PHYSICAL REVIEW VOLUME 125, NUMBER 6 MARCH 15, 1962

Classical Equations of Motion for a Polarized Particle in an Electromagnetic Field

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(Received April 12, 1961)

This paper gives the classical relativistic equations of motion for a particle with intrinsic angular momentum in an external electromagnetic field, including the effects of first-order field gradients. The system considered especially is a nucleus in its ground state. The preferred value of the electric quadrupole moment Q is found to be $-(2I-1)\mu(\hbar/mc)$, where I is the spin and μ the magnetic moment of the particle.

I. INTRODUCTION

Classical relativistic equations of motion for a particle with intrinsic angular momentum have been given by Frenkel[1] and by Kramers[2] using a Lorentz six-vector to describe the polarization. Bargmann, Michel, and Telegdi[3] recently found equations of motion using a Lorentz four-vector for the polarization. The two approaches are equivalent, as has been shown in detail by Ford and Hirt.[4] All these authors consider the situation where derivatives of the external electromagnetic fields are negligible, in which case the charge and magnetic moment are sufficient to distinguish the classical particle. The purpose of the present paper is to extend the argument to include the effects of first-order field gradients, in which case the electric quadrupole moment is also needed to describe the particle. The four-vector description of the polarization is used since it seems to be simpler.

In order to find the equations of motion, a system of nucleons, interacting with each other and with the external fields, is considered. The internal variables are eliminated in such a way that coupled differential equations for the position and polarization of the particle are obtained.

Some new considerations come in when a composite particle is treated. The system considered especially here is that of a nucleus in its ground state. First of all, quantum-mechanical nonrelativistic equations of motion for the position and polarization are found under the following main assumptions: (1) The internal motions of the nucleons are nonrelativistic. (2) All effects of the higher energy states may be disregarded (3) Terms of second degree in the external fields are negligible. Secondly, the classical nonrelativistic equations of motion are written down. Finally, the relativistic equations of motion are inferred using an argument essentially the same as that of Bargmann, Michel, and Telegdi.

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[2] H. A. Kramers, *Quantum Mechanics* (North-Holland Publishing Company, Amsterdam, 1957), Sec. 57.

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[4] G. W. Ford and C. W. Hirt (private communication).

II. NONRELATIVISTIC EQUATIONS OF MOTION

The Hamiltonian for the system is

$$H=\sum_\alpha(2m_\alpha)^{-1}[\mathbf{p}_\alpha-e_\alpha c^{-1}\mathbf{A}(\mathbf{x}_\alpha)]\cdot[\mathbf{p}_\alpha-e_\alpha c^{-1}\mathbf{A}(\mathbf{x}_\alpha)] +\sum_\alpha e_\alpha\phi(\mathbf{x}_\alpha)-\sum_\alpha(g_\alpha e_p/2m_pc)\mathbf{s}_\alpha\cdot\mathbf{B}(\mathbf{x}_\alpha)+h, \quad (1)$$

where α is the serial number of the individual nucleons in the nucleus. Here e_p and m_p are the charge and mass of the proton and $\mathbf{s}_\alpha$ is $\frac{1}{2}\hbar\boldsymbol{\sigma}_\alpha$. The symbols $\mathbf{A}$, ϕ, $\mathbf{B}$ are for the externally applied fields, possibly time dependent. The h term is for the internal electromagnetic and nuclear-force effects. It is supposed that h depends on the spins and relative coordinates only.

The center-of-mass coordinate is defined by

$$\mathbf{x}_0=\sum_\alpha m_\alpha\mathbf{x}_\alpha/m,$$

where $m=\sum m_\alpha$ is the total mass of the nucleus. The internal coordinates are defined by

$$\mathbf{x}_\alpha'=\mathbf{x}_\alpha-\mathbf{x}_0.$$

They are not all independent, since

$$\sum m_\alpha\mathbf{x}_\alpha'=0.$$

The total canonical momentum is

$$\mathbf{p}_0=\sum \mathbf{p}_\alpha$$

and internal canonical momenta are defined by

$$\mathbf{p}_\alpha'=\mathbf{p}_\alpha-(m_\alpha/m)\mathbf{p}_0.$$

They also are not all independent:

$$\sum \mathbf{p}_\alpha'=0.$$

The external variables $\mathbf{x}_0$, $\mathbf{p}_0$ are conjugate to each other,

$$[p_{0i},x_{0j}]=-i\hbar\delta_{ij},$$

and they commute with all the variables $\mathbf{x}_\alpha'$, $\mathbf{p}_\alpha'$, $\mathbf{s}_\alpha$. The only nonzero commutators among the primed variables are given by

$$[p_{\alpha i}',x_{\beta j}']=i\hbar\delta_{ij}[(m_\alpha/m)-\delta_{\alpha\beta}]. \quad (2)$$

The operator

$$\mathbf{J}=\sum_\alpha(\mathbf{x}_\alpha\times\mathbf{p}_\alpha+\mathbf{s}_\alpha)$$

is the total angular displacement operator in the sense that

$$[J_i,v_j]=i\hbar\epsilon_{ijk}v_k,$$

where $\mathbf{v}$ is any vector formed from $\mathbf{x}_\alpha$, $\mathbf{p}_\beta$, and $\mathbf{s}_\gamma$. It may be written as

$$\mathbf{J}=\mathbf{x}_0\times\mathbf{p}_0+\mathbf{J}',$$

where

$$\mathbf{J}'=\sum_\alpha(\mathbf{x}_\alpha'\times\mathbf{p}_\alpha'+s_\alpha).$$

Since $\mathbf{x}_0$ and $\mathbf{p}_0$ commute with the internal variables it is clear that $\mathbf{J}'$ is the angular displacement operator for the variables $\mathbf{x}_\alpha'$, $\mathbf{p}_\beta'$, $\mathbf{s}_\gamma$. Therefore one may apply all the theorems that follow from angular momentum algebra to $\mathbf{J}'$ and these variables.

In the absence of external fields the Hamiltonian reduces to

$$H=(2m)^{-1}\mathbf{p}_0\cdot\mathbf{p}_0+\sum(2m_\alpha)^{-1}\mathbf{p}_\alpha'\cdot\mathbf{p}_\alpha'+h(\mathbf{x}',s).$$

Assuming that h is reflection and rotation invariant, one considers then a set of functions $\psi_m(\mathbf{x}_\alpha')$ with the properties

$$\begin{aligned}\left[\sum(2m_\alpha)^{-1}\mathbf{p}_\alpha'\cdot\mathbf{p}_\alpha'+h\right]\psi_m&=E\psi_m,\\ J'^2\psi_m&=\hbar^2I(I+1)\psi_m,\\ J_z'\psi_m&=\hbar m\psi_m,\\ \psi_m(-\mathbf{x}_\alpha')&=\lambda\psi_m(\mathbf{x}_\alpha'),\end{aligned}\tag{3}$$

where E, I, and λ are the energy, spin, and parity of the ground state of the nucleus. It is assumed below that these functions, perhaps perturbed by the external fields, form a complete set for describing the internal nuclear effects.

The Hamiltonian of Eq. (1) implies these equations of motion in the Heisenberg picture:

$$\begin{aligned}md\mathbf{x}_0/dt&=\mathbf{p}_0-\sum e_\alpha c^{-1}\mathbf{A}(\mathbf{x}_\alpha),\\ md^2\mathbf{x}_0/dt^2&=\sum e_\alpha\mathbf{E}(\mathbf{x}_\alpha)+\sum(e_\alpha/2m_\alpha c)[\mathbf{p}_\alpha-e_\alpha c^{-1}\mathbf{A}(\mathbf{x}_\alpha)]\\ &\quad\times\mathbf{B}(\mathbf{x}_\alpha)-\sum(e_\alpha/2m_\alpha c)\mathbf{B}(\mathbf{x}_\alpha)\\ &\quad\times[\mathbf{p}_\alpha-e_\alpha c^{-1}A(\mathbf{x}_\alpha)]\\ &\quad+\sum(g_\alpha e_p/2m_pc)\nabla_\alpha(\mathbf{s}_\alpha\cdot\mathbf{B}).\end{aligned}$$

Here the second-degree terms in the fields are to be disregarded and an expansion of the fields is to be made,

$$\mathbf{E}(\mathbf{x}_\alpha)=\mathbf{E}_0+(\mathbf{x}_\alpha'\cdot\nabla_0)\mathbf{E}_0,\quad \mathbf{B}(\mathbf{x}_\alpha)=\mathbf{B}_0+(\mathbf{x}_\alpha'\cdot\nabla_0)\mathbf{B}_0,$$

where $\mathbf{E}_0$ denotes $\mathbf{E}(\mathbf{x}_0)$. Also only the blocks of these operators which are effective between the states ψ_m need be retained. Many of the terms drop out because of the parity consideration and the acceleration equation simplifies to

$$\begin{aligned}md^2x_{0i}/dt^2&=eE_{0i}+(e/2mc)\epsilon_{ijk}(B_{0k}p_{0j}+p_{0j}B_{0k})\\ &\quad+\sum(e_\alpha/2m_\alpha c)\epsilon_{ijk}(x_{\alpha l}'p_{\alpha j}'+p_{\alpha j}'x_{\alpha l}')(\partial B_{0k}/\partial x_{0l})\\ &\quad+\sum(g_\alpha e_p/2m_pc)s_{\alpha j}(\partial B_{0j}/\partial x_{0i}),\end{aligned}\tag{4}$$

where $e=\sum e_\alpha$ is the total charge on the nucleus. From Eq. (2) one finds that

$$\begin{aligned}p_{\alpha j}'x_{\alpha l}'+x_{\alpha j}'p_{\alpha l}'\\ =im_\alpha\hbar^{-1}\left[\sum(2m_\beta)^{-1}\mathbf{p}_\beta'\cdot\mathbf{p}_\beta'+h,\ x_{\alpha j}'x_{\alpha l}'\right],\end{aligned}$$

and so, as a consequence of Eq. (3), this quantity is zero in the block being considered. The last two terms in Eq. (4) now may be combined together in terms of the total g factor for the nucleus. For the states under consideration, as an application of the Wigner-Eckart theorem, the g factor is defined by

$$\begin{aligned}\sum[(e_\alpha/2m_\alpha c)\mathbf{x}_\alpha'\times\mathbf{p}_\alpha'+(g_\alpha e_p/2m_pc)\mathbf{s}_\alpha]\\ =(ge_p/2m_pc)\,\mathbf{J}'.\end{aligned}\tag{5}$$

The result is

$$\begin{aligned}md^2\mathbf{x}_0/dt^2&=e\mathbf{E}_0+(e/2c)[(d\mathbf{x}_0/dt)\times\mathbf{B}_0\\ &\quad-\mathbf{B}_0\times(d\mathbf{x}_0/dt)]+(ge_p/2m_pc)\nabla_0(\mathbf{J}'\cdot\mathbf{B}_0),\end{aligned}\tag{6}$$

for the acceleration equation.

This process leads to these equations of motion for $\mathbf{J}'$ in the Coulomb gauge:

$$\begin{aligned}dJ_i'/dt&=-\sum(e_\alpha/mc)\epsilon_{ikm}x_{\alpha m}'x_{\alpha l}'p_{0j}(\partial^2A_{0j}/\partial x_{0k}\partial x_{0l})\\ &\quad+\sum e_\alpha\epsilon_{ikm}x_{\alpha m}'x_{\alpha l}'(\partial^2\phi_0/\partial x_{0k}\partial x_{0l})\\ &\quad+\epsilon_{ijk}(ge_p/2m_pc)J_j'B_{0k}.\end{aligned}$$

It is clear that $\mathbf{J}'$ is not an appropriate variable to describe the internal effects because it depends on the gauge. A better choice is the physical internal angular momentum, defined by

$$\begin{aligned}\mathcal{J}_i'&=\sum\left[m_\alpha\epsilon_{ijk}x_{\alpha j}'(dx_{\alpha k}'/dt)+s_{\alpha i}\right]\\ &=J_i'-\sum e_\alpha c^{-1}\epsilon_{ijk}x_{\alpha j}'A_k(\mathbf{x}_\alpha).\end{aligned}$$

The time derivative of this extra term can be evaluated by the same methods, and then one finds

$$\begin{aligned}d\mathcal{J}_i'/dt&=\sum(e_\alpha/2mc)x_{\alpha r}'x_{\alpha l}'[p_{0i}(\partial B_{0r}/\partial x_{0l})\\ &\quad+(\partial B_{0r}/\partial x_{0l})p_{0i}-p_{0r}(\partial B_{0i}/\partial x_{0l})-(\partial B_{0i}/\partial x_{0l})p_{0r}]\\ &\quad-\sum e_\alpha\epsilon_{ikm}x_{\alpha m}'x_{\alpha l}'(\partial E_{0k}/\partial x_{0l})+\epsilon_{ijk}(ge_p/2m_pc)J_j'B_{0k}.\end{aligned}$$

The quadrupole moment q for the states in question may be defined by

$$\begin{aligned}\sum e_\alpha(x_{\alpha i}'x_{\alpha j}'-\tfrac{1}{3}\delta_{ij}x_\alpha'^2)\\ =[J_i'J_j'+J_j'J_i'-\tfrac{2}{3}\delta_{ij}\hbar^2I(I+1)](q/2I\hbar).\end{aligned}\tag{7}$$

The connection between this notation and Blatt and Weisskopf's[5] is

$$q=Q/\hbar(2I-1).$$

Now the internal coordinates can be eliminated in favor of q and the charge radius r,

$$\sum e_\alpha x_\alpha'^2=er^2,$$

so that the equation of motion becomes

$$\begin{aligned}d\mathcal{J}_i'/dt&=(q/4mc\hbar I)(J_r'J_l'\\ &\quad+J_l'J_r')[p_{0i}(\partial B_{0r}/\partial x_{0l})+\cdots]\\ &\quad-(q/2I\hbar)\epsilon_{ikm}(J_m'J_l'+J_l'J_m')(\partial E_{0k}/\partial x_{0l})\\ &\quad+\epsilon_{ijk}(ge_p/2m_pc)J_j'B_{0k}\\ &\quad-(1/3c)[er^2-\hbar(I+1)q]\{(2m)^{-1}[p_{0l}(\partial B_{0i}/\partial x_{0l})\\ &\quad+(\partial B_{0i}/\partial x_{0l})p_{0l}]+\partial B_{0i}/\partial t\},\end{aligned}$$

[5] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 28.

where the homogeneous Maxwell equations,

$$\nabla_0\cdot\mathbf{B}_0=0,\quad \nabla_0\times\mathbf{E}_0+c^{-1}\partial\mathbf{B}_0/\partial t=0,$$

have been used. This result suggests that one should define the polarization operator $\mathbf{O}$ for the nucleus to be

$$O_i=\{\mathcal{J}_i'+(1/3c)[er^2-\hbar(I+1)q]B_{0i}\}/I\hbar, \tag{8}$$

since it has equations of motion independent of the charge radius:

$$\begin{aligned}\frac{dO_i}{dt}=\frac{q}{4c}(O_rO_l+O_lO_r)\bigg(&\frac{dx_{0i}}{dt}\frac{\partial B_{0r}}{\partial x_{0l}}+\frac{\partial B_{0r}}{\partial x_{0l}}\frac{dx_{0i}}{dt}-\frac{dx_{0r}}{dt}\frac{\partial B_{0i}}{\partial x_{0l}}\\ &-\frac{\partial B_{0i}}{\partial x_{0l}}\frac{dx_{0r}}{dt}\bigg)+\tfrac{1}{2}q\epsilon_{imk}(O_mO_l+O_lO_m)\frac{\partial E_{0k}}{\partial x_{0l}}\\ &+\epsilon_{ijk}\frac{ge_p}{2m_pc}O_jB_{0k},\end{aligned} \tag{9}$$

where again second-degree terms in the fields have been disregarded. Equation (6), with $\mathbf{J}'$ replaced by $I\hbar\mathbf{O}$ on the right, and Eq. (9) form a set of coupled differential equations for $\mathbf{x}_0$ and $\mathbf{O}$ of the desired type.

The corresponding classical equations are found simply by disregarding commutators. The results are

$$\frac{md^2\mathbf{x}_0}{dt^2}=e\mathbf{E}_0+\frac{e}{c}\frac{d\mathbf{x}_0}{dt}\times\mathbf{B}_0+\mu\nabla_0(\mathbf{O}\cdot\mathbf{B}_0), \tag{10}$$

$$\begin{aligned}\frac{d\mathbf{O}}{dt}=&\frac{\mu}{I\hbar}\mathbf{O}\times\mathbf{B}_0\\ &+q(\mathbf{O}\cdot\nabla_0)\left[\mathbf{O}\times\left(\mathbf{E}_0+\frac{1}{c}\frac{d\mathbf{x}_0}{dt}\times\mathbf{B}_0\right)\right],\end{aligned} \tag{11}$$

where $\mu=(ge_pI\hbar/2m_pc)$ is the magnetic moment.

III. RELATIVISTIC EQUATIONS OF MOTION

In making the relativistic generalization of Eqs. (10) and (11) it is convenient to use as dependent variables the dimensionless velocity u_α and the polarization T_α. The velocity is defined by

$$u_\alpha=c^{-1}dx_\alpha/d\tau=c^{-1}\gamma dx_\alpha/dt,$$

where

$$\gamma=(1-v^2)^{-\frac{1}{2}},$$

τ is the proper time, and $\mathbf{v}$ is $d\mathbf{x}/cdt$. (In this section the zero subscripts are left off and the Greek indices run from 1 to 4, x_4 being ict.) The polarization is defined to be a Lorentz four-vector with components $(\mathbf{O},0)$ in the instantaneous rest system of the particle. This means that T_α and $\mathbf{O}$ are related by

$$\mathbf{T}=\mathbf{O}+\gamma^2(\gamma+1)^{-1}\mathbf{O}\cdot\mathbf{v}\mathbf{v},\quad T_4=i\gamma\mathbf{O}\cdot\mathbf{v}. \tag{12}$$

The problem is to find covariant equations of motion which reduce to Eqs. (10) and (11) when terms proportional to c^{-2} are disregarded and which are compatible with the requirements

$$u_\nu u_\nu=-1, \tag{13}$$

$$u_\nu T_\nu=0. \tag{14}$$

These last two equations are implied by the definitions of u and T.

It is easily seen that the equations

$$mc\frac{du_\nu}{d\tau}=eF_{\nu\rho}u_\rho+\mu\left(u_\sigma\frac{\partial F_{\sigma\rho}{}^D}{\partial x_\nu}T_\rho+u_\nu u_\tau u_\sigma\frac{\partial F_{\sigma\rho}{}^D}{\partial x_\tau}T_\rho\right), \tag{15}$$

$$\begin{aligned}\frac{dT_\nu}{d\tau}=&\frac{\mu}{I\hbar}F_{\nu\sigma}T_\sigma+\left(\frac{\mu}{I\hbar}-\frac{e}{mc}\right)u_\nu u_\mu F_{\mu\sigma}T_\sigma\\ &+qT_\tau\frac{\partial F_{\nu\sigma}{}^D}{\partial x_\tau}T_\sigma+\left(q+\frac{\mu}{mc}\right)u_\nu u_\mu T_\tau\frac{\partial F_{\mu\sigma}{}^D}{\partial x_\tau}T_\sigma,\end{aligned} \tag{16}$$

do have the correct nonrelativistic limit. Here F and F^D are the electromagnetic field tensor and its dual,

$$F_{ij}=\epsilon_{ijk}B_k,\quad F_{i4}=-F_{4i}=-iE_i,\quad F_{44}=0,$$
$$F_{ij}{}^D=\epsilon_{ijk}E_k,\quad F_{i4}{}^D=-F_{4i}{}^D=iB_i,\quad F_{44}{}^D=0.$$

Also Eq. (15) implies that

$$\tfrac{1}{2}mc\frac{d}{d\tau}(u_\nu u_\nu)=(1+u_\nu u_\nu)\mu u_\tau u_\sigma\frac{\partial F_{\sigma\rho}{}^D}{\partial x_\tau}T_\rho,$$

so if Eq. (13) applies at the start it is valid forever. It is consistent therefore to postulate that Eqs. (13), (15), and (16) all apply. Then one finds that $d(u_\nu T_\nu)/d\tau$ is a factor times $u_\nu T_\nu$ so that Eq. (14) is also consistent. Equations (13) to (16) therefore all together give the solution of the problem. It is seen that

$$d(T_\nu T_\nu)/d\tau=0,$$

so that the size of the polarization vector $\mathbf{O}$ is an integral of motion.

It is clear that the auxiliary conditions make the equations of motion redundant and that actually only three orbit equations and three polarization equations are needed. If $\mathbf{v}$ and $\mathbf{O}$ are used as dependent variables, the equations of motion are

$$\begin{aligned}mcd(\gamma\mathbf{v})/dt=e\mathbf{E}+e\mathbf{v}\times\mathbf{B}+\mu\gamma[\nabla+\mathbf{v}\times(\mathbf{v}\times\nabla)\\ +\mathbf{v}(\partial/c\partial t)]\mathbf{O}\cdot(\mathbf{M}+\gamma\mathbf{N}),\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{d\mathbf{O}}{dt}=&\frac{\mu}{I\hbar}\frac{1}{\gamma}\mathbf{O}\times(\mathbf{M}+\gamma\mathbf{N})-\frac{e}{mc}\mathbf{O}\times\mathbf{N}\\ &+\frac{\mu}{mc}\frac{1}{(\gamma+1)}\mathbf{O}\times(\mathbf{v}\times\nabla)\mathbf{O}\cdot(\mathbf{M}+\gamma\mathbf{N})\\ &+q\mathbf{O}\cdot\left[\nabla+\frac{\gamma}{\gamma+1}\mathbf{v}\times(\mathbf{v}\times\nabla)+\mathbf{v}\frac{\partial}{c\partial t}\right]\mathbf{O}\\ &\times(\mathbf{E}+\gamma\mathbf{v}\times\mathbf{M}),\end{aligned} \tag{18}$$

where

$$\mathbf{M}=\mathbf{B}+\gamma(\gamma+1)^{-1}\mathbf{E}\times\mathbf{v}, \tag{19}$$

$$\mathbf{N}=\gamma(\gamma+1)^{-1}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\times\mathbf{v}. \tag{20}$$

Here the partial derivatives ∇, $\partial/\partial t$ operate on the fields $\mathbf{E}$, $\mathbf{B}$ as functions of space and time.

IV. DISCUSSION

Although a nuclear system was considered especially in Sec. II, it is clear that the same arguments apply to an atomic system as long as the assumptions listed in the Introduction are valid. Also Eqs. (10) and (11) hold for an electron or nucleon if q is set equal to zero and $\mathbf{O}$ replaced by $\boldsymbol{\sigma}$; the results of Sec. III can therefore be applied for those cases also.

It is already known[3] that in the classical treatment of polarization there is a special magnetic moment for which the second term on the right in Eq. (16) is zero. This moment is

$$\mu=e\hbar I/mc. \tag{21}$$

Actual nuclear moments are about this size or smaller. Analogously, there is a special value of the quadrupole moment for which the last term in Eq. (16) is zero. One finds

$$Q=-(2I-1)\mu(\hbar/mc). \tag{22}$$

Actual nuclear moments are usually large compared to this value.

There is a limitation on the usefulness of a classical approximation carried out to include field-gradient terms this way. Considering Eq. (10) for example, one sees that, in treating the second term on the right classically, the commutator between the terms $ec^{-1}B_{0i}$ and dx_{0j}/dt is disregarded. This commutator amounts to $(e\hbar/mc)\partial B_{0i}/\partial x_{0j}$. However, the third term on the right amounts to $(g e_p I\hbar/2m_pc)\partial B_{0i}/\partial x_{0j}$. Therefore the quantum-mechanical effect of the distortion of the wave packet by the external fields is of the same order as the effect of field gradients on the average position of the packet. This is exemplified by Niels Bohr's argument, quoted by Mott,[6] against the possibility of observing a Stern-Gerlach effect for electrons. If the charge or the magnetic field gradient is zero, this limitation is not there and, in any case, it is felt that these equations will be useful in estimating and visualizing the effects of field gradients.

One may ask when it is sensible to disregard the second-degree field terms, especially since it is the fields in the rest system of the particle that are pertinent. To discuss this, it is convenient to use units such that m_p, e_p, c are 1. Nuclear angular momenta and magnetic moments are of the order of $\hbar=137$. The unit of distance is $(e_p^2/m_pc^2)=1.5\times10^{-16}$ cm so nuclear distances are 10^3 or 10^4. However, the unit of electric or magnetic field is $(m_p^2c^4/e_p^3)=2\times10^{22}$ gauss. Thus laboratory fields are much smaller than can ever be compensated for by factors depending on the nuclear structure and it is sufficient to keep only linear terms in such fields. On the other hand, the electric field of a nucleus near the surface is about 10^{-6} and, for phenomena depending on such fields, the higher degree terms may not be neglected.

Classical equations of motion for particles with intrinsic quadrupole (and higher order) moments have been found by Havas,[7] using an entirely classical approach. The problem he considered is somewhat different than the one treated here and his equations of motion differ from those given in Sec. III mainly in two respects: (1) He obtains quadrupole contributions in both the translational and rotational equations whereas here, since only first-order field gradients are retained, there is a quadrupole contribution only in the rotational equation. (2) Here, elementary particles without quadrupole moments are considered and the quadrupole moment of the composite particle is an aspect of the charge distribution of the system. On the other hand, Havas allows for particles possessing intrinsic quadrupole moments of different types, not necessarily limits of distributions such as those considered here, and thus he is led to several alternative sets of equations of motion.

ACKNOWLEDGMENTS

The author is grateful to Dr. Gabriel Barton for very valuable discussions and to Professor J. Robert Oppenheimer for the hospitality of the Institute for Advanced Study.

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