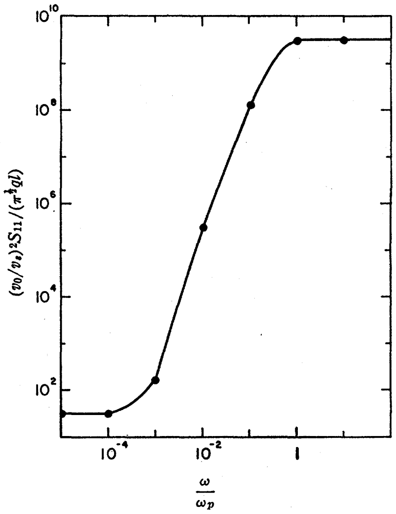

FIG. 1. The attenuation of longitudinal waves in a longitudinal magnetic field versus frequency is shown in the short-wavelength limit. The curves are plotted for the case when $\omega_0/\omega_p=10^{-3}$ and $\omega_1/\omega_p=10^{-1}$. The abscissa is the ratio of the sound frequency to the plasma frequency.

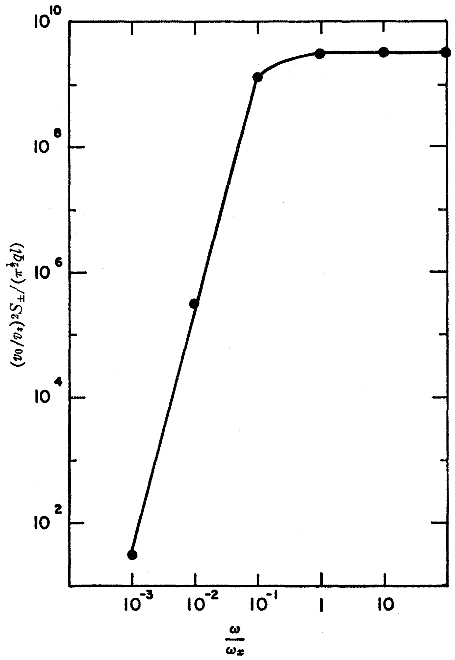

FIG. 2. The attenuation of transverse waves in a longitudinal magnetic field vs frequency is shown in the short-wavelength limit and for $\omega\tau>1$. The curves are for the case of strong deformation forces, i.e., $V_{1\pm}/mv_0^2\gg 1$.

longitudinally polarized waves, the first two terms are completely negligible compared to the third term. The behavior of the absorption with frequency is the same as it was in the case of longitudinal waves with the exception that the first region of unobservable attenuation is missing. This results because of the strong coupling, in the presence of strong deformation potential forces, of transverse polarized waves to longitudinal forces as will be discussed later. This behavior of the transverse waves is shown in Fig. 2.

B. Long Wavelength Case

When $ql\ll 1$, we must use the asymptotic form of the error function $\operatorname{erfc}(x)$ to obtain the limiting forms of σ_{11}' and $\sigma_{\pm}'$. For large X,[32]

$$\operatorname{erfc}(x)=(\exp(-X^2)/\pi^{\frac{1}{2}}X)[1-1/2X^2]. \quad (4.12)$$

Then for all values of the magnetic field we obtain

$$\sigma_{11}'=\frac{1-i\omega\tau}{1+\frac{1}{2}i\omega\tau(v_0/v_s)^2}, \quad (4.13a)$$

$$\sigma_{\pm}'=\frac{1}{1-i(\omega\pm\omega_c)\tau}. \quad (4.13b)$$

Relation (4.13b) also holds for $ql\gg 1$ and $\omega_c>qv_0$; the question of whether the long-wavelength limit holds is thus a matter of either the wavelength being long compared to the mean free path or the most probable Larmor radius for the case of transverse waves.

Substituting (4.13a) into (4.2a), we obtain

$$S_{11}=\frac{[1+\frac{1}{2}(ql)^2][1+(\omega/\omega_p)^2(v_0/v_s)^2V_{11}/mv_0^2]^2}{[1+\frac{1}{2}(v_0/v_s)^2(\omega/\omega_p)^2]^2+(\omega\tau)^2[1-1/(\omega_p\tau)^2]^2}. \quad (4.14)$$

The absorption as a function of frequency again falls into three regions of interest. In the first region, $\omega<\omega_0$,

$$S_{11}=\tfrac{1}{2}(ql)^2 \quad (4.15)$$

so the attenuation is negligible, going to zero linearly with τ. In the second region where $\omega_0<\omega<\omega_1$, S_{11} becomes

$$S_{11}=(\omega/\omega_p)^4(v_0/v_s)^4(V_{11}/mv_0)^2, \quad (4.16)$$

and is a rapidly increasing function of frequency, the absorption varying as $(\omega/\omega_p)^4$. In the third region of interest, $\omega>\omega_1$, the attenuation saturates at a value of $S_{11}=4(V_{11}/mv_0^2)^2$ provided that $ql>2v_s/v_0$. If, however, $ql<2v_s/v_0$, then $S_{11}=(\omega\tau)^2(v_0/v_s)^4(V_{11}/mv_0)^2$ and the attenuation increases with the square of the frequency and goes to zero linearly with τ. The magnitude of the saturation value of S_{11} when $ql>2v_s/v_0$ is of the order 10^8. The behavior of S_{11} as a function of frequency for $ql\ll 1$ is shown in Fig. 3.

The attenuation for the transverse waves can be obtained by substituting (4.13a) and (4.13b) into (4.2b). As in the case when $ql\gg 1$, the term containing

[32] Reference 29, p. 342.

the deformation potential dominates the whole frequency range when there are strong deformation forces. We then have

$$S_{\pm}=\frac{(\omega/\omega_p)^4(v_0/v_s)^4(V_{\pm}/mv_0^2)^2}{[1+\frac{1}{2}(v_0/v_s)^2(\omega/\omega_p)^2]^2+(\omega\tau)^2[1-1/(\omega_p\tau)^2]^2}. \tag{4.17}$$

In both (4.11) and (4.17), the dominating term is independent of the magnetic field so that we can say that (4.11) holds for $ql\gg 1$ and (4.17) holds for $ql\ll 1$ for all values of the magnetic field for which a semiclassical treatment is valid.

V. TRANSVERSE MAGNETIC FIELD

When the direction of propagation of the sound wave is transverse to the magnetic field, $q_z=0$ and we can use a coordinate system in which $x\leftrightarrow 1$, $y\leftrightarrow 2$, and $z\leftrightarrow 3$. We then obtain for the components of the conductivity tensor

$$\begin{aligned}
\sigma_{11}&=\frac{2\sigma_0(1-i\omega\tau)}{(ql)^2}\left[1-\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\frac{(1-i\omega\tau)I_n(\frac{1}{2}X^2)}{1+i(n\omega_c-\omega)\tau}\right],\\
\sigma_{22}&=\frac{\sigma_0}{2}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}[1+i(n\omega_c-\omega)\tau]^{-1}\left[X^2I_n(\tfrac{1}{2}X^2)+2(1-X^2)\frac{dI_n(\frac{1}{2}X^2)}{d(\frac{1}{2}X^2)}+X^2\frac{d^2I_n(\frac{1}{2}X^2)}{d(\frac{1}{2}X^2)^2}\right],\\
\sigma_{33}&=\sigma_0\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\frac{I_n(\frac{1}{2}X^2)}{1+i(n\omega_c-\omega)\tau},\\
\sigma_{12}&=-\sigma_{21}=(1-i\omega\tau)\frac{\sigma_0}{ql}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\frac{dI_n(\frac{1}{2}X^2)/d(\frac{1}{2}X^2)-I_n(\frac{1}{2}X^2)}{1+i(n\omega_c-\omega)\tau},\\
\sigma_{13}&=\sigma_{31}=\sigma_{23}=\sigma_{32}=0.
\end{aligned} \tag{5.1}$$

The expressions for σ_{11} and σ_{12} have been rewritten from the form

$$\begin{aligned}
\sigma_{11}&=\frac{2\sigma_0}{X^2}\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\frac{n^2I_n(\frac{1}{2}X^2)}{1+i(n\omega_c-\omega)\tau},\\
\sigma_{12}&=i\sigma_0\exp(-\tfrac{1}{2}X^2)\sum_{n=-\infty}^{\infty}\frac{n[dI_n(\frac{1}{2}X^2)/d(\frac{1}{2}X^2)-I_n(\frac{1}{2}X^2)]}{1+i(n\omega_c-\omega)\tau},
\end{aligned} \tag{5.2}$$

which is obtained directly from (3.6). This is done by noting that $\sum_{n=-\infty}^{\infty}I_n(z)=e^z$ which follows from the relations $\sum_{n=-\infty}^{\infty}J_n^2(z)=1$ and (A2) in the Appendix, and also that $I_n(z)=I_{-n}(z)$.[33]

As in Sec. IV, we may proceed to write down the diagonal components of the tensor $\mathbf{S}$:

$$S_{11}=\mathrm{Re}\left\{\frac{[1+(\omega/\omega_p)^2(v_0/v_s)^2V_{11}/mv_0^2][1-iql(v_0/v_s)(V_{11}/mv_0^2)\sigma_{11}']}{\sigma_{11}'-i\gamma}\right\}, \tag{5.3a}$$

$$S_{22}=\mathrm{Re}\left\{\frac{(1+i\beta)^2(\sigma_{11}'-i\gamma)}{(\sigma_{11}'-i\gamma)(\sigma_{22}'+i\beta)+\sigma_{12}'^2}+\frac{(\omega/\omega_p)^4(v_0/v_s)^4(V_{12}/mv_0^2)^2}{\sigma_{11}'-i\gamma}\right\}, \tag{5.3b}$$

$$S_{33}=\mathrm{Re}\left\{\frac{(1+i\beta)^2}{\sigma_{33}'+i\beta}+\frac{(\omega/\omega_p)^4(v_0/v_s)^4(V_{13}/mv_0^2)^2}{\sigma_{11}'-i\gamma}\right\}. \tag{5.3c}$$

We need now only evaluate the components of the conductivity tensor $\boldsymbol{\sigma}'$ for the cases of interest and substitute them into (5.3a–c).

[33] Reference 28, Vol. II, p. 1323.

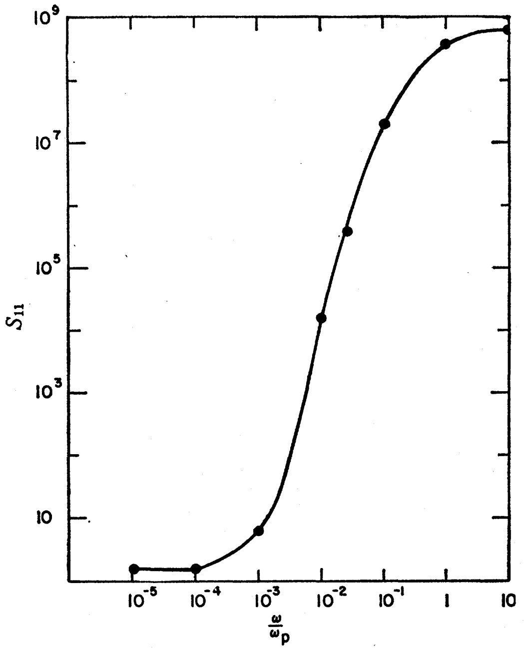

FIG. 3. The attenuation of longitudinal waves when the magnetic field is also in the longitudinal direction is plotted in the long-wavelength limit $ql<1$.

A. Cyclotron Resonance and the Low-Field Limit

We expect cyclotron resonance effects when the sound-wave frequency is of the order of the cyclotron frequency, i.e., $\omega\approx\omega_c$. In this case the frequency denominators in the conductivity components (5.1) can become small, and the possibility of oscillatory behavior arises. Under this condition, X will become much greater than unity since $X=qv_0/\omega_c=(\omega/\omega_c)\times(v_0/v_s)$. Thus we can use the asymptotic form of the hyperbolic Bessel function[33]:

$$I_n(a)=e^a/(2\pi a)^{\frac{1}{2}}. \tag{5.4}$$

This expression is only valid for $a=\frac{1}{2}X^2>n$; when n exceeds a, the hyperbolic Bessel functions become small. Hence, if we take the asymptotic form of $I_n(a)$, (5.4), in evaluating (5.1), we make an error of the form of the final term in the following equation:

$$\sum_{n=-\infty}^{\infty}\frac{I_n(\frac{1}{2}X^2)}{1+i(n\omega_c-\omega)\tau} =\frac{\exp(\frac{1}{2}X^2)}{\pi^{\frac{1}{2}}X}\left\{\sum_{n=-\infty}^{\infty}\frac{1}{1+i(n\omega_c-\omega)\tau} -O\left[\sum_{n=\frac{1}{2}X^2}^{\infty}\frac{2(1-i\omega\tau)}{(1-i\omega\tau)^2+n^2(\omega_c\tau)^2}\right]\right\}. \tag{5.5}$$

The last term may be estimated by replacing the summation by an integration over n and the term is found to be of the order of $1/X(ql)^2$ whereas the first term is of order $1/ql$. We are interested in the case $\omega\tau$ large; therefore we may take ql to be very large and retain only the first term which may be evaluated directly, noting that[34]

$$\sum_{n=-\infty}^{\infty}\frac{1}{b+in}=\pi\left[\frac{1}{b\pi}+2b\pi\sum_{n=1}^{\infty}\frac{1}{(b\pi)^2+(n\pi)^2}\right] =\pi\coth(b\pi). \tag{5.6}$$

Using (5.5) and (5.6), we obtain the following for the components of $\boldsymbol{\sigma}$:

$$\sigma_{11}=\frac{2\sigma_0}{(ql)^2}(1-i\omega\tau)\left[1-\frac{\pi^{\frac{1}{2}}(1-i\omega\tau)}{ql}\coth\frac{(1-i\omega\tau)\pi}{\omega_c\tau}\right],$$

$$\sigma_{22}=\sigma_{33}=\frac{\sigma_0}{ql}\pi^{\frac{1}{2}}\coth\frac{(1-i\omega\tau)\pi}{\omega_c\tau}, \tag{5.7}$$

$$\sigma_{12}=-\sigma_{21}=-\frac{\sigma_0\pi^{\frac{1}{2}}}{(qlX)^2}\coth\frac{(1-i\omega\tau)\pi}{\omega_c\tau}.$$

As the field goes to zero, $\coth[(1-i\omega\tau)\pi/\omega_c\tau]$ approaches unity, and taking this fact into account we obtain

$$\sigma_{11}'=-\frac{2iv_s}{qlv_0}\{1-\pi^{\frac{1}{2}}[(1-i\omega\tau)/ql]\coth[(1-i\omega\tau)\pi/\omega_c\tau]\},$$

$$\sigma_{22}'=\sigma_{33}'=\frac{\pi^{\frac{1}{2}}}{2ql}\coth\frac{(1-i\omega\tau)\pi}{\omega_c\tau}, \tag{5.8}$$

$$\sigma_{12}'=-\sigma_{21}'=\frac{i\pi^{\frac{1}{2}}}{qlX^2}\frac{v_s}{v_0}\frac{\coth[(1-i\omega\tau)\pi/\omega_c\tau]}{1-i\omega\tau}.$$

The terms in σ_{12}' are negligible compared to those in σ_{11}', σ_{22}', and therefore, in this limit, the expressions for the two transverse waves become identical. Accordingly, we have, for $\omega\tau>1$,

$$S_{11}=\tfrac{1}{2}\pi^{\frac{1}{2}}ql[1+(\omega/\omega_p)^2(v_0/v_s)^2(V_{11}/mv_0^2)]^2[1+\tfrac{1}{2}(\omega/\omega_p)^2(v_0/v_s)^2]^{-2}\left\{\frac{\tanh(\pi/\omega_c\tau)\sec^2(\omega\pi/\omega_c)}{\tanh^2(\pi/\omega_c\tau)+\tan^2(\omega\pi/\omega_c)}\right\}, \tag{5.9a}$$

$$S_{jj}=\tfrac{1}{2}\pi^{\frac{1}{2}}ql(v_0/v_s)^4(\omega/\omega_p)^4(V_{1j}/mv_0^2)^2[\cdots]^{-2}\{\cdots\}, \tag{5.9b}$$

[34] Reference 29, p. 136.

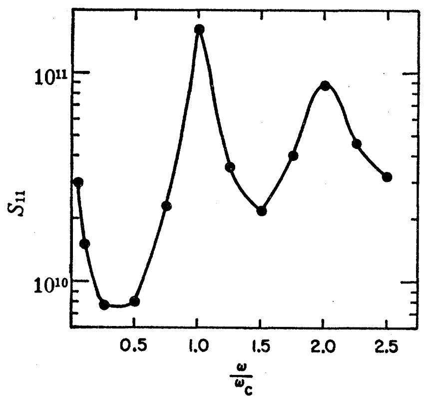

FIG. 4. The attenuation of longitudinal waves in a transverse magnetic field is shown as a function of the ratio of sound frequency to cyclotron frequency. The product of the sound frequency and the electron scattering time $\omega\tau$ is taken equal to ten and we also have taken the sound wavelength to be smaller than the Debye wavelength, $\lambda < \lambda_D$.

where V_{1j}, $j=2, 3$, is the deformation potential in the direction transverse to $\mathbf{q}$. As was the case in Sec. IV, the terms other than those containing the deformation potential have been neglected in calculating the expressions for the absorption of transverse waves (5.9b). When $\omega_c\tau > 1$ we will observe cyclotron resonance, while when $\omega_c\tau < 1$, the resonances will be damped out. The resonances occur whenever $\omega/\omega_c = n$, i.e., at the harmonics of the cyclotron frequency. In Fig. 4, (5.9a) is plotted against ω/ω_c with $\omega\tau = 10$. A somewhat similar behavior can be expected for the transverse waves (5.9b).

In the low-field limit where ω_c goes to zero, the conductivity components (5.8) reduce to those derived in Sec. IV for $ql \gg 1$. Thus, for zero magnetic field, (5.8) agrees with the expressions for the longitudinal field case when $ql \gg 1$.

B. High-Field Limit

In the limit of high magnetic fields, $\omega_c > qv_0$, the conductivity components have the behavior:

$$\begin{aligned} \sigma_{11} &= \sigma_0 \frac{1-i\omega\tau}{(\omega_c\tau)^2}, \\ \sigma_{22} &= \sigma_0 \frac{X^2}{1-i\omega\tau}, \\ \sigma_{33} &= \frac{\sigma_0}{(1-i\omega\tau)}, \\ \sigma_{12} &= -\sigma_{21} = -\sigma_0/ql, \end{aligned} \tag{5.10}$$

which leads to the expressions

$$\begin{aligned} \sigma_{11}' &= \frac{(1-i\omega\tau)}{(\omega_c\tau)^2 + \frac{1}{2}iqlv_0/v_s}, \\ \sigma_{22}' &= \frac{X^2}{(1-i\omega\tau)} + \frac{i(v_0/v_s)(\omega_c\tau)^2}{ql[(\omega_c\tau)^2 + \frac{1}{2}iqlv_0/v_s](1-i\omega\tau)}, \\ \sigma_{33}' &= \frac{1}{(1-i\omega\tau)}, \\ \sigma_{12}' &= \frac{(\omega_c\tau)^2}{ql[(\omega_c\tau)^2 + \frac{1}{2}iqlv_0/v_s]}. \end{aligned} \tag{5.11}$$

Using (5.11) to evaluate the component of $\mathbf{S}$ appropriate for longitudinal waves, we find that

$$S_{11} = \frac{(\omega_c\tau)^2[1 + (\omega/\omega_p)^2(v_0/v_s)^2 V_{11}/mv_0^2]^2}{[1 + \frac{1}{2}(\omega/\omega_p)^2(v_0/v_s)^2]^2 + (\omega\tau)^2[1 + (\omega_c/\omega_p)^2]^2}. \tag{5.12}$$

As in the case of a longitudinally polarized wave in a longitudinal magnetic field, we find three regions of frequency behavior characterized by the frequencies ω_0 and ω_1, which were defined in Sec. IV. However, in addition, we find a distinct magnetic field dependence, which includes a high field peak in the first two frequency ranges. We obtain by differentiation the value of the magnetic field at which (5.12) has a maximum:

$$\left(\frac{\omega_c}{\omega_p}\right)^2 = \left[1 + \left(\frac{1 + \frac{1}{2}(v_0/v_s)^2(\omega/\omega_p)^2}{\omega\tau}\right)^2\right]^{\frac{1}{2}}. \tag{5.13}$$

When $\omega\tau \gg 1$, we obtain a high-field peak at

$$\omega_c = \omega_p; \tag{5.14}$$

while, when $\omega\tau \ll 1$, the value of the peak occurs at

$$\frac{\omega_c}{\omega_p} = \left[\frac{1 + \frac{1}{2}(v_0/v_s)^2(\omega/\omega_p)^2}{\omega\tau}\right]^{\frac{1}{2}}. \tag{5.15}$$

We can see that there will not be any high-field peak in the third frequency region $\omega > \omega_1$, when $\omega\tau > 1$, because (5.14) cannot be satisfied for $\omega_c > (v_0/v_s)\omega$ when ω is in this region. The condition (5.15), however, is consistent with $\omega_c > (v_0/v_s)\omega$ for small values of $\omega\tau$, and in this case, a high field peak could be observed in the third frequency region. The peak value of S_{11}, when $\omega\tau \gg 1$, is

$$S_{11} = \frac{1}{4}\left(\frac{\omega_p}{\omega}\right)^2 \left[1 + \left(\frac{\omega}{\omega_p}\right)^2 \left(\frac{v_0}{v_s}\right)^2 \frac{V_{11}}{mv_0^2}\right]^2. \tag{5.16}$$

Thus, the frequency dependence at the high field peak depends upon whether $\omega<\omega_0$ or $\omega>\omega_0$. For $\omega<\omega_0$, the frequency dependence goes as $(\omega_p/\omega)^2$, while for $\omega>\omega_0$, the dependence goes as $(\omega/\omega_p)^2$. In Figs. 5 and 6, S_{11} is plotted against the magnetic field for $\omega\tau>1$ and $\omega\tau<1$, respectively.

For the absorption of the transverse waves, we find, as previously, that we can neglect the terms not containing the deformation potential for strong deformation forces. We therefore obtain for the transverse waves

$$S_{jj}=\frac{(\omega_c\tau)^2(\omega/\omega_p)^4(v_0/v_s)^4(V_{1j}/mv_0^2)^2}{[1+\frac{1}{2}(\omega/\omega_p)^2(v_0/v_s)^2]^2+(\omega\tau)^2[1+(\omega_c/\omega_p)^2]^2}, \tag{5.17}$$

where $j=2$, 3. The behavior of the absorption with magnetic field is the same as (5.12) except that the lowest of the three frequency ranges is missing.

VI. DISCUSSION

In this section, an effort will be made to present a physical picture in terms of which the various phenomena discussed can be understood. We also attempt to estimate which phenomena can be observed at the present time.

The rapid rise in attenuation over a limited frequency range in the case of propagation parallel to the magnetic field can be understood in terms of the breakdown in screening of the longitudinal currents. In the first frequency region, defined in Sec. IV by the condition $\omega<\omega_0$, the deformation potential forces are well screened and the attenuation takes on a very low value. In metals there is still considerable absorption when there is screening but this is no longer the case in semiconductors, because of the small number of carriers involved. As the frequency increases beyond ω_0, the screening of the longitudinal currents breaks down and the attenuation begins to rise quite rapidly, approximately as ω^5. When we reach the critical frequency ω_1, the absorption begins to increase more slowly, increasing only linearly with ω, as the electrostatic potential due to the variation in electron charge density can be ignored and the electric fields vanish. This can be better understood if we rewrite the condition $\omega>\omega_1$ in the form

$$\lambda<\lambda_D, \quad \lambda_D=v_0/\omega_p, \tag{6.1}$$

where λ is the sound wavelength and λ_D is the Debye length. Here, λ_D has the physical meaning of the distance within which the charge density variation of the electrons is small in magnitude. Therefore, in this region, collective effects can be ignored, i.e., the breakdown of the screening is complete, and the attenuation arises only from the unscreened deformation forces.

In the case of the transverse waves in a longitudinal field, we find the attenuation follows the same kind of behavior when there are strong deformation forces because the deformation forces are longitudinal in character even when generated by transverse waves. Thus, the deformation potential couples the transverse polarized waves only to the longitudinal currents. Therefore we find that the frequency region $\omega<\omega_0$, which is due to the screening of the currents arising from deformation forces by the currents due to electromagnetic forces, is missing from the transverse waves.

We now wish to examine the regions in which the formulas developed in Sec. IV for the long- and short-wavelength cases are applicable. From cyclotron resonance[26] and mobility[27] data at low temperatures, we

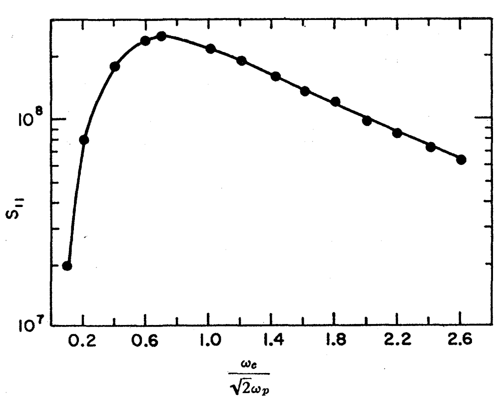

FIG. 5. The attenuation of longitudinal waves in transverse magnetic fields is shown as a function of the ratio of the cyclotron frequency to the plasma frequency in the high-field limit, $\omega_c>\omega v_0/v_s$ and with $\omega\tau>1$. The position of the peak at $\omega_c=\omega_p$ is clearly evident.

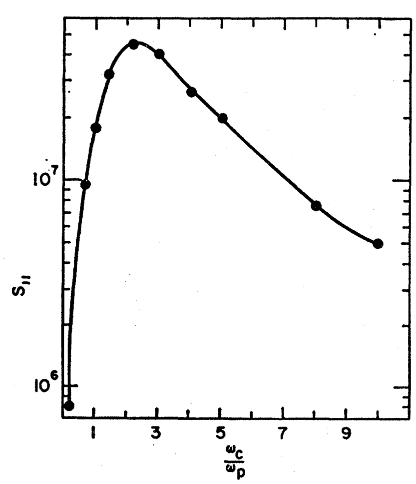

FIG. 6. The attenuation of longitudinal waves in transverse magnetic fields is shown in the high-field limit when $\omega\tau<1$. The peak in the absorption is now a function of $\omega\tau$.

find that values of τ as long as 10^{-10} sec are now available. Therefore the boundary between the cases $ql<1$ and $ql>1$ lies at a frequency $\omega=10^9$ cps which lies near the microwave range. With present day techniques for production of acoustic waves in semiconductors, frequencies lying on both sides of this boundary are available and both regions can be investigated experimentally. It is, in fact, possible to reach frequencies such that $\omega\tau>1$ (i.e., $\omega>10^{10}$ cps) with present-day apparatus. Taking germanium with a deformation potential of 20 ev, an effective mass of $m^*/m=0.1$, a dielectric constant of 16, and a carrier concentration of 10^{13} cm^{-3} at 10°K, we find that $\omega_0\approx 10^7$ cps and $\omega_1\approx 10^9$ cps. Thus only the long-wavelength limit $ql\ll 1$ holds for $\omega<\omega_0$, while the other two frequency regions can be reached either in the long-wavelength limit $ql\ll 1$ or in the short wavelength limit $ql\gg 1$. In both cases, the attenuation will be observable only when $\omega>\omega_0$, as the discussion in Sec. IV demonstrates.

Our results agree with those derived by Mikoshiba[21] for longitudinally polarized waves in zero magnetic field as the presence of the magnetic field does not affect the waves which are polarized and propagating in the direction of the magnetic field. Our results for the transverse polarized waves, would differ from his, even in the limit of zero magnetic field, as he did not consider the case of the coupling of transverse polarized waves to longitudinal currents via a strong transverse deformation potential.

It is noted in passing that the results obtained using Weinreich's phenomenological approach[16] agree with our expressions for the absorption when $ql\ll 1$ and $\omega_0<\omega<\omega_1$. This is because his approach ignores the screening effects compared to effects arising from the deformation potential and is only valid for wavelengths longer than the mean free path.

The background absorption from effects other than the electronic contribution to the attenuation decreases to a very low level at low temperatures. In quartz, in which the order of magnitude of the absorption is about the same as the background in semiconductors, the attenuation at 20°K falls to less than 10^{-1} db/cm.[30] Thus the electronic contribution to the absorption should be observable for $\omega>\omega_0$.

When the magnetic field is transverse to the direction of propagation, we have found that there are cyclotron resonances when the sound frequency is equal to an integral multiple of the cyclotron frequency. For the resonances to occur, we must also require $\omega_c\tau>1$. If this condition is not satisfied, the cyclotron resonances will be damped out. The condition $\omega_c\tau>1$ requires an $\omega_c>10^{10}$ cps, i.e., a magnetic field greater than 300 gauss using the previously quoted values of the effective mass and the relaxation time for germanium. Since sound waves of this frequency have already been produced,[30] we could obtain the first few harmonics of the cyclotron frequency.

In the region in which cyclotron resonances can be experimentally obtained, i.e., $\omega_c\tau>1$, $\omega\gtrsim\omega_1$, we have $S_{11}\sim 10^{11}$ for the peak at $\omega=\omega_c$ and $S\sim 10^{10}$ for the valley at $\omega=\frac{3}{2}\omega_c$. Therefore in this region, the attenuation has a high value and both the peaks and valleys of the cyclotron resonances should be clearly observable.

In the high-field limit $\omega_c>qv_0=\omega v_0/v_s$, we find a peak in the attenuation as a function of magnetic field. The position of the peak depends upon whether $\omega\tau$ is greater than or less than unity. When $\omega\tau>1$, the high-field peak occurs at magnetic fields such that $\omega_c/\omega_p=1$. Thus when $\omega\tau>1$, the determination of the position of the peak permits a determination of carrier concentration independent of relaxation processes. On the other hand, when $\omega\tau<1$, we still obtain a high-field peak, but the position of this peak depends upon $\omega\tau$.

To satisfy the requirement $\omega\tau>1$, we must work at frequencies $\omega>10^{10}$ cps and magnetic fields greater than 30 000 gauss (i.e., $\omega_c>10^{12}$ cps) and therefore we would require a greater concentration of carriers so that $\omega_p>10^{12}$ cps.[26] It is possible to obtain such concentrations at low temperatures by exciting electrons from donor states by light. The relaxation aspect of the high-field peak can be observed at lower concentrations of carriers, field strengths, and frequencies.

At frequencies, field strengths, and carrier concentrations such that the high-field peak can be observed without relaxation effects, S_{11} has the value 10^{10} at the peak and falls to the value $S_{11}\sim 5\times 10^9$ at $\omega_c=2\omega_p$, so that the peak is still quite broad. In the relaxation region, $\omega\tau<1$, the attenuation is not observable for $\omega<\omega_0$ (i.e., $S_{11}\sim 10^5$) but in the frequency range $\omega_0<\omega<\omega_1$, $S_{11}\approx 10^9$ at the peak, and the attenuation has a very high value. For cyclotron frequencies greater than the plasma frequency, the attenuation varies with frequency and magnetic field as $(\omega/\omega_c)^2$ and has the maximum value $S_{11}\sim 10^9$ when $\omega\tau>1$.

The high-field peak arises from the same sort of collective behavior discussed by Harrison in the case of semimetals[7] and semiconductors.[35] We can obtain a physical insight into the problem by a Drude-Lorentz treatment since when (5.14) holds, relaxation effects can be ignored. For the equation of motion of the electrons we have

$$d\mathbf{v}/dt=-e(\boldsymbol{\mathcal{E}}+(\mathbf{v}/c)\times\mathbf{H})-iq^2(v_0/\omega)v_x\hat{x}, \quad (6.2)$$

where $\boldsymbol{\mathcal{E}}$ arises solely from the electronic current via (2.1); $\mathbf{q}$ is the wave number of a density fluctuation and is in the x direction; $\mathbf{H}$ is in the z direction; and the last term represents the diffusion of electrons. The natural frequency of (6.2) for a longitudinally polarized wave is

$$\omega_r^2=\omega_p^2+(qv_0)^2+\frac{\omega_c}{1+(\omega_p/cq)^2}. \quad (6.3)$$

[35] M. J. Harrison (unpublished work).

This resonance cannot be reached by sound waves since $v_0 > v_s$ and there is no intersection between the dispersion curves for the magnetoplasma mode (6.3) and the sound wave $\omega = qv_s$. However, some manifestation of this collective motion of the electron gas must occur when the magnetic field is such that the cyclotron frequency is of the order of magnitude of the plasma frequency. We note that the high-field peak found in our calculations is just such a manifestation of the collective behavior. Unlike the case in metals and semimetals, the peak is not a relaxation effect since it requires only the condition $\omega\tau > 1$ which can be obtained at present in semiconductors.

The results derived by Pokatilov[22] for transverse magnetic fields agree with our results when $\gamma > |\sigma_{11}'|$. In this case, all the effects due to screening and to the electron density gradient can be neglected and the carriers respond just to the unscreened deformation forces.

It may be of interest to indicate why our calculations do not show any direct magnetoplasma resonances even though, for semiconductors, sound waves are available with frequencies equal to the plasma frequency. A Drude-Lorentz treatment, such as was given earlier in connection with the high-field peak, is indicated. We treat the case of density fluctuations with wave number $\mathbf{q}$ parallel to $\mathbf{H}$, where $\mathbf{H}$ lies in the z direction. Our equation of motion now is

$$d\mathbf{v}/dt = -e(\boldsymbol{\mathcal{E}} + (\mathbf{v}/c)\times\mathbf{H}) - iq^2(v_0^2/\omega)v_z\hat{\mathbf{z}}, \quad (6.4)$$

where the terms have the same meaning as in (6.2). For a wave polarized in the direction of propagation we obtain for the natural frequency of (6.4),

$$\omega_r^2 = \omega_p^2 + (qv_0)^2. \quad (6.5)$$

As was the case with (6.3), the dispersion curves for the sound wave and the plasma mode do not intersect because $v_0 > v_s$. If, on the other hand, we assume a transverse polarized wave, we obtain for the natural frequency

$$\omega_r = \omega_c / [1 + (\omega_p/cq)^2]. \quad (6.6)$$

Therefore, for transverse polarized waves in a longitudinal magnetic field, it is possible to excite a magnetoplasma oscillation using sound waves. Unfortunately, in our case the coupling to the longitudinal currents via the deformation potential is so strong that we lose our magnetoplasma resonance everywhere where the attenuation is observable. However, in the case of metals, where the deformation forces are not so strong compared to the electromagnetic forces, this magnetoplasma resonance can be found at attainable fields and frequencies. A peak corresponding to this resonance in fact appears in the calculation of Cohen, Harrison, and Harrison[5] for a metal, although the identification with the above magnetoplasma resonance was not made by them.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Professor Morrel H. Cohen for the many fruitful discussions which led to the writing of this paper. He also wishes to thank Dr. N. Mikoshiba and Philip E. Bloomfield for their helpful comments.

APPENDIX

In Sec. III, we found it necessary to evaluate the following types of integrals:

$$U_1 = \int_0^\infty dw\, w \exp(-w^2) J_n^2(Xw), \quad \text{(A1a)}$$

$$U_2 = \int_0^\infty dw\, w^3 \exp(-w^2) J_n^2(Xw), \quad \text{(A1b)}$$

$$U_3 = (d/dX) U_1, \quad \text{(A1c)}$$

$$V_m = \int_{-\infty}^{\infty} \frac{du\, u^m \exp(-u^2)}{1 + i(n\omega_c - \omega + q_z v_0 u)\tau}, \quad \text{(A1d)}$$

$$m = 0, 1, 2.$$

The integrals U_1, U_2, and U_3 can be evaluated directly by using the relation[36]

$$\int_0^\infty dw\, w \exp(-w^2) J_n(aw) J_n(bw) = \tfrac{1}{2} \exp[-(a^2+b^2)/4] I_n(ab/2), \quad \text{(A2)}$$

where I_n is the hyperbolic Bessel function. We then obtain

$$U_1 = \tfrac{1}{2} \exp(-\tfrac{1}{2}X^2) I_n(\tfrac{1}{2}X^2), \quad \text{(A3a)}$$

$$U_2 = \tfrac{1}{2} \lim_{a,b\to X} \frac{d^2}{da\,db}\left[\exp\left(-\frac{a^2+b^2}{4}\right) I_n\left(\frac{ab}{2}\right)\right]$$

$$= \tfrac{1}{8} \exp(-\tfrac{1}{2}X^2)\left[X^2 I_n(\tfrac{1}{2}X^2) + 2(1-X^2)\frac{dI_n(\tfrac{1}{2}X^2)}{d(\tfrac{1}{2}X^2)} + X^2 \frac{d^2 I_n(\tfrac{1}{2}X^2)}{d(\tfrac{1}{2}X^2)^2}\right], \quad \text{(A3b)}$$

$$U_3 = \tfrac{1}{2} X \exp(-\tfrac{1}{2}X^2)\left[\frac{dI_n(\tfrac{1}{2}X^2)}{d(\tfrac{1}{2}X^2)} - I_n(\tfrac{1}{2}X^2)\right]. \quad \text{(A3c)}$$

To evaluate integrals of the type V_m we must first rewrite them in the form

$$V_m = \frac{1}{\tau} \int_0^\infty d\psi \int_{-\infty}^{\infty} du\, u^m \exp(-u^2) \times \exp\{-[i(n\omega_c - \omega + q_z v_0 u) + 1/\tau]\psi\}, \quad \text{(A4)}$$

[36] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, New York, 1958), 2nd ed., p. 395.

and introduce a change in variables, $\Omega=u+\frac{1}{2}iq_zv_0\psi$. Using this change, (A4) becomes

$$V_m=\frac{1}{\tau}\int_0^\infty d\psi\,\exp\left\{-\left[\left(i(n\omega_c-\omega)+\frac{1}{\tau}\right)\psi+\left(\frac{q_zv_0}{2}\right)^2\psi^2\right]\right\}\int_{-\infty}^\infty d\Omega(\Omega-\tfrac{1}{2}iq_zv_0\psi)^m\exp(-\Omega^2). \quad \text{(A5)}$$

By using the evaluation of integrals of the type[37]

$$\int_{-\infty}^\infty d\Omega\,\Omega^{2m}\exp(-\Omega^2)=\frac{1\times3\times\cdots(2m-1)}{2^m}\pi^{\frac{1}{2}}, \quad \text{(A6a)}$$

$$\int_{-\infty}^\infty d\Omega\,\Omega^{2m+1}\exp(-\Omega^2)=0, \quad \text{(A6b)}$$

we can bring the integrals V_m, $m=0$, 1, 2 into the following form:

$$V_0=\frac{\pi}{q_zl}F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_zl}\right), \quad \text{(A7a)}$$

$$V_1=-\frac{i\pi}{(q_zl)^2}\left[[1+i(n\omega_c-\omega)\tau]F\left(\frac{1+i(n\omega_c-\omega)\tau}{q_zl}\right)-\frac{q_zl}{\pi^{\frac{1}{2}}}\right], \quad \text{(A7b)}$$

$$V_2=\frac{\pi}{(q_zl)^3}[1+i(n\omega_c-\omega)\tau][\cdots], \quad \text{(A7c)}$$

where the function $F(x)$ was defined in (3.7). Using the above relations, (A3) and (A7), we can now obtain expressions (3.6) from the expression (3.5).

[37] *Handbook of Chemistry and Physics*, edited by C. D. Hodgman (Chemical Rubber Publishing Company, Cleveland, Ohio, 1955), 37th ed., p. 275.