Effect of π-K Interaction on Nucleon Form Factors; Processes $\pi\pi \rightarrow KK$ and $\pi\pi \rightarrow \pi\pi$*

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π-K coupled solutions are applied to evaluate the relative contributions of $2K$ and 2π intermediate states to the isovector part of the nucleon's electromagnetic form factor. Spin and unphysical branch cuts on the left are neglected. It is assumed that the K and nucleon N are coupled indirectly through the K-π and π-N couplings only. Approximate integrations are carried out for different limiting cases. Except in the case when π-π self-coupling and π-K coupling are both weak and the results do not show a definite trend, all other cases show that the $2K$-state contribution to the nucleon vector form factor is small. Maximum cross sections and resonance energies for the processes $\pi\pi \rightarrow \pi\pi$ and $\pi\pi \rightarrow K\bar{K}$ are also evaluated.

I. INTRODUCTION

THE assumption of a resonance in the π-π scattering channel in which angular momentum and isospin are equal to unity enhances the pion form factor and makes it possible to bring the theoretical dispersion calculation of the isovector part of the nucleon's magnetic form factor into agreement with experiment.[1] The detailed calculations so far have taken only the lowest mass 2π intermediate state into account in evaluating the absorptive amplitude. It is of interest to study how important the corrections are to such an analysis when heavier mass intermediate states are included in the calculation of the absorptive amplitude. In particular, we may ask how their presence affects the parameters assigned to the π-π resonance in order to bring the calculated nucleon form factor into agreement with experiment.

Owing to the enormous difficulties in calculating many-particle intermediate states,[2] we confine our discussion to two-particle states, and in particular to the $K\bar{K}$ state.[3] In this paper we study the relative contributions of the $2K$ to 2π intermediate states in the vector form factor under various assumptions about the π-K interaction. Because of mutual interaction between π, K, and N, this is indeed a three-channel coupled problem. A general form of solution for the coupled-channel problem which satisfies unitarity and analyticity requirements in the physical region, but ignores crossing symmetry and the intrinsic spin of the particles, has been obtained by Bjorken.[4] We make an approximation in applying his results in such a way that the K-N scattering amplitude $\langle KK|NN\rangle$ exists only through a π-N coupling $\langle\pi\pi|NN\rangle$. This is shown graphically in Fig. 1.

The related problem of two coupled channels was studied recently by Baker and Zachariasen,[5] who included the two intermediate π-π and $N\bar{N}$ channels in computing the nucleon structure. With certain simplifying assumptions in the two-particle scattering amplitudes they obtained an exact solution which showed the contribution of the heavier $N\bar{N}$ intermediate state to be small compared with that of the 2π state. A similar result is obtained here for the three-channel coupled problem using the Bjorken solution for more general scattering amplitudes. Except in the case when π-π self-coupling and π-K coupling are both weak and the results do not show a definite trend, all cases show that the $2K$-state contribution to the nucleon vector form factor is small. Maximum cross sections and resonance energies for the processes $\pi\pi \rightarrow \pi\pi$ and $\pi\pi \rightarrow K\bar{K}$ are also evaluated. In order to simplify the calculations as much as possible, we suppress nucleon spin throughout and perform approximate integrations for various limiting cases.

II. CALCULATIONS

We define the partial wave amplitude $T_{ij}^{(l)}(\omega)$ in the center-of-mass system for spinless particles by

$$S_{ij}=\delta_{ij}+4\pi i\left(\frac{\rho_i\rho_j}{\omega^2 q_i q_j}\right)^{\frac{1}{2}}(2\pi)^4\delta^4(P_i-P_j)\times\sum\nolimits_l(2l+1)T_{ij}^{(l)}(\omega)P_l(\cos\theta),\quad(1)$$

where ω is the center-of-mass energy, P_i and P_j are the total final and initial four-momenta, q_i and q_j are

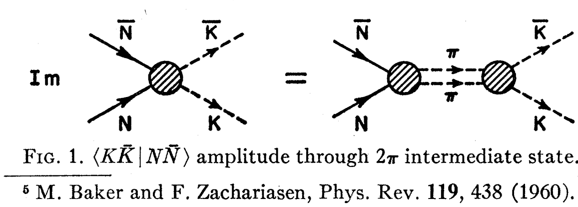

FIG. 1. $\langle K\bar{K}|N\bar{N}\rangle$ amplitude through 2π intermediate state.

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[1] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958); W. R. Frazer and J. R. Fulco, *ibid.* **117**, 1609 (1960). For a complete reference see S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, New York, 1960).

[2] R. Blankenbecler and J. Tarski (to be published).

[3] The $K\bar{K}$ state can form either $I=0$ or $I=1$ (I being total isospin), and therefore contributes both to the scalar and vector nucleon form factors.

[4] J. D. Bjorken, Phys. Rev. Letters **4**, 473 (1960); J. D. Bjorken and M. Nauenberg, Phys. Rev. **121**, 1250 (1961).

[5] M. Baker and F. Zachariasen, Phys. Rev. **119**, 438 (1960).

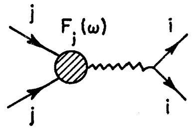

FIG. 2. One-photon process of T_{ij}.

the magnitudes of three-momenta in the center-of-mass system, θ is the scattering angle, and $\rho_{i,j} \equiv q_{i,j}^3/\Omega(\omega)$ is a phase-space factor $[\Omega(\omega)$ arbitrary$]$ which vanishes below the relevant threshold. We normalize to one particle per unit volume. The unitarity condition gives

$$\mathrm{Im}T_{ij}^{(l)}(\omega)=\sum_k \rho_k T_{ik}^{(l)*}(\omega)T_{kj}^{(l)}(\omega). \tag{2}$$

T_{ij} is a general matrix element. For our application we consider that the initial state j consists of two identical particles of the jth kind, and the final state i consists of two identical particles of the ith kind. By considering the i particles to be structureless with unit form factors, and the j and k particles to have form factors $F_j(\omega)$ and $F_k(\omega)$, respectively, as shown in Fig. 2, Eq. (2) gives

$$\mathrm{Im}F_j(\omega)=\sum_k \rho_k F_k^*(\omega)T_{kj}(\omega). \tag{3}$$

The solution of the coupled Eqs. (2) and (3) has been found by Bjorken:

$$\begin{aligned} T_{ij}=T_{ji}&=\sum_k r_{ik}(\omega)D_{jk}/D,\\ F_j(\omega)&=\sum_k f_k D_{jk}/D, \end{aligned} \tag{4}$$

where

$$D=\det d_{ij},$$

$$d_{ij}=\delta_{ij}-\frac{q_i^2}{\pi}\int_{m_i^2}^{\infty}\frac{d\omega'^2\,\rho_i(\omega'^2)r_{ij}(\omega'^2)}{q_i'^2(\omega'^2-\omega^2-i\epsilon)},$$

$$q_i^2=\omega^2-m_i^2,$$

D_{jk} being the minor: $\delta_{ik}D=\sum_j d_{ij}D_{jk}$. m_i is the rest mass of the i particle, ω is the center-of-mass energy of one particle, $r_{ij}(\omega)$ are arbitrary analytic functions, and f_k are constants. The functions d_{ij} as expressed above are slightly different from those of reference 4 since we represent them here in the complex ω^2 plane and make a subtraction so that the integrals are more convergent for large ω^2.

As we are interested in the effect of π-K interaction, we apply the solution expressed in Eq. (4) to the π-K channels only. The calculation of the nucleon form factor proceeds as follows: We denote the π, K, N channels by indices 1, 2, 3, respectively, and the rest mass of the ith particle by m_i. The nucleon form factor F_3 is given by the dispersion integral,

$$F_3(\omega^2)=\frac{1}{\pi}\int_{m_1^2}^{\infty}\frac{\mathrm{Im}F_3(\omega'^2)}{\omega'^2-\omega^2-i\epsilon}\,d\omega'^2. \tag{5}$$

Equation (2) gives $\mathrm{Im}F_3$:

$$\mathrm{Im}F_3=\rho_1F_1^*T_{13}+\rho_2F_2^*T_{23}, \tag{6}$$

in which F_1 and F_2 are the solutions of Eq. (4). To find T_{13} and T_{23}, we write T_{23} from Eq. (2)

$$\mathrm{Im}T_{23}=\rho_1T_{21}^*T_{13},\quad \text{for}\quad \omega>m_1, \tag{7}$$

where we have retained only the 2π intermediate state in the sum. Below the $2K$ threshold this is the only term present in the approximation made here which neglects states with more than two particles. Since $\mathrm{Im}T_{23}$ is real, T_{21} and T_{13} must have the same phase:

$$T_{13}(\omega)=\eta(\omega)T_{21}(\omega), \tag{8}$$

where $\eta(\omega)$ is real between the 2π and $2K$ thresholds. In the range considered, we approximate $\eta(\omega)$ by an effective real constant η. This would imply that the ratio T_{13}/F_1 is constant when π-K coupling is very small. Incidentally, this result agrees with that of Bowcock, Cottingham, and Lurié.[6] In our approximation, the left unphysical branch cut of $T_{23}(\omega^2)$ will be neglected. As we consider K-N being coupled through K-π and π-N, the right-hand branch cut extends from the 2π threshold to infinity:

$$T_{23}(\omega^2)=\frac{1}{\pi}\int_{m_1^2}^{\infty}\frac{\mathrm{Im}T_{23}(\omega'^2)d\omega'^2}{\omega'^2-\omega^2-i\epsilon}. \tag{9}$$

Equations (5)–(9) give the nucleon form factors in terms of the π-K coupled solution (4) for which we shall obtain an explicit expression by a proper choice of the functions $r_{ij}(\omega)$ and $\Omega(\omega)$. It is known in potential scattering or, in our case, the π-π scattering without π-K interaction, the phase shift S_l of the lth partial wave is proportional to q^{2l+1} $(l\geq 1)$ for small q. In our case, the π-π scattering is p wave, so $\delta\sim q^3$. It is seen from Eq. (4) that in the absence of a π-K interaction, $r_{12}=r_{21}=0$, the phase shift is just the phase angle of the complex conjugate of d_{11}. For small q, then

$$\delta=\rho_1 r_{11}=[q^3{}_1/\Omega(\omega)]r_{11}(\omega).$$

Hence the phase condition is automatically satisfied if r_{11}/Ω approaches a finite constant as $q_1\rightarrow 0$. To find the form of $\Omega(\omega)$, we arbitrarily confine ourselves so that in the absence of π-K interaction, our pion form factor is equivalent to that of Frazer and Fulco, though the results of Bowcock *et al.* might be more favorable experimentally. We shall see that the simple choice,

$$\Omega(\omega)=\omega, \tag{10}$$

is sufficient. The functions $r_{ij}(\omega)$ remain undetermined except for their analytical behavior in the solution of Bjorken. We shall approximate them by constants within the range significant to the dispersion integrals:

$$r_{ij}(\omega)=\lambda_{ij}/m_i^2, \tag{11}$$

λ_{ij} being dimensionless constants. For simplicity we

[6] J. Bowcock, W. N. Cottingham, and D. Lurié, Nuovo cimento **16**, 918 (1960); Phys. Rev. Letters **5**, 386 (1960).

take $\lambda_{22}=0$. r_{ii} as proportional to the phase shift of the cross section σ_{ii} when r_{ii} is small and the coupling to other particles is neglected. Therefore, $r_{22}=0$ corresponds to neglect of the direct K-K interaction, and the $KK\rightarrow KK$ cross section is the indirect result of the π-K interaction. With the choice of Eqs. (10) and (11), the integral for d_{ij} is logarithmically divergent and we must introduce a second parameter, cutting off the ∞ upper limit in Eq. (4) at Λ^2. This replaces the neglected contributions to the distant part of the branch cut from the heavy-mass states.

It is convenient to transform the variable ω^2 to dimensionless form by introducing variables $x_i=\omega^2/m_i^2$, $i=1$, 2. With Eqs. (10) and (11) and a cutoff Λ, we then find from Eq. (4)

$$\begin{aligned} d_{ij}(x_i)&=\delta_{ij}-\lambda_{ij}(x_i-1)I_i(x_i),\\ I_i(x_i)&=g_i(x_i)+i(1-1/x_i)^{\frac{1}{2}} \quad \text{for} \quad x_i>1\\ &=g_i(x_i) \quad \text{for} \quad 0<x_i<1, \end{aligned} \tag{12}$$

where $g_i(x_i)$ is real, and is given by

$$\begin{aligned} g_i(x_i)&=\frac{2}{\pi}\ln\frac{2\Lambda}{m_i}-\frac{1}{\pi}\left(1-\frac{1}{x_i}\right)^{\frac{1}{2}}\ln\frac{1+(1-x_i^{-1})^{\frac{1}{2}}}{1-(1-x_i^{-1})^{\frac{1}{2}}}, \quad x_i>1\\ &=\frac{2}{\pi}\ln\frac{2\Lambda}{m_i}-\frac{2}{\pi}(x_i^{-1}-1)^{\frac{1}{2}}\cot^{-1}(x_i^{-1}-1)^{\frac{1}{2}}, \quad 0<x_i<1. \end{aligned} \tag{13}$$

T_{ij} is also found from Eq. (4), by making use of the symmetry of T_{ij} ($T_{ij}=T_{ji}$),[4]

$$\begin{aligned} T_{11}&=[\lambda_1+\Delta^2(x_2-1)I_2(x_2)]/m_1^2D,\\ T_{22}&=\Delta^2(x_1-1)I_1(x_1)/m_2^2D,\\ T_{12}&=T_{21}=\Delta/m_1m_2D,\\ D&=1-(x_1-1)I_1(x_1)[\lambda_1+\Delta^2(x_2-1)I_2(x_2)],\\ \lambda_1&\equiv\lambda_{11}, \quad \Delta^2\equiv\lambda_{12}\lambda_{21}, \quad \lambda_{12}/m_1^2=\lambda_{21}/m_2^2. \end{aligned} \tag{14}$$

The two constants f_k in Eq. (4) of the form factors $F_k(\omega)$ are uniquely determined from the two equations of the normalization condition $F_k(0)=1$. We find

$$f_1=1+\lambda_1I_1(0)+\lambda_{21}I_2(0),$$
$$f_2=1+\lambda_{12}I_1(0),$$

and

$$F_i(x_1)=N_i(x_i)/D,$$

where

$$\begin{aligned} N_1&=1+\lambda_1+r\Delta\Lambda_2-(r\Delta+\Delta^2\Lambda_1)(1-x_2)I_2(x_2),\\ N_2&=1+\Delta\Lambda_1/r+(\Delta/r+\Delta^2\Lambda_2-\lambda_1)(x_1-1)I_1(x_1),\\ r&\equiv m_2/m_1=3.54,\\ \Lambda_i&\equiv(2/\pi)(\ln 2\Lambda/m_i-1). \end{aligned} \tag{15}$$

D is the same as in Eq. (14).

The case $r_{11}=\lambda_{11}'/(\omega^2+\omega_0^2)$, $\lambda_{12}=\lambda_{21}=\lambda_{22}=0$ corresponds to the pion form factor of Frazer and Fulco. Their best choice $\Gamma=0.4$, $\nu_r=1.5$ corresponds to $\lambda_{11}'=167$, $\omega_0^2=625m_1^2$. However, if we choose $r_{11}=\lambda_{11}'/\omega_0^2=0.267/m^2$, and a cutoff $\Lambda=25m_1$, we find that the pion form factor is the same except for a factor $(625+\omega^2/m_1^2)/(625-1)$. This factor is essentially unity for the range of ω^2 of interest in the physical region.

The dispersion integral for the nucleon form factors Eqs. (5) to (9) cannot be integrated explicitly in terms of elementary functions of the parameters λ_{ij} and the cutoff Λ. Approximate integration in three limiting cases is carried out according to (I) $\Delta^2/\lambda_1\gg1$, (II) $\Delta^2/\lambda_1\ll1$, and (III) $\Delta^2/\lambda_1\approx1$. Consistent with the results of references 1 and 4, we assume that the parameters λ_1, Δ, and Λ are such that there is a strong π-π resonance between the 2π and $2K$ thresholds originating from the vanishing of the real part of D. The functions $|1/D|^2$ and $(q_i^3/\omega)|1/D|^2$ are then represented by δ functions in approximate integrations. This approximation will be good if the resonance is very strong or the rest of the integrand is sufficiently smooth.

The resonance condition is $\mathrm{Re}D=0$, or

$$1-(x_1-1)g_1(x_1)[\lambda_1-\Delta^2(1-x_2)g_2(x_2)]=0.$$

If the resonance is to occur below the $2K$ threshold as we have assumed, $x_2<1$, and g_1 and g_2 are positive. λ_1 should always be positive irrespective of the sign of Δ in order that the above equation be satisfied. Indeed, λ_1 must satisfy

$$\lambda_1>\Delta^2(1-x_{2r})g_2(x_{2r}), \tag{16}$$

where x_{2r} is the resonance position. Positive λ_1 indicates that π-π interaction potential is attractive. $g_1(x_1)$ and $g_2(x_2)$ are slowly varying functions of x_1 and x_2. For $\Lambda=25m_1$, g_1 decreases from 2.5 at $x_1=0$ to 0 at cutoff; g_2 increases from 1 at $x_2=0$ to 1.7 at $x_2=1$ and then decreases to 0 at cutoff. They are nearly constant within the significant interval of integration. This is helpful in approximate integration. g_1, g_2, Λ_1, and Λ_2 contain Λ as logarithmic functions, and are insensitive to the cutoff. However, the magnitude of the resonance peak, which is proportional to $|1/D(x_{1r})|$, is sensitive to Λ (x_{1r} is the resonance position). This occurs because at resonance

$$\left|\frac{1}{D(x_{1r})}\right|^2=\left|\frac{1}{\mathrm{Im}D(x_{1r})}\right|=\left|\frac{\mathrm{Re}I_1(x_{1r})}{\mathrm{Im}I_1(x_{1r})}\right|=\frac{g_1(x_{1r})}{(1-x_{1r}^{-1})^{\frac{1}{2}}}. \tag{17}$$

For x_{1r} values not too close to the 2π threshold a large enhancement requires a large $g_1(x_{1r})$, which can be achieved only for large Λ^2. In the following calculation, the cutoff is taken to be $25m_1$ when the result is not sensitive to Λ but some numerical value is needed; otherwise Λ is arbitrary.

III. 2K AND 2π CONTRIBUTIONS TO NUCLEON FORM FACTORS

A. $\Delta^2/\lambda_1 \gg 1$

The π-K coupling is much larger than π-π coupling. From Eqs. (5) and (6) it is seen that F_3 consists of two terms. They are the contributions from the 2π and $2K$ intermediate states, respectively, and will be denoted by F_{31} and F_{32}. Putting Eq. (8) and the first term of Eq. (6) into (5), we get

$$F_{31}(x_1)=\frac{\eta\Delta}{\pi r}\int_1^\infty \frac{dx_1'}{x_1'-x_1}\left(\frac{(x_1'-1)^3}{x_1'}\right)^{\frac{1}{2}} \frac{N_1(x_1')}{|D(x_1')|^2}, \quad x_1<0.$$

The peak position of $|1/D|^2$ is slightly below the $2K$ threshold, and will be approximated by the δ function

$$\frac{1}{|D(x_2)|^2}\simeq\frac{\pi}{\sqrt{3}r^2\Delta^2 g_1 g_2}\delta\left(x_2-1+\frac{\lambda_1-(2r^2g_1)^{-1}}{\Delta^2 g_2}\right). \quad (18)$$

g_1 and g_2 take the value at $x_2=1$ in the above expression. We then get

$$F_{31}(x_2)=\left|\frac{\eta r N_1}{\sqrt{3}\Delta g_1 g_2}\right|\left(\frac{1}{1-x_2}\right), \quad x_2<0, \quad (19)$$

where N_1 is to be taken at

$$x_2=1-\frac{\lambda_1-(2r^2g_1)^{-1}}{\Delta^2 g_2}.$$

To calculate F_{32}, we first calculate T_{23} from Eqs. (7) and (9). By the same δ function (18), we find the real part of T_{23} [we need not know $T_{23}(x_2)$ for $x_2<1$]:

$$\mathrm{Re}T_{23}(x_2)=\frac{\eta}{\sqrt{3}m_2^2g_1g_2}\bigg|_{x_2=1}\frac{1}{1-[\lambda_1-(2r^2g_1)^{-1}]/\Delta^2g_2-x_2}, \quad x_2\geqslant 1. \quad (20)$$

By substituting the second term of Eq. (6) into Eq. (5),

$$F_{32}(x_2)=\frac{m_2^2}{\pi}\int_1^\infty \frac{dx_2'}{x_2'-x_2}\left(\frac{(x_2'-1)^3}{x_2'}\right)^{\frac{1}{2}} F_2^*(x_2')T_{23}(x_2'), \quad x_2<0. \quad (21)$$

When T_{23} $(=\mathrm{Re}T_{23}+i\,\mathrm{Im}T_{23})$ is put into the above expression, we find that F_{32} fails to be real for $x_2<0$. F_{31} also has complex quantities originating from the integration above the $2K$ threshold in the dispersion integral.

For an exact theory the imaginary parts of F_{32} and F_{31} should cancel each other so that the total form factor F_3 $(=F_{31}+F_{32})$ is still real for $x_1<0$. In our calculation $F_{31}+F_{32}$ fails to be real. This arises from our approximation, in that we have not applied the complete three-channel π-K, K-N, N-π coupled solution. Owing to the large resonance below $2K$ threshold, the integration above $2K$ threshold which gives rise to the imaginary part of F_{31} is small. Therefore, in our approximation only the real parts of F_{31} and F_{32} are to be considered. The phase of $F_2(x_2)$ in the integral (21) is about 180° within the significant region of integration, and we approximate $F_2^*(x_2)$ by $-|F_2(x_2)|$. As we are integrating from above the $2K$ threshold, $1/(|D|^2)$ cannot be approximated by the δ function (18) which peaks below the $2K$ threshold. So we approximate

$$|D|^2\approx L_1^2x_1^2[\lambda+\Delta'^2(x_2-1)]^2, \quad x_2\geqslant 1,$$
$$\lambda\equiv\lambda_1-1/L_1^2r^2, \quad \Delta'^2\equiv\Delta^2 g_2, \quad L_1\equiv(1+g_1^2)^{\frac{1}{2}}. \quad (22)$$

A term $(x_2-1)^3/x_2$ is neglected compared with $(x_2-1)^2$ since the most important contributions come from x_2 values near 1. Equation (20) can also be expressed as

$$\mathrm{Re}T_{23}\simeq-\left|\frac{\eta\Delta^2}{\sqrt{3}m_2^2g_1}\right|_{x_2=1}\frac{x_1L_1}{|D|}, \quad x_2>1,$$

so

$$F_{32}=\left|\frac{\eta\Delta^2}{\sqrt{3}\pi r^2 g_1}\right|_{x_2=1}\times\int_1^{(\Lambda/m_2)^2}\frac{dx_2'}{x_2'-x_2}\frac{|N_2|}{L_1x_2'^{\frac{3}{2}}}\frac{(x_2'-1)^{\frac{3}{2}}}{[\lambda+\Delta'^2(x_2-1)]^2}.$$

As before, we represent the last factor by a δ function and find

$$F_{32}\simeq\frac{2\eta}{\sqrt{3}\pi r^2\Delta^4}\left(\frac{\Lambda}{m_2}\right)\left|\frac{N_2}{g_1g_2^2L_1}\right|_{x_2=1}\frac{1}{1-x_2}. \quad (23)$$

Combined with (19), this gives

$$\frac{F_{32}}{F_{31}}\simeq\frac{2\sqrt{2}}{\pi g_2 L_1}\bigg|_{x_2=1}\left(\frac{\Lambda}{m_2}\right)\frac{1}{\Delta^2}\simeq\frac{\Lambda}{\pi m_2\Delta^2}, \quad (24)$$

where we have substituted $|N_2/N_1|_{x_2=1}\simeq\sqrt{2}r\Delta$ for large Δ. Therefore, in our finite cutoff theory, $\Lambda/m_2=7$, for instance, the $2K$ contribution is negligible for large Δ^2.

B. $\Delta^2/\lambda_1 \ll 1$

The π-K coupling is much smaller than the π-π coupling. The resonance position is primarily determined by λ_1, and can be at any position between 2π and $2K$ thresholds. The calculation is the same as before; we write down only the results:

$$F_{31}=\frac{\eta\Delta(1+\lambda_1\Lambda_1)A_1'}{\pi r}\frac{1}{x_{1r}'-x_1}, \quad x_1<0,$$

where A_1' and x_{1r}' are the parameters in δ-function representation:

$$\left(\frac{(x_1-1)^3}{x_1}\right)^{\frac{1}{2}} \frac{1}{|D|^2} \simeq A_1'\delta(x_1-x_{1r}').$$

$$F_{32} = -\frac{6.4\,\eta\Delta^2}{(\pi\lambda_1 r)^2}\left(\frac{1}{2-x_2}\right).$$

$$\frac{F_{32}}{F_{31}} = -\frac{2\Delta}{\lambda_1^2(1+\lambda_1\Lambda_1)A_1'}\left(\frac{x_{1r}'/r^2-x_2}{2-x_2}\right), \quad x_2<0. \tag{25}$$

A_1' is primarily determined by λ_1, and is finite for vanishing Δ. Therefore, the above ratio is small for small Δ. This is also obvious from physical considerations because we have assumed that K-N coupling is through K-π coupling only.

C. $\Delta^2/\lambda_1 \approx 1$

The π-K and π-π coupling are nearly equal. The calculations are the same as before. By approximating

$$\left(\frac{(x_1-1)^3}{x_1}\right)^{\frac{1}{2}} \frac{1}{|D|^2} \simeq A_1\delta(x_1-x_{1r}),$$
$$\left(1-\frac{1}{x_2}\right)^{\frac{3}{2}} \frac{1}{1+(x_2-1)g_2} \simeq A_2\delta(x_2-x_{2r}), \tag{26}$$

we find

$$F_{31} \cong \frac{\eta\Delta A_1}{\pi r}\frac{N_1(x_{1r})}{x_{1r}-x_1}, \quad x_1<0$$
$$F_{32} \cong \frac{\eta A_1 A_2}{(\pi L_1 r)^2}\frac{|N_2(x_{2r})|}{r^2 x_{2r}-x_{1r}}\frac{1}{x_{2r}-x_2}, \quad x_2<0. \tag{27}$$

A_2 and x_{2r} are found to be approximately 2.5 and 2.2, respectively. However, x_{1r} depends on λ_1 and is found for five numerical cases. These values are tabulated below, together with the ratio F_{32}/F_{31}.

λ_1	0.06	0.2	1	10	∞
x_{1r}	12.6	6.7	3.5	1.6	1
$\dfrac{F_{32}(x_1)}{F_{31}(x_1)}$	$0.53\dfrac{12.6-x_1}{28-x_1}$	$0.6\dfrac{6.7-x_1}{28-x_1}$	$0.35\dfrac{3.5-x_1}{28-x_1}$	$0.15\dfrac{1.6-x_1}{28-x_1}$	0

The case $\lambda_1=1/17$ represents the smallest possible value of λ_1 for a resonance below the $2K$ threshold. In the case $\lambda_1=\infty$, we have taken Λ higher than $25m_1$, so that Λ_2-1 is positive and appreciably different from zero. Within the range of experiment, $0>x_1>-10$, the ratio F_{32}/F_{31} varies between 0 and 0.31 for different cases.

IV. $\pi\pi \to K\bar{K}$ AND $\pi\pi \to \pi\pi$ TOTAL CROSS SECTIONS

The total p-wave cross section σ_{ij} of two i particles produced from two j particles is obtained from Eq. (1):

$$\sigma_{ij} = 12\pi(q_i^3 q_j/\omega^2)|T_{ij}|^2.$$

The evaluation of $(\sigma_{ij})_{\max}$ and its peak position is straightforward, and is carried out approximately for the three limiting cases discussed above.

A. $\Delta^2/\lambda_1 \gg 1$

$$(\sigma_{21})_{\max} \simeq 27/\Delta(\lambda_1-0.04)^{\frac{1}{2}}\ \text{mb}$$

at

$$(x_2)_{\max} \simeq 1+1.7(\lambda_1-0.04)^{\frac{1}{2}}/\Delta^2,$$

$$(\sigma_{11})_{\max} \simeq 60\ \text{mb} \quad \text{at} \quad (x_2)_{\max} \simeq 1-(\lambda_1-0.04)^{\frac{1}{2}}/1.7\Delta^2.$$

σ_{11} drops to zero for x_2 slightly smaller than $(x_2)_{\max}$ and remains constant at 30 mb up to the 2π threshold.

B. $\Delta^2/\lambda_1 \ll 1$

$$(\sigma_{21})_{\max} \simeq 4.5(\Delta/\lambda_1)^2\ \text{mb} \quad \text{at} \quad (x_2)_{\max} \simeq 2.5,$$
$$(\sigma_{11})_{\max} \simeq 750\lambda_1\ \text{mb} \quad \text{at} \quad (x_1)_{\max} \simeq 1+(1/2\lambda_1).$$

C. $\Delta^2/\lambda_1 \approx 1$

Numerical evaluation for three different λ_1 gives the following results:

λ_1	0.2	1	10
$(\sigma_{21})_{\max}$(mb)	0.47	0.084	0.0082
$(x_2)_{\max}$	1.4	1.5	1.55
$(\sigma_{11})_{\max}$(mb)	120.0	430.0	300.0
$(x_1)_{\max}$	7.0	4.0	2.5

V. CONCLUSIONS AND DISCUSSION

We have seen that for either a very large or a small π-K coupling parameter Δ, the $2K$-state contribution to the nucleon form factor is small compared with that of the 2π state. When λ_1 and Δ^2 are nearly equal the $2K$ contribution is still small for large λ_1. As λ_1 decreases F_{32} becomes appreciable, especially at high momentum transfer (large $|x_1|$). The highest ratio we obtained for small λ_1 and $x_1=-10$ was $F_{32}/F_{31}=0.31$. This is unfortunately neither large enough nor small enough to show a definite trend. An accurate numerical integration might shift the number in either direction. The $\pi\pi\to K\bar{K}$ total cross section is small, of the order 1/2 mb or less, in all cases.

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