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Scattering of Fast Neutrons by Magnesium*

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Angular distributions for the scattering of fast neutrons by magnesium have been obtained for incident neutron energies of 2.0, 3.0, 4.0, and 5.0 Mev. The results for the elastic scattering have been compared with optical model calculations. The results for the inelastic scattering have been compared with calculations based on Hauser-Feshbach theory, and with direct interaction predictions. Over the range of conditions investigated, compound nucleus formation appears to be the dominant mechanism for the inelastic scattering.

INTRODUCTION

THIS paper is the third in a series whose purpose is to utilize the techniques of neutron spectroscopy to test the various theories of nuclear reactions as applied in particular to inelastic neutron scattering. The first two papers, hereafter referred to as I [1] and II,[2] contain material on elements in the iron region and on U^{238}, respectively. This paper continues the investigation with a study of the neutron excitation of the first state in Mg^{24} which is at 1.37 Mev.

The earlier investigations have shown that inelastic scattering of neutrons of energy less than 3 Mev is described at least to first approximation by the compound nucleus model of nuclear reactions. In particular, comparisons have been made between experiment and the detailed predictions of the Hauser-Feshbach[3] theory with respect to the angular distributions and yields of inelastically scattered neutrons and of de-excitation gamma rays.

Before presenting the new material it is appropriate at this point to review the status of those comparisons in the light of developments which have occurred in the interim.

In I there were two major discrepancies between the experimental results and the predictions of the Hauser-Feshbach theory. First, the observed anisotropy of the de-excitation gamma rays for the first state in Fe^{56} was found to be only about one half the calculated value, and second, the magnitude of the inelastic cross section was about twice that calculated. The first discrepancy has been substantially eliminated by the discovery of an error in the theory,[4] and the agreement has been noted[4] and confirmed.[5]

The second discrepancy is very substantially reduced by using the diffuse-edge version of the optical model potential as in II for calculating nuclear penetrabilities rather than the square-edge model on which the earliest calculations were based. Thus, Table I summarizes the situation with respect to 90° neutron scattering cross sections for the Fe^{56} and Ti^{48} results reported in I, and gives a comparison with new calculations using the rounded edge well and the parameters of Beyster,[6] and of Campbell, Feshbach, Porter, and Weisskopf.[7]

The remaining disagreement seems clearly within the range within which the optical model parameters may be adjusted without seriously damaging the agreement of the optical model with its predictions in other directions. The small anisotropy of the inelastically scattered neutrons predicted by the square well parameters is preserved in the calculations based on the rounded edge, and is in agreement with experiment.

The disagreement noted in II was an observed anisotropy of the inelastically scattered neutrons corresponding to excitation of the 44-kev level in U^{238} which was a factor of 2 to 3 greater than the calculated value. The large anisotropy has been confirmed in the similar case of the first-excited state in Th^{232},[8] but it has been pointed out by Moldauer[9] that inclusion of a spin-orbit term in the optical model potential can modify the neutron penetrabilities sufficiently to reconcile theory and experiment.

Smaller discrepancies with details of the Hauser-Feshbach theory have also been noted in I, particularly with respect to departures of the angular distribution for inelastically scattered neutrons from symmetry about 90°. However, the variation of these departures with

TABLE I. A comparison of the experimental[a] values of 90° inelastic neutron scattering cross sections for Fe^{56} and Ti^{48} with values calculated[b] by using the parameters of Beyster[c] and of Campbell *et al.*[d]

Element	Level (Mev)	E_n (Mev)	$\sigma_{in}(90°)$ (mb/sr) Experiment[a]	Calc[c]	Calc[d]
Fe^{56}	0.85	2.25	80±4	72	78
Ti^{48}	1.00	2.25	75±4	72	104
Fe^{56}	0.85	2.45	85±3	73	75
Ti^{48}	1.00	2.45	85±3	74	104

[a] See reference 1.
[b] John Wills, Los Alamos Scientific Laboratory, 1958 (private communication).
[c] See reference 6.
[d] See reference 7.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] L. Cranberg and J. S. Levin, Phys. Rev. **103**, 343 (1956).
[2] L. Cranberg and J. S. Levin, Phys. Rev. **109**, 2063 (1958).
[3] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[4] G. R. Satchler, Phys. Rev. **104**, 1198 (1956); **111**, 1747 (1958).
[5] R. B. Day and M. Walt, Phys. Rev. **117**, 1330 (1960).
[6] J. R. Beyster, Los Alamos Report LA-2099, 1957 (unpublished).
[7] E. J. Campbell, H. Feshbach, C. E. Porter, and V. F. Weisskopf, Massachusetts Institute of Technology Report MIT No. 73 (unpublished).
[8] A. C. Smith (private communication).
[9] P. Moldauer (private communication).

respect to mass number and incident neutron energy pointed to an interpretation in terms of failure to satisfy in detail the requirements of the "statistical assumption" of the theory. This interpretation is also consistent with subsequent high-resolution measurements of the total cross sections of Fe and Ti over the same range of energy as was used in the scattering measurements,[10] in which fluctuations by a factor of 2 with a periodicity of about 10 kev were observed. Recently considerations developed by several authors[11] emphasized the role of fluctuations in compound nucleus processes.

The present investigation was motivated by the appearance of a set of calculations by Yoshida[12] on excitation of the first-excited state of Mg^{24} at 1.37 Mev by inelastic neutron scattering, and by subsequent calculations by Glendenning.[13] These calculations are based on various forms of the direct interaction model. The characteristic feature which distinguishes the predictions of this model from those of the compound nucleus is the occurrence of large anisotropies with, in most cases, strong fore-aft asymmetry in the angular distribution of the inelastically scattered neutrons. On the other hand, the absolute magnitudes of the predictions are subject to large uncertainties associated with choice of the calculational parameters. These uncertainties preclude a quantitative separation of the roles of compound nucleus formation and direct interaction, but one can anticipate that at least a qualitative determination can be made as to which is the dominant mechanism on the basis of the experimental anisotropy, and this is the prime motive underlying the present investigation.

Preliminary results of this experiment have been reported earlier.[14] The scattering of fast neutrons by magnesium was observed for incident energies of 2.0, 3.0, 4.0, and 5.0 Mev. Angular distributions were obtained for elastic scattering and for the inelastic neutrons due to excitation of the 1.37-Mev level in Mg^{24}.

PROCEDURE AND RESULTS

The experimental technique was the same as was used previously[1] to study the angular distribution of neutrons scattered inelastically due to excitation of discrete levels in Fe, Ti, and Ni. The large Los Alamos Van de Graaff accelerator was used with the $D(d,n)He^3$ reaction to produce the 5.0-Mev neutrons, and with the $T(p,n)He^3$ reaction to produce the lower energy neutrons. The energy thicknesses of the gas targets were 120, 94, and 78 kev, corresponding to the neutron energies of 2.0, 3.0, and 4.0 Mev, respectively. At 5.0 Mev, the target thickness was 288 kev.

The energy spreads in the primary neutron beam are comparable to those used in the work previously reported in I. In view of the discussion above concerning fluctuation effects observed in I and what is known about the systematics of the dependence of level density on mass number and excitation, it is to be expected that fluctuation effects should also be in evidence under the conditions of these measurements. The magnitude of these effects could not be anticipated, however, so that it was considered worthwhile simply to explore the problem with target thicknesses which were experimentally convenient.

The neutrons were scattered by a hollow cylindrical scatterer, of length 2 in., i.d. $\frac{3}{8}$ in., o.d. $\frac{3}{4}$ in., designed to reduce multiple scattering, and placed about 10 cm from the neutron source. The scattered neutrons were observed with a plastic proton-recoil scintillator and an RCA 5819 photomultiplier. This detector was inside a large lead-lined paraffin-lithium carbonate collimator. A heavy copper cone shielded the detector from the source. The scattering angle was varied from 20° to 150°.

The pulsed-beam time-of-flight technique[1,15] was used to identify separately the elastic neutrons, the inelastic neutrons, and the gamma rays resulting from the inelastic scattering. Figure 1 shows a typical time spectrum for the case of 3.0-Mev neutrons incident on magnesium. The scattering angle was 75° and the flight path was 1.3 m. The time per channel was 2.3 mμ sec. The sample-out run has been subtracted from the sample-in run. The double presentation is a consequence of the fact that there are two neutron bursts for each reference (stop) pulse.

Complete angular distributions for the elastically scattered neutrons were obtained at each of the four incident energies, 2.0, 3.0, 4.0, and 5.0 Mev. The angular distributions for the neutrons scattered inelastically due

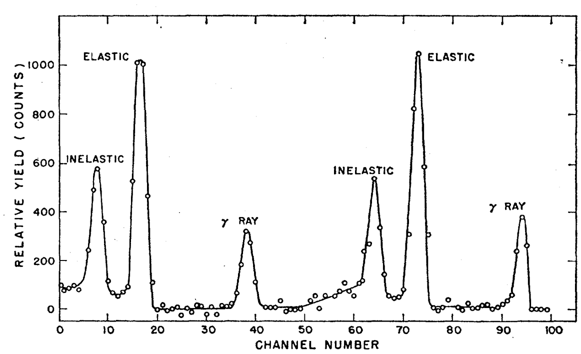

FIG. 1. A typical time spectrum, for the case of 3.0-Mev neutrons incident on magnesium. The scattering angle was 75° and the flight path was 1.3 m.

[10] L. Cranberg, J. S. Levin, and D. B. Thomson (unpublished).
[11] L. Dresner, *Proceedings of the International Conference on the Neutron Interactions with the Nucleus*, Atomic Energy Commission Report TID-7547 (U. S. Government Printing Office, Washington, D. C., 1957), p. 71. T. Ericson, Phys. Rev. Letters **5**, 430 (1960).
[12] Shiro Yoshida, Progr. Theoret. Phys. (Kyoto) **19**, 169 (1958).
[13] N. K. Glendenning, Phys. Rev. **114**, 1297 (1959) and private communication.
[14] D. B. Thomson, L. Cranberg, and J. S. Levin, Bull. Am. Phys. Soc. **3**, 365 (1958); D. B. Thomson, Ph.D. thesis, University of Kansas, 1960 (unpublished).
[15] W. Weber, C. W. Johnstone, and L. Cranberg, Rev. Sci. Instr. **27**, 166 (1956).

to excitation of the 1.37-Mev level were obtained at 3.0, 4.0, and 5.0 Mev. At 2.0 Mev, the inelastic peak was too weak to make practical an accurate angular distribution measurement but the differential inelastic cross section was obtained at 45°. The observed angular distributions were corrected for multiple scattering with the aid of computer calculations. These corrections were small, the largest being of the order of 5%. The effect of multiple scattering is to fill in the valleys and decrease the forward peaks of the angular distributions. The corrections were the largest percentage at the valleys.

At 5.0 Mev the inelastic peak partially overlapped the elastic peak, making it necessary to separate the two peaks by comparison with a "true" shape of a line corresponding to pure elastic scattering. The most accurate means of doing this is to obtain the line shape with another scatterer for which the elastic line can be completely resolved from the inelastic line corresponding to excitation of the lowest-lying excited state. At the time this work was done no suitable sample was available and the pure elastic line shape was approximated by the line shape obtained by observing the unscattered neutron beam at 0°. Multiple scattering in the sample and energy shifts due to center-of-mass effect impair the validity of this approximation under certain conditions, but rough calculation indicates that at 5.0 Mev and for the resolution conditions of these measurements the approximation is a valid one. The errors assigned to the inelastic points at extreme forward angles are large, however, due to the relative increase in the elastic differential cross section at those angles.

The absolute cross sections were obtained for both the elastic and inelastic neutrons by comparison with the n-p cross section at 45°, at each energy. A CH_2 sample was used to observe the n-p scattering. Attenuation corrections, due to the finite sample thicknesses, were made by a method described earlier.[16] The elastic scattering angular distributions are shown in Fig. 2 and listed in Table II. The angular distributions for the

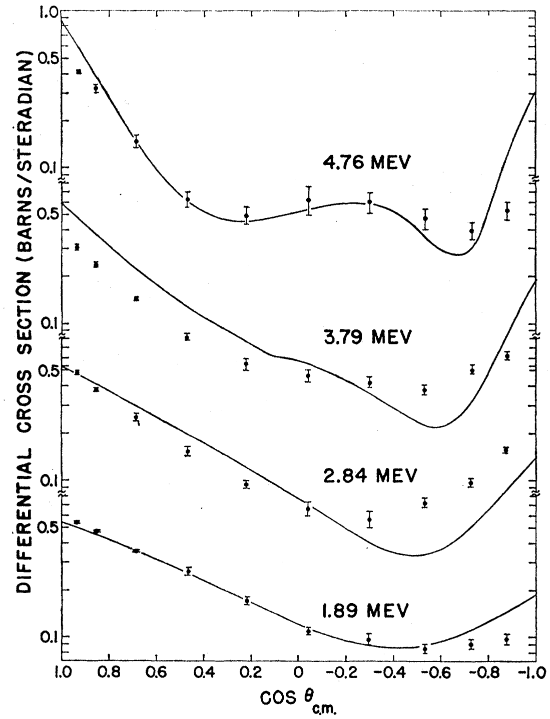

FIG. 2. Angular distributions for the elastic scattering of fast neutrons by magnesium. The incident energy listed for each curve is the energy average over the angular extent of the sample, in the center-of-mass system. The curves shown are the results of the optical model and Hauser-Feshbach calculations discussed in the text.

inelastic scattering are shown in Fig. 3 and listed in Table III. Neutron energies in these two figures and tables are given in the center-of-mass system. Total

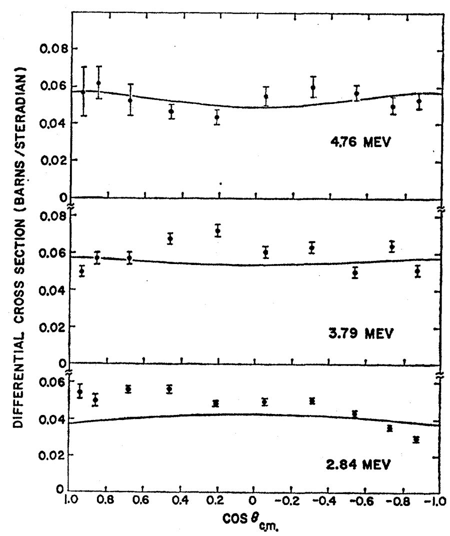

FIG. 3. Angular distributions for the inelastic scattering of fast neutrons by Mg^{24}, due to excitation of the 1.37-Mev level. The sample was natural magnesium; cross sections are corrected for the abundance of Mg^{24}. The incident energy listed for each curve is the energy average over the angular extent of the sample, in the center-of-mass system. The curves shown are the results of the optical model and Hauser-Feshbach calculations discussed in the text.

TABLE II. Differential elastic cross section for the scattering of fast neutrons by magnesium.

	Differential cross section			
$\cos\theta_{c.m.}$	1.89 Mev (barns/s)	2.84 Mev (barns/s)	3.79 Mev (barns/s)	4.76 Mev (barns/s)
0.9347	0.542	0.493	0.308	0.449
0.8552	0.482	0.380	0.239	0.327
0.6858	0.357	0.256	0.145	0.148
0.4682	0.263	0.156	0.083	0.064
0.2196	0.172	0.095	0.056	0.050
−0.0422	0.110	0.067	0.047	0.063
−0.2977	0.0977	0.057	0.042	0.061
−0.5312	0.0840	0.073	0.038	0.047
−0.7278	0.0904	0.099	0.052	0.040
−0.8764	0.0968	0.158	0.063	0.054

[16] L. Cranberg and J. S. Levin, Los Alamos Report No. LA-2177, 1959 (unpublished).

TABLE III. Differential inelastic cross section for exciting the 1.37-Mev level of Mg^{24}.

2.84 Mev		3.79 Mev		4.76 Mev	
$\cos\theta_{c.m.}$	$\sigma(\theta)$ (barns/s)	$\cos\theta_{c.m.}$	$\sigma(\theta)$ (barns/s)	$\cos\theta_{c.m.}$	$\sigma(\theta)$ (barns/s)
0.9327	0.0546	0.9334	0.0497	0.9337	0.0568
0.8511	0.0497	0.8526	0.0568	0.8533	0.0618
0.6775	0.0559	0.6803	0.0573	0.6816	0.0522
0.4558	0.0568	0.4602	0.0682	0.4620	0.0468
0.2045	0.0483	0.2093	0.0726	0.2119	0.0437
−0.0581	0.0499	−0.0523	0.0608	−0.0500	0.0552
−0.3126	0.0508	−0.3076	0.0638	−0.3051	0.0601
−0.5429	0.0437	−0.5388	0.0503	−0.5371	0.0572
−0.7355	0.0361	−0.7329	0.0647	−0.7318	0.0502
−0.8802	0.0302	−0.8788	0.0515	−0.8783	0.0538

elastic and total inelastic cross sections were obtained by integrating the angular distributions. These are listed in Table IV. The total cross section for each incident energy has been obtained by transmission measurements, and these values are listed in Table IV. For comparison, Table IV also shows the sum of the total elastic and the total inelastic cross sections corrected for isotopic abundance of Mg^{24}.

CALCULATIONS AND DISCUSSION

A Hauser-Feshbach calculation of the excitation of the 1.37-Mev excited state of Mg^{24} was made with the aid of a program prepared by Wills[17] for the IBM-704 EDPM. The transmission coefficients required by the theory were calculated by assuming a complex potential of the Woods-Saxon form with the imaginary part of the potential proportional to the real part. No spin orbit term was included. With the exception of the radius, the values assumed for the parameters were those suggested by Campbell *et al.*,[7] i.e., $V_0=-52$ Mev, $W_0=-3.12$ Mev, $\zeta=0.06$, and $a=0.52$ f. The radius was taken to be $r=1.22A^{\frac{1}{3}}$ f. Since the resolution used in the experimental measurements was not sufficiently good to eliminate contributions from some of the levels in Mg^{25} and Mg^{26}, calculations were also made for these isotopes. The level schemes that were assumed were those given in Nuclear Data Sheets[18] NRC 59–6–10a, NRC 59–6–27, and NRC 59–5–12. The predicted contributions of these minor isotopes were then added to the results of the calculations for Mg^{24}. These corrections amounted to about 3% for 3-Mev incident neutron energy, and nearly 20% for 4- and 5-Mev incident neutron energy. The results of these calculations are compared to the experimental data in Fig. 3.

The angular distributions of shape-elastic scattering were also calculated for the same potential. To these results were added the predictions for compound elastic scattering that were obtained in the Hauser-Feshbach calculations. The results of these calculations are compared to the measured elastic angular distributions in Fig. 2.

The character of the agreement between the experimental results for the elastic scattering and the calculations for the various values of parameters chosen is of about the same quality as the agreement exhibited in the curve-fitting investigations which underlie our choices of optical model parameters. In any case quantitative agreement is probably not to be expected in view of the remarks above concerning fluctuation effects.

It may be significant that so far as the inelastic results are concerned the best agreement between experiment and calculation is obtained for 5-Mev incident neutron energy. At that energy the level density is greatest and the energy spread in the primary neutron beam is also greatest. Thus the 5-Mev results are least vulnerable to the effects of fluctuations in the cross section for compound nucleus formation. Also, it has been the general experience[6,7] that neutron scattering data can be better fitted with smoothly varying optical model parameters at the higher energies, so that one has more confidence at the higher energies in the accuracy of the penetrabilities calculated on the basis of those parameters.

Departure from symmetry about 90° for inelastic scattering at the lower incident energies is quite consistent with the observations reported in I for iron, and suggests the same interpretation—namely, fluctuation effects, which should be even more prominent for magnesium than for iron, other things being equal. In view of what has been said about fluctuation effects and the weaknesses of optical model fits at lower neutron energies, the agreement between calculation and experiment in the average inelastic cross sections must be regarded as gratifying.

One special calculation has been made on a direct interaction model by Glendenning[13] for neutron scattering from Mg at 6.0 Mev. This calculation, together with those published by Yoshida,[12] provides a basis for comparison of the results with several representative versions of the direct interaction theory. The discrepancies between these predictions and the experimental results are qualitatively significant. Glendenning's result shows a peak-to-valley ratio in the angular distribution of 4:1, in contrast with an observed anisotropy at about 5 Mev of the order of 10%. Similar remarks apply to Yoshida's results. As stated above, however, in the direct interaction calculations there is large uncertainty in the absolute magnitude of the predicted cross sections due to uncertainties in the choice of the relevant parameters. Thus, there is no possibility of establishing a contradiction. The results do indicate, however, that direct interaction models which predict strong anisotropy cannot account for a substantial fraction of the observed excitation cross sections.

[17] John Wills, Los Alamos Scientific Laboratory, 1958 (private communication).

[18] *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C.).

TABLE IV. Measured total cross sections for fast neutrons scattered by magnesium.

Incident energy (Mev)	Total inelastic cross section for exciting the 1.37-Mev level in Mg^{24} (barns)	Total elastic cross section for magnesium (barns)	Sum of the total elastic and inelastic[c] cross section (barns)	Total cross section obtained by transmission measurements (barns)
2.0	0.23 ± 0.023[a]	2.52 ± 0.25	2.70 ± 0.25	$2.57^{d}\pm0.08$
3.0	0.61 ± 0.06	1.93 ± 0.19	2.41 ± 0.20	$2.67^{d}\pm0.08$
4.0	0.77 ± 0.077	1.10 ± 0.11	1.71 ± 0.13	$1.85^{e}\pm0.08$
5.0	0.65 ± 0.065	1.22 ± 0.12	2.07 ± 0.14	$2.02^{d}\pm0.06$
5.0	0.43 ± 0.06[b]			

[a] Obtained from a measurement at 45° and assuming isotropy.
[b] Total cross section for exciting the group of levels near 4.2 Mev in Mg^{24}, obtained from a measurement at 90° and assuming isotropy.
[c] Corrected for isotopic abundance of Mg^{24}.
[d] Present work.
[e] *Neutron Cross Sections*, compiled by D. J. Hughes and R. Schwartz, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), 2nd. ed.

Haddad and Phillips[19] have reported similar conclusions for the $C^{12}(n,n')C^{12*}$ reaction, at incident energies between 6 and 7 Mev. At 14 Mev, however, Clarke and Cross[20] have observed a substantial forward peaking, indicative of a direct interaction mechanism, in the angular distribution for the excitation of the 1.37-Mev level in Mg^{24}. Their result is consistent with the 14-Mev work of Rosen and Stewart,[21] who also observed forward peaking of those inelastically scattered neutrons that leave the target nuclei in low-lying states of excitation.

The general conclusion that emerges, then, from this work as well as from the earlier work reported in I and II is that over the range of conditions investigated compound nucleus formation appears to be the mechanism which dominates the reaction process for inelastic neutron scattering, and that the Hauser-Feshbach theory provides at least a semi-quantitative description of the details of the scattering process. A more precise test of that theory requires not only more extensive and more accurate data, particularly under conditions where fluctuation effects are small, but also a more comprehensive and more intensive program of investigation of optical model parameters than has yet been carried out.

[19] E. Haddad and D. D. Phillips, Bull. Am. Phys. Soc. 4, 358 (1959).
[20] W. G. Cross and R. L. Clarke, Bull. Am. Phys. Soc. 4, 258 (1959).
[21] L. Rosen and L. Stewart, Phys. Rev. **107**, 824 (1957).

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