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# Recoil Model in Quantum Field Theory

FRANZ SCHNEIDER
*University of Vienna, Vienna, Austria*
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The static-source model is generalized to allow for the recoils and the mutual degrees of freedom of the "source" particles. This is done by adding to the Hamiltonian corresponding to the field and its sources the nonrelativistic Hamiltonian for the motion of the heavier particles. The resulting system can be quantized in a familiar way. The scattering of mesons by nucleons and deuterons is calculated in this picture with the interaction of the pair theory in first approximation. The description of the deuteron is effected by a system of functions containing one bound state only.

## 1. INTRODUCTION

THE well-known static-source model is conveniently used for the description of systems consisting of particles of different mass. Representing the light particles by the corresponding free-field Hamiltonian, the heavier ones are considered as stationary potential distributions of finite extension acting as the sources of the field. This treatment is unsatisfactory in two respects: It does not allow for (1) the recoils of the heavy particles and (2) any forces acting between them. Neglecting creation and annihilation processes for the heavy particles seems a restriction far less important, especially at medium or low kinetic energies. It should therefore be possible to describe their motion by a nonrelativistic operator added to the Hamiltonian of the static-source model.

## 2. QUANTIZATION OF THE RECOIL MODEL

The system under consideration shall consist of two parts: an indefinite number of light particles—say, for example, mesons—and a definite number $N$ of relatively heavy particles, e.g., nucleons. Suppressing any interactions for the moment, we apply the Hamiltonian

$$H=H_F+H_N,$$

where

$$H_F=\int d\mathbf{r}:\Pi^2(\mathbf{r})+[\nabla\Phi(\mathbf{r})]^2+m^2\Phi^2(\mathbf{r}): \tag{1}$$

is the energy of the free field and

$$H_N=\sum_{j=1}^{N}\mathbf{p}_j^2/2m_j \tag{2}$$

is the nonrelativistic energy of the nucleons in Cartesian coordinates. The symbol : : has no meaning when applied to $c$ numbers as above, but indicates the normal product whenever operators are involved, to cancel the vacuum energy. The quantization can be carried out, representing the canonically conjugate variables by operators and postulating the usual commutators,

$$[q_t,p_s]=i\delta_{t,s},\quad t,\,s=1\cdots 3N,$$
$$[\Phi(\mathbf{r}),\Pi(\mathbf{r}')]=i\delta(\mathbf{r}-\mathbf{r}').$$

The states of the system form a Hilbert space in which the operators are acting. In the usual representation,

$$\mathbf{p}_j=-i\nabla_j,$$
$$\Phi(\mathbf{r})=\textstyle\sum_{\mathbf{k}}(2V\omega_k)^{-\frac{1}{2}}(a_{\mathbf{k}}+a_{-\mathbf{k}}^\dagger)e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{3}$$
$$[a_{\mathbf{k}},a_{\mathbf{l}}^\dagger]=\delta_{\mathbf{k},\mathbf{l}},$$

with the corresponding expression for $\Pi(\mathbf{r})$, we have

$$H=-\textstyle\sum_j\Delta_j/2m_j+\sum_{\mathbf{k}}\omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}}.$$

The eigenstates $|E\rangle$ are obtained from the equation

$$H\,|E\rangle=E\,|E\rangle, \tag{4}$$

where $E$ characterizes completely the quantum states of both mesons and nucleons. Since there is no interaction, the solutions of (4) can be built up from the momentum eigenfunctions of the nucleons in configuration space and the eigenfunctions of $H_F$ in the many-particle representation:

$$|E\rangle=\Psi(\mathbf{r}_j)|n_{\mathbf{p}}\rangle=V^{-N/2}e^{i(\mathbf{p}_1\cdot\mathbf{r}_1+\cdots+\mathbf{p}_N\cdot\mathbf{r}_N)}\,|n_{\mathbf{p}}\rangle, \tag{5}$$
$$H_F\,|n_{\mathbf{p}}\rangle=\textstyle\sum_{\mathbf{p}}\omega_p n_{\mathbf{p}}\,|n_{\mathbf{p}}\rangle. \tag{6}$$

The most general-state vectors in the Schrödinger picture are obtained by superposition:

$$|A_s,t\rangle=\textstyle\sum_E C_E(t)\,|E\rangle; \tag{7}$$

they are solutions of the equation

$$i(\partial/\partial t)|A_s,t\rangle=H|A_s,t\rangle. \tag{8}$$

The expectation value of any operator $O$ is

$$o=\langle A_s|O|A_s\rangle,$$

where integration over $d\mathbf{r}_1\cdots d\mathbf{r}_N$ is implied and

$$\langle A_s|A_s\rangle=1.$$

All operators can be built up from the corresponding operators for the nucleons and for the meson field. For example,

$$\mathbf{P}=-\sum_{j=1}^{N}i\nabla_j+\sum_{\mathbf{k}}\mathbf{k}a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$$

is the operator for the total momentum of the system.

We are now going to consider interactions. Let $V_{jl}=V(\mathbf{r}_j-\mathbf{r}_l)$ be the potential of the forces between

the nucleons $j$ and $l$ and $W(\mathbf{r}_j)$ the interaction between the nucleon $j$ and the meson field. The total Hamiltonian of the system is now

$$H=H_F+H_N+\tfrac{1}{2}\sum_{j\neq l} V_{jl}+\sum_j W(\mathbf{r}_j)=H_0+H', \quad (9)$$

where

$$H'=\sum_j W(\mathbf{r}_j). \quad (10)$$

The quantization can be carried out in the same way as for the free particles, postulating commutators for canonically conjugate variables. However, the spectrum of eigenfunctions will be changed by the interaction, their exact evaluation being impossible in general.

Defining $H'$ in the above way, including the interaction between the nucleons and the field only, should be advantageous for the following reasons: The eigenfunctions of $H_0$ can again be separated with

$$|E_0\rangle=\Psi_0(\mathbf{r}_1\cdots\mathbf{r}_N)\ |n_\mathbf{p}\rangle, \quad (11)$$

where the second factor on the right is known and we are left only with the calculation of the eigenfunctions $\Psi_0$ of $H_N+\frac{1}{2}\sum V_{jl}$ to any approximation. This problem being solved, $H'$ may be dealt with by perturbation theory. This procedure can be justified since, on account of the mass difference, the nucleons and the corresponding functions $\Psi_0$ should not be disturbed by the interaction as much as the mesons. Therefore, the $\Psi_0$, being evaluated only approximately, cannot be too much disturbed by $H'$, the total error being less than when starting from the functions (5) known exactly. The calculation of scattering cross sections will be carried out in the interaction picture. The transition from the Schrödinger picture can be effected by the unitary transformation

$$|A_I,t\rangle=e^{iH_0t}\ |A_s,t\rangle$$

for the state vectors. Defining the scattering operator $S$ by[1]

$$|A_I,+\infty\rangle=S\ |A_I,-\infty\rangle,$$

we have the matrix element

$$M_{a,a'}=\langle E_{a'}|S|E_a\rangle \quad (12)$$

for the transition from the initial state $|E_a\rangle$ to the final state $|E_{a'}\rangle$. The scattering operator is

$$S_n=\sum_{l=0}^{n} S^{(l)}$$

to the $n$th approximation, where

$$S^{(n)}=\frac{(-i)^n}{n!}\int_{-\infty}^{\infty} dt_1\cdots dt_N\ T\{H'(t_1)\cdots H'(t_n)\}, \quad (13)$$

and

$$H'(t)=e^{-\epsilon|t|}e^{iH_0t}H'e^{-iH_0t}.$$

$T$ means the time-ordered product. As $\epsilon$ is always considered to be vanishingly small, we have, for $t\rightarrow\pm\infty$, $H'(\pm\infty)=0$ and $H=H_0$, and the initial and final states can be composed of eigenstates of $H_0$, e.g.,

$$|A_I,+\infty\rangle=\sum_{E_0} C_{E_0}|E_0\rangle.$$

To get the scattering probability, the initial state must be chosen appropriately. All possible final states of the nucleons and the corresponding meson states should then be summed over. In our calculations, however, we shall suppress for simplicity states with more than one outgoing meson. Denoting outgoing states by the prime, we have $\mathbf{p}_j$ and $\mathbf{p}_j'$ for the momenta of the nucleon $j$ and $\mathbf{p}$, $\mathbf{p}'$ for the meson, respectively. Remembering that

$$\sum_{\mathbf{p}_j}=(2\pi)^{-3}V\int d\mathbf{p}_j,$$

we get the scattering probability,

$$W=V^{N+1}(2\pi)^{-3N-3}\int d\mathbf{p}_1'\cdots d\mathbf{p}_N'd\mathbf{p}'\,|M_{a,a'}|^2\Big/\int dtd\mathbf{r}$$

and the cross section,

$$d\sigma/d\Omega=WV^2\omega_p/pd\Omega. \quad (14)$$

### 3. SCATTERING OF MESONS BY NUCLEONS

In the following, mesons and nucleons shall always interact according to the pair theory,

$$W(\mathbf{r}_j)=\frac{g}{2}\int d\mathbf{r}d\mathbf{r}'\ U_j(\mathbf{r})U_j(\mathbf{r}'):\Phi(\mathbf{r})\Phi(\mathbf{r}'):. \quad (15)$$

Considering the static-source model, we have

$$H=H_F+W(\mathbf{r}_1).$$

The field equation,

$$\left(\frac{\partial^2}{\partial t^2}-\Delta+m^2\right)\Phi(\mathbf{r},t)=-gU(\mathbf{r})\int d\mathbf{r}'\ U(\mathbf{r}')\Phi(\mathbf{r}',t), \quad (16)$$

with the boundary condition

$$\Phi\underset{r\rightarrow\infty}{\longrightarrow} e^{i(\mathbf{p}\cdot\mathbf{r}-\omega_pt)}+r^{-1}e^{ipr}f(\vartheta,t),$$

can be solved exactly to give[2]

$$\Phi(\mathbf{r},t)=e^{-i\omega_pt}\Bigg(e^{i\mathbf{p}\cdot\mathbf{r}}+g\frac{\bar{U}(-\mathbf{p})}{1+g\mathcal{G}}\times\int\frac{d\mathbf{k}\,e^{i\mathbf{k}\cdot\mathbf{r}}\bar{U}(\mathbf{k})}{(2\pi)^3[(p+i\epsilon)^2-k^2]}\Bigg), \quad (17)$$

[1] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950).

[2] E. Henley and W. Thirring, *Elementary Quantum Field Theory* [McGraw-Hill Book Company, Inc., New York (to be published)], p. 139.

where $\bar{U}$ is the Fourier transform, $\omega_p=(m^2+p^2)^{\frac{1}{2}}$,

$$\mathcal{g}=(2\pi)^{-3}\int d\mathbf{k}\,\bar{U}(-\mathbf{k})\bar{U}(\mathbf{k})[k^2-(p+i\epsilon)^2]^{-1}, \quad (18)$$

and lim $\epsilon\to 0$ is to be understood. Defining a vector $\mathbf{p}'$ of magnitude $p$ pointing in the direction of the scattered particle, we get

$$\lim_{r\to\infty}(2\pi)^{-3}\int d\mathbf{k}\,\bar{U}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}[k^2-(p+i\epsilon)^2]^{-1} = \bar{U}(\mathbf{p}')e^{ipr}/4\pi r,$$

and the scattering cross section is[3]

$$\frac{d\sigma}{d\Omega}=\left|\frac{g\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')}{4\pi(1+g\mathcal{g})}\right|^2. \quad (19)$$

Clearly, for a potential with central symmetry $U(r)$, the scattering cross section has central symmetry too, since the Fourier transform of the potential does. With respect to

$$\int d\mathbf{r}\,e^{-i\mathbf{k}\mathbf{r}}U(\mathbf{r}-\mathbf{r}_1)=e^{-i\mathbf{k}\cdot\mathbf{r}_1}\int d\mathbf{r}\,e^{-i\mathbf{k}\cdot\mathbf{r}}U(\mathbf{r}),$$

a displacement $\mathbf{r}_j$ of the nucleon from the origin of coordinates results in a mere phase factor, $e^{-i\mathbf{k}\cdot\mathbf{r}_j}$, and is meaningless for the angular distribution of scattering. At this point, unsymmetric potentials corresponding to a deformation of the nucleon during the collision shall be considered briefly. Let the potential be stretched or compressed continuously in the direction of the incoming particles,

$$U(\mathbf{r},n)=nU((x^2+y^2+n^2z^2)^{\frac{1}{2}}),$$

giving

$$\bar{U}(\mathbf{k},n)=n\int d\mathbf{r}\,U((x^2+y^2+n^2z^2)^{\frac{1}{2}})e^{-i\mathbf{k}\cdot\mathbf{r}}=\bar{U}(\mathbf{k}^*) \quad (20)$$

for the Fourier transform, where

$$\mathbf{k}^*=(k_1,k_2,k_3/n). \quad (21)$$

It is easily seen that $\mathcal{g}$ is still a constant with respect to the scattering angle $\vartheta$, though its value might depend on $n$. Remembering that

$$d\sigma/d\Omega\sim|\bar{U}(\mathbf{p}^{*\prime})|^2,$$

and

$$\mathbf{p}^{*\prime}=p(\sin\vartheta\cos\varphi,\,\sin\vartheta\sin\varphi,\,\cos\vartheta/n), \quad (22)$$

we learn that the scattering cross section is no longer a constant. There is no $p$, $f$, $\cdots$ scattering, however, since by (20) the absolute value of $\bar{U}(\mathbf{k}^*)$ must be independent of the sign of $n$, that is, by (21) and (22) independent of uneven exponentials of $\cos\vartheta$. For $n$ larger (smaller) than unity, the scattering cross section has a minimum (maximum) at $\vartheta=90°$.

[3] J. M. Blatt, Phys. Rev. **72**, 470 (1947).

In the recoil model, the Hamiltonian

$$H=H_F-\Delta/2m_1+W(\mathbf{r}_1)+m_1$$

shall be applied. Using

$$|E_0\rangle=V^{-\frac{1}{2}}e^{-i\mathbf{p}_1\cdot\mathbf{r}_1}|\mathbf{p}\rangle$$

and

$$e^{iH_0t}|E_0\rangle=e^{iEt}|E_0\rangle,$$

we have the first approximation of the matrix element

$$M_{E,E'}{}^{(1)}=\langle E_0'|S^{(1)}|E_0\rangle=\frac{-i}{V}\int d\mathbf{r}_1\,e^{i(\mathbf{p}_1-\mathbf{p}_1')\cdot\mathbf{r}_1}\times\int dt\,e^{-\epsilon|t|}e^{i(E'-E)t}\langle\mathbf{p}'|W(\mathbf{r}_1)|p\rangle.$$

Since

$$\langle\mathbf{p}'|W(\mathbf{r}_1)|\mathbf{p}\rangle=(2V)^{-1}(\omega_p'\omega_p)^{-\frac{1}{2}}\times g\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}_1}, \quad (23)$$

and

$$\int_{-\infty}^{\infty}dt\,e^{-\epsilon|t|}e^{ixt}=2\pi\delta(x),$$

we have

$$M_{E,E'}{}^{(1)}=\frac{-ig\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')}{2V^2(\omega_p\omega_{p'})^{\frac{1}{2}}}\times(2\pi)^4\delta(\mathbf{p}_1+\mathbf{p}-\mathbf{p}_1'-\mathbf{p}')\delta(E'-E).$$

In the laboratory system, the nucleon is initially at rest. Remembering that

$$[2\pi\delta(x)]^2=2\pi\delta(x)\int dx',$$

we get the scattering probability

$$W^{(1)}=\frac{V^2}{(2\pi)^6}\int d\mathbf{p}_1'd\mathbf{p}'\frac{|M_{E,E'}|^2}{\int dtd\mathbf{r}} =\left(\frac{g}{4\pi V}\right)^2\int d\mathbf{p}'\,\frac{\bar{U}^4}{\omega_p\omega_{p'}}\delta(E'-E)\bigg|_{\mathbf{p}_1'=\mathbf{p}-\mathbf{p}'}.$$

Now

$$d\mathbf{p}'=p'^2dp'd\Omega,$$

the integration running over positive values of $p'$ only. The energy delta function can then be transformed:

$$E'-E=(m_1^2+p_1'^2)^{\frac{1}{2}}+(m^2+p'^2)^{\frac{1}{2}}-m_1-(m^2+p^2)^{\frac{1}{2}} \quad (24)$$

has the positive zero

$$p_n=p\frac{[m^2+m_1(m^2+p^2)^{\frac{1}{2}}]\cos\vartheta+E(m_1^2-m^2\sin^2\vartheta)^{\frac{1}{2}}}{E^2-p^2\cos^2\vartheta}\leq p, \quad (25)$$

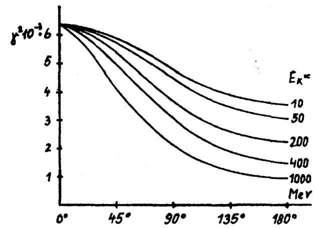


FIG. 1. Differential cross section $d\sigma^{(1)}/d\Omega$ in first approximation. Scattering of mesons by nucleons for various meson kinetic energies. $\gamma$ is the coupling constant.

and we get with

$$\delta(E'-E)=\delta(p'-p_n)/f'(p_n), \tag{26}$$

$$f'(p_n)=p_n(m^2+p_n^2)^{-\frac{1}{2}}+(p_n-p\cos\vartheta)\times(m_1^2+p^2+p_n^2-2pp_n\cos\vartheta)^{-\frac{1}{2}}>0, \tag{27}$$

the scattering cross section in the lab system in first approximation

$$\frac{d\sigma^{(1)}}{d\Omega}=g^2\frac{\bar{U}^2(-\mathbf{p})\bar{U}^2(\mathbf{p}_n)p_n^2}{16\pi^2 p(m^2+p_n^2)^{\frac{1}{2}}f'(p_n)}. \tag{28}$$

Letting $m_1$ go to infinity, $p_n\rightarrow p$ and we get

$$\frac{d\sigma^{(1)}}{d\Omega}\rightarrow\left(\frac{g}{4\pi}\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')\right)^2,$$

which turns out to be just the first approximation of (19). The same is always true for $\vartheta=0°$. For the scattering of slow mesons, $p\ll m$,

$$\frac{d\sigma^{(1)}}{d\Omega}\approx\frac{g^2\bar{U}^4}{16\pi^2}\left(\frac{7+\cos\vartheta}{8}\right)^2,$$

while for high-energy mesons, $p\gg m_1$,

$$\frac{d\sigma^{(1)}}{d\Omega}\approx\frac{g^2\bar{U}^4}{16\pi^2}\frac{m_1}{p}\quad\text{for}\quad\vartheta=90°,$$

and four times less for $\vartheta=180°$. The scattering cross section has been calculated with $\bar{U}=1$, $m_1=939$ Mev, and $m=135$ Mev for different kinetic energies $E_K=\omega_p-m$, the results being compounded in Fig. 1.

#### 4. MESON-DEUTERON SCATTERING

The Hamiltonian is $(m_1=m_2)$

$$H=-(\Delta_1+\Delta_2)/2m_1+V(\mathbf{r}_1-\mathbf{r}_2)+2m_1+H_F+W(\mathbf{r}_1)+W(\mathbf{r}_2). \tag{29}$$

Passing to relative coordinates in the deuteron subspace,

$$\mathbf{x}=\mathbf{r}_1-\mathbf{r}_2,\quad \mathbf{y}=(\mathbf{r}_1+\mathbf{r}_2)/2,$$

we have

$$H_0=-\Delta(\mathbf{x})/m_1-\Delta(\mathbf{y})/4m_1+V(\mathbf{x})+2m_1+H_F, \tag{30}$$

and the equation

$$H_0\,|E_0\rangle=E\,|E_0\rangle$$

can be separated with

$$|E_0\rangle=V^{-\frac{1}{2}}e^{i(\mathbf{p}_1+\mathbf{p}_2)\cdot\mathbf{y}}\varphi(\mathbf{x})\,|\mathbf{p}\rangle,$$

to

$$[\Delta(\mathbf{x})-m_1V(\mathbf{x})+m_1E']\varphi(\mathbf{x})=0, \tag{31}$$

where

$$E'+2m_1=E-(\mathbf{p}_1+\mathbf{p}_2)^2/4m_1-\textstyle\sum_{\mathbf{p}}\omega_p n_{\mathbf{p}}$$

is the relativistic energy of the nucleons in the c.m. system.

We are now going to consider Eq. (16) no longer as an equation for the field operators, but for the wave function $\varphi(\mathbf{x})$ of the deuteron in relative coordinates, $g\rightarrow G$

$$(q^2+\Delta)\varphi(\mathbf{x})=GU(\mathbf{x})\int d\mathbf{r}\,U(\mathbf{r})\varphi(\mathbf{r}). \tag{32}$$

Whether this "ansatz" is permissible or not shall not be discussed, since the determination of deuteron eigenfunctions is not the main purpose of this treatise. Nevertheless, (32) possesses a complete system of exact solutions, containing at most one bound state[4] and remaining everywhere steady. For a Yukawa distribution,

$$U(\mathbf{r})=a^2e^{-ar}/4\pi r,$$

we have

$$\varphi_s(\mathbf{x})\sim e^{i\mathbf{q}\cdot\mathbf{x}}-K(e^{iqx}-e^{-ax})/x, \tag{33}$$

$$K=\frac{Ga^4}{4\pi(a^2+q^2)^2}\left(\frac{Ga^3}{8\pi(a-iq)^2}+1\right)^{-1}, \tag{34}$$

for the scattering states, according to (17). For the bound state, the kinetic energy $E'=q^2/m_1$ is negative, and $q=(m_1E')^{\frac{1}{2}}=ic$, where $c$ is real, and we have from (33)

$$\varphi_b(\mathbf{x})\sim(e^{-cx}-e^{-ax})/x. \tag{35}$$

Inserting this result into (32) gives

$$G=-8\pi(c+a)^2a^{-3}.$$

Since the binding energy of the deuteron is $B=E'=-2.23$ Mev, we get $c=45.5$ Mev. Further, we put $a=7c$.[5] Equating

$$e^{i(\mathbf{p}_1\cdot\mathbf{r}_1+\mathbf{p}_2\cdot\mathbf{r}_2)}=e^{i\mathbf{q}\cdot\mathbf{x}}e^{i\mathbf{P}\cdot\mathbf{y}}$$

with $\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2$ gives $\mathbf{q}=(\mathbf{p}_1-\mathbf{p}_2)/2$ and we get finally for the eigenfunctions of $H_0$

$$|E_b\rangle=\left(\frac{ac(a+c)}{2\pi(a-c)^2V}\right)^{\frac{1}{2}}\frac{e^{-cx}-e^{-ax}}{x}e^{i\mathbf{P}\cdot\mathbf{y}}\,|\mathbf{p}\rangle \tag{36}$$

and

$$|E_s\rangle=\frac{1}{V}\left(e^{i\mathbf{q}\mathbf{x}}-K\frac{e^{iqx}-e^{-ax}}{x}\right)e^{i\mathbf{P}\cdot\mathbf{y}}\,|\mathbf{p}\rangle, \tag{37}$$

respectively.

[4] E. Henley and W. Thirring, reference 2, p. 144.
[5] S. Fernbach, T. A. Green, and K. M. Watson, Phys. Rev. 84, 1084 (1951).

Considering elastic collisions, we get

$$M_{b,b'}{}^{(1)}=\langle E_{b'}|S^{(1)}|E_b\rangle$$
$$=\frac{-iN}{V}\int d\mathbf{r}_1 d\mathbf{r}_2\left(\frac{e^{-c|\mathbf{r}_1-\mathbf{r}_2|}-e^{-a|\mathbf{r}_1-\mathbf{r}_2|}}{|\mathbf{r}_1-\mathbf{r}_2|}\right)^2$$
$$\times e^{i(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')\cdot(\mathbf{r}_1+\mathbf{r}_2)/2}\langle\mathbf{p}'|W(\mathbf{r}_1)+W(\mathbf{r}_2)|\mathbf{p}\rangle$$
$$\times\int dt\, e^{-\epsilon|t|}e^{i(E_{b'}-E_b)t},$$

$$N=ac(a+c)/2\pi(a-c)^2,$$

which can be integrated to give with (23)

$$M_{b,b'}{}^{(1)}=\frac{-2igN\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')}{V^2(\omega_p\omega_{p'})^{\frac{1}{2}}}(2\pi)^5$$
$$\times\delta(E_{b'}-E_b)\delta(\mathbf{P}+\mathbf{p}-\mathbf{P}'-\mathbf{p}')\Sigma,$$

where

$$\Sigma=\frac{1}{u}\left(\tan^{-1}\frac{u}{2c}+\tan^{-1}\frac{u}{2a}-2\tan^{-1}\frac{u}{c+a}\right),\tag{38}$$
$$\mathbf{u}=(\mathbf{p}-\mathbf{p}')/2.$$

As the deuteron shall not be split up during the collision, its initial and final states are differing only with respect to the motion of its center of mass. We may as well treat the deuteron like one particle of mass $2m_1$ being initially at rest, $\mathbf{P}=0$, and get

$$W^{(1)}=V^2(2\pi)^{-6}\int\left(d\mathbf{P}'d\mathbf{p}'|M_{b,b'}{}^{(1)}|^2\bigg/\int dtd\mathbf{r}\right).$$

As

$$E_{b'}-E_b=(4m_1{}^2+P'^2)^{\frac{1}{2}}+(m^2+p'^2)^{\frac{1}{2}}$$
$$+B-2m_1-(m^2+p^2)^{\frac{1}{2}}-B$$

is analogous to (24), we have finally, replacing $m_1$ by $2m_1$,

$$p_n=p\frac{[m^2+2m_1(m^2+p^2)^{\frac{1}{2}}]\cos\vartheta+[2m_1+(m^2+p^2)^{\frac{1}{2}}](4m_1{}^2-m^2\sin^2\vartheta)^{\frac{1}{2}}}{[2m_1+(m^2+p^2)^{\frac{1}{2}}]^2-p^2\cos^2\vartheta},\tag{39}$$

and the scattering cross section for elastic collisions in the lab system is

$$\frac{d\sigma_{\text{el}}{}^{(1)}}{d\Omega}=\frac{[2g\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}_n)p_n N\Sigma]^2}{p(m^2+p_n{}^2)^{\frac{1}{2}}f'(p_n)}.\tag{40}$$

For the scattering of slow mesons,

$$p\ll c,\quad u\ll c,\quad \Sigma\approx\Sigma_0=(a-c)^2/2ac(a+c),$$

and

$$d\sigma_{\text{el}}{}^{(1)}/d\Omega\approx(2g\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')Q/4\pi)^2,$$

with

$$\frac{p_n{}^2}{p(m^2+p_n{}^2)^{\frac{1}{2}}f'(p_n)}\approx Q^2=\left(\frac{14+\cos\vartheta}{15}\right)^2,$$

while in the static-source model or for $\vartheta=0°$ always $p_n=p$, $u=0$, $\Sigma=\Sigma_0$, and $Q=1$. For high-energy scattering ($p\gg c$), $u$ is increasing, $\Sigma$ and the scattering cross section decreasing very rapidly with $\vartheta$. This behavior is obvious: For small momenta being interchanged, the deuteron is acting like a single nucleon of mass $2m_1$, the scattering being determined by the factor $Q$. For fast mesons, even a small deflection causes considerable momentum exchange and the scattering cross section is determined by the "internal structure-term" $\Sigma$ of the deuteron. The deuteron will probably be split up, and the elastic scattering vanishes.

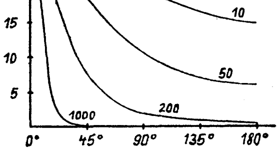


FIG. 2. Differential cross section for the elastic scattering of mesons by deuterons. $E_K$ is the incident meson energy in the lab system.

The calculated data are available from Fig. 2.

Considering inelastic scattering, the deuteron will be split up into its components after the collision. In the lab system,

$$M_{b,s}{}^{(1)}=\langle E_s|S^{(1)}|E_b\rangle=\frac{-igN^{\frac{1}{2}}\bar{U}(-\mathbf{p})\bar{U}(\mathbf{p}')}{(V^5\omega_p\omega_{p'})^{\frac{1}{2}}}(2\pi)^5$$
$$\times\delta(E_s-E_b)\delta(\mathbf{p}-\mathbf{P}'-\mathbf{p}')\tau,$$

$$\tau=\frac{1}{c^2+v^2}+\frac{1}{c^2+w^2}-\frac{1}{a^2+v^2}-\frac{1}{a^2+w^2}$$
$$-\frac{2K^*}{u}\left(\tan^{-1}\frac{u}{c+iq'}-\tan^{-1}\frac{u}{a+iq'}\right.\tag{41}$$
$$\left.-\tan^{-1}\frac{u}{c+a}+\tan^{-1}\frac{u}{2a}\right),$$

$$\mathbf{v}=\mathbf{u}-\mathbf{q}',\quad \mathbf{w}=\mathbf{u}+\mathbf{q}'.$$

As $d\mathbf{p}_1d\mathbf{p}_2=d\mathbf{P}d\mathbf{q}$, we have

$$W^{(1)}=V^3(2\pi)^{-9}\int d\mathbf{P}'d\mathbf{q}'d\mathbf{p}'|M_{b,s}{}^{(1)}|^2\bigg/\int dtd\mathbf{r}.$$

With

$$\mathbf{p}_{1,2}'=(\mathbf{p}-\mathbf{p}')/2\pm\mathbf{q}',$$

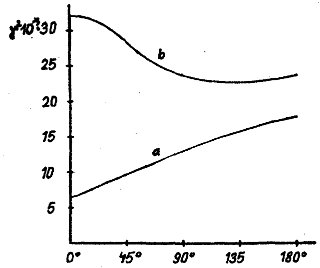


FIG. 3. Differential cross section for inelastic (a) and for both elastic and inelastic (b) scattering of 50-Mev mesons (lab) by deuterons (first approximation).

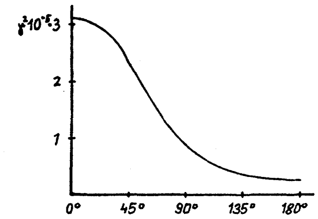


FIG. 4. Differential cross section for inelastic scattering of 2.5-Mev mesons (lab) by deuterons in first approximation.

we have

$$f(q')=E_s-E_b=\left[m_1^2+\left(\frac{\mathbf{p}-\mathbf{p}'}{2}+\mathbf{q}'\right)^2\right]^{\frac{1}{2}}+\left[m_1^2+\left(\frac{\mathbf{p}-\mathbf{p}'}{2}-\mathbf{q}'\right)^2\right]^{\frac{1}{2}}+(m^2+p'^2)^{\frac{1}{2}}-(m^2+p^2)^{\frac{1}{2}}-B-2m_1. \quad (42)$$

Since $p'$ is present in the first three terms and $q'$ only twice, we take

$$\delta(E_s-E_b)=\delta(q'-q_n)/f'(q_n),$$

remembering that the integration runs over positive real values of $q'$ only,

$$\int d\mathbf{q}'=\int_0^\infty q'^2dq'\int_0^\pi \sin\alpha d\alpha\times 2\pi,$$

where $\alpha$ is defined by

$$(\mathbf{u}\pm\mathbf{q}')^2=u^2+q'^2\pm 2uq'\cos\alpha.$$

Then we have

$$q_n=\frac{F}{2}\left(\frac{F^2-4m_1^2-4u^2}{F^2-4u^2\cos^2\alpha}\right)^{\frac{1}{2}},$$

$$f'(q_n)=(q_n+u\cos\alpha)(m_1^2+u^2+q_n^2+2uq_n\cos\alpha)^{-\frac{1}{2}}+(q_n-u\cos\alpha)(m_1^2+u^2+q_n^2-2uq_n\cos\alpha)^{-\frac{1}{2}}, \quad (43)$$

with

$$F=(m^2+p^2)^{\frac{1}{2}}+2m_1+B-(m^2+p'^2)^{\frac{1}{2}}.$$

Furthermore, (42) tells us that $F>2u$ and the denominator in (43) is always positive. Thus, for real $q_n$, the condition

$$F^2-4m_1^2-4u^2\geq 0 \quad (44)$$

must be satisfied. This condition, resulting from energy and momentum conservation only, can be used to evaluate the minimum kinetic energy for inelastic scattering,

$$E_M=\frac{2m_1+m}{2m_1}|B|=2.39\text{ Mev}>|B|,$$

and the boundaries for the integration over $p'$. The zeros of (44) are

$$\bar{p}_{1,2}\approx\frac{p\cos\vartheta(2m_1\omega_p+m^2)\pm[p^2\cos^2\vartheta(2m_1\omega_p+m^2)^2+(4m_1^2p^2+8m_1^2B\omega_p+4m_1Bm^2)(4m_1^2+4m_1\omega_p+p^2\sin^2\vartheta)]^{\frac{1}{2}}}{4m_1^2+4m_1\omega_p+p^2\sin^2\vartheta},$$

where $\bar{p}_1$ gives the upper limit, the lower boundary being $\bar{p}_2$ as long as it is positive, which proves right for very small kinetic energies only, otherwise being zero. Finally, we have

$$\frac{d\sigma_{\text{inel}}^{(1)}}{d\Omega}=\frac{g^2N^2}{4\pi^2p}\int_0^\pi\int_0^{p_1}\frac{\sin\alpha d\alpha\, p'^2dp'}{(m^2+p'^2)^{\frac{1}{2}}f'(q_n)}\times\bar{U}^2(-\mathbf{p})\bar{U}^2(\mathbf{p}')q_n^2|\tau|^2,$$

where the last integrations cannot be done.

The scattering cross section has been calculated numerically for mesons of 2.5 and 50 Mev kinetic energy by Simpson's formula. Figure 3 shows the angular distribution of the scattered particles which is determined by $|\tau|^2$, thus depending mainly on the internal structure of the deuteron. Figure 3 also gives the total cross section for both elastic and inelastic scattering.

Figure 4 shows data for the 2.5-Mev mesons.

## 5. CONCLUSIONS

The recoil model allows for the recoils of all particles involved, while creation and annihilation processes are restricted to the lightweight components only. So this picture should be useful for the description of systems where the total kinetic energy does not much surpass the rest energy of the heavy particles.

## ACKNOWLEDGMENTS

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