Interference Effects in the Decay of Neutral K Mesons: $K^0 \to \pi + \pi + \gamma$

S. BARSHAY* AND C. ISO†
Physics Department, Brandeis University, Waltham, Massachusetts
(Received November 13, 1961)

An analysis is made of interference effects between the K_1^0 and K_2^0 components of a neutral K beam undergoing the $\pi^+ + \pi^- + \gamma$ decay mode. It is shown that a time dependence of the angle between the photon polarization and the normal to the decay plane in the rest system of the K particle results from the interference between the K_1^0 and K_2^0 channels. If we can assume that for π-π scattering at the energies relevant here only $I=0$, D wave and $I=1$, P-wave elastic phase shifts may be large, then these interference terms can be uniquely written in terms of the π-π phase shifts and Δm, the $K_1^0 - K_2^0$ mass difference. This effect may serve as a good method to determine of the sign, as well as the magnitude, of the $K_1^0 - K_2^0$ mass difference. It may also be useful to determine the π-π scattering phase shifts.

I. INTRODUCTION

THE neutral K meson has some unusual properties, the double lifetime behavior[1] and the regeneration phenomenon[2] owing to the fact that the production of K particles is governed by charge independence while the 2π decay mode of the K^0 meson is an eigenstate of the CP operator. A further phenomenon peculiar to the neutral K particles is the time dependence of the interference effects in the following decay modes of these mesons: $K^0 \to \pi^\pm + \mu^\mp (e^\mp) + \nu$,[3] $K^0 \to \pi^+ + \pi^- + \pi^0$,[4] and $K^0 \to \gamma + \gamma$.[5] The time dependences of the oscillatory effects occur with a frequency given by the mass difference $\Delta m = m_1 - m_2$ between the K_1^0 and K_2^0.

In this note we point out another interference effect in neutral K decay phenomenon. We consider the decay mode: $K^0 \to \pi^+ + \pi^- + \gamma$. In Sec. II we discuss the matrix elements and the interference phenomena generally. In Sec. III we assume that only the $I=0$, D wave and $I=1$, P-wave $\pi - \pi$ elastic scattering phase shifts may be large at the energies relevant to this decay. With this assumption we can simplify the general expression and we obtain an expression from which one can determine the sign as well as the magnitude of the $K_1^0 - K_2^0$ mass difference. This interference may also be useful in determining $\pi - \pi$ elastic scattering phase shifts. In particular, if it is possible to study $K^0 \to \pi^0 + \pi^0 + \gamma$ experimentally, we can discuss $I=2$, D-wave $\pi - \pi$ scattering.

II. GENERAL THEORY

Neutral K particles are produced as K^0 mesons or $\bar{K}^0$ mesons. The neutral K meson decays are best characterized by K_1^0 and K_2^0 mesons:

$$|K_1^0\rangle = \tfrac{1}{2}(|K^0\rangle + |\bar{K}^0\rangle), \quad |K_2^0\rangle = -\tfrac{1}{2}i(|K^0\rangle - |\bar{K}^0\rangle). \quad (1)$$

To discuss the time dependence of the $\pi + \pi + \gamma$ phenomena associated with an initially produced K^0 beam, we consider the time-dependent state vector:

$$\begin{aligned}|\psi(t)\rangle &= (e^{-\lambda_1 t/2}|K_1^0\rangle + e^{-\lambda_2 t/2} e^{i(\Delta m)t} i |K_2^0\rangle) e^{-im_1 t}/\sqrt{2} \\ &= |\Phi(t)\rangle e^{-im_1 t},\end{aligned} \quad (2)$$

where

$$\Delta m \equiv m_1 - m_2 \quad (3)$$

is the mass difference between the K_1^0 meson and the K_2^0 meson, and

$$\begin{aligned}(\lambda_1)^{-1} &= \text{mean lifetime of } K_1^0 \text{ meson} \\ &= (1.00 \pm 0.04) \times 10^{-10} \text{ sec} \\ (\lambda_2)^{-1} &= \text{mean lifetime of } K_2^0 \text{ meson} \\ &= (6.1 \pm 1.4) \times 10^{-8} \text{ sec.}\end{aligned} \quad (4)$$

The transition matrix element at time t which takes the neutral K into the $\pi + \pi + \gamma$ state denoted by f is given by

$$\langle f|T|\Phi(t)\rangle = (T_{1,f} e^{-\lambda_1 t/2} + iT_{2,f} e^{-\lambda_2 t/2} e^{i(\Delta m)t})/\sqrt{2}, \quad (5)$$

where

$$\begin{aligned}T_{1,f} &= \langle f|T_1|K_1^0\rangle, \\ T_{2,f} &= \langle f|T_2|K_2^0\rangle.\end{aligned} \quad (6)$$

The corresponding decay probability $\Gamma_f(t)$ at time t is proportional to $|\langle f|T|\Phi(t)\rangle|^2$.

In this note we assume CP invariance in weak interactions as well as in strong and electromagnetic interactions. Then the final $\pi + \pi + \gamma$ state is characterized by $CP = +1$ if it originates from K_1^0 decay and $CP = -1$ if it originates from K_2^0 decay. The most general Lorentz-invariant transition matrix elements for the $\pi + \pi + \gamma$ decay of K_1^0 and K_2^0 are as follows:

$$\begin{aligned}T_{1,f} &\equiv \langle f|T_1|K_1^0\rangle \\ &= \langle f|[P_0 A_1(\mathbf{k}_1 - \mathbf{k}_2)\cdot\boldsymbol{\varepsilon} + iP_e B_1(\mathbf{k}_1 \times \mathbf{k}_2)\cdot\boldsymbol{\varepsilon}]|K_1^0\rangle, \\ T_{2,f} &\equiv \langle f|T_2|K_2^0\rangle \\ &= \langle f|[iP_0 A_2(\mathbf{k}_1 \times \mathbf{k}_2)\cdot\boldsymbol{\varepsilon} + P_e B_2(\mathbf{k}_1 - \mathbf{k}_2)\cdot\boldsymbol{\varepsilon}]|K_2^0\rangle,\end{aligned} \quad (7)$$

where P_0 and P_e are, respectively, projection operators for states of odd and even isopin, I, of the $\pi - \pi$ system. $\mathbf{k}_1$, $\mathbf{k}_2$, and $\boldsymbol{\varepsilon}$ denote the momentum of one final π meson, the momentum of the other final π meson, and polariza-

* Partially supported by the National Science Foundation.
† On leave from Department of Physics, Tokyo University of Education. Supported by the National Science Foundation, and Air Force Office of Scientific Research.
[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).
[3] S. B. Treiman and R. G. Sachs, Phys. Rev. **103**, 1545 (1956).
[4] S. B. Treiman and S. Weinberg, Phys. Rev. **116**, 239 (1959).
[5] J. Dreitlein and H. Primakoff, Phys. Rev. **124**, 268 (1961).

tion of the photon, respectively. The amplitudes A_1, A_2, B_1 and B_2 are in general complex functions of k_1^2, k_2^2 and $(\mathbf{k}_1\cdot\mathbf{k}_2)$.

Now we rewrite $(\mathbf{k}_1\times\mathbf{k}_2)\cdot\boldsymbol{\varepsilon}$ and $(\mathbf{k}_1-\mathbf{k}_2)\cdot\boldsymbol{\varepsilon}$ as

$$(\mathbf{k}_1\times\mathbf{k}_2)\cdot\boldsymbol{\varepsilon}=k_1k_2\sin\Theta\cos\theta, \tag{8}$$

$$\begin{aligned}(\mathbf{k}_1-\mathbf{k}_2)\cdot\boldsymbol{\varepsilon}&=|k_1-k_2|\cos\phi\sin\theta\\ &=\frac{2}{|\mathbf{k}_1+\mathbf{k}_2|}k_1k_2\sin\Theta\sin\theta,\end{aligned} \tag{9}$$

where angles θ, Θ, and ϕ are defined as follows (see Fig. 1):

$$\begin{aligned}\cos\theta&=\frac{(\mathbf{k}_1\times\mathbf{k}_2)\cdot\boldsymbol{\varepsilon}}{|\mathbf{k}_1\times\mathbf{k}_2||\boldsymbol{\varepsilon}|},\quad \cos\Theta=\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)}{k_1k_2},\\ \cos\phi&=\frac{(\mathbf{k}_1+\mathbf{k}_2)\times(\mathbf{k}_1\times\mathbf{k}_2)\cdot(\mathbf{k}_1-\mathbf{k}_2)}{|\mathbf{k}_1\times\mathbf{k}_2||\mathbf{k}_1+\mathbf{k}_2||\mathbf{k}_1-\mathbf{k}_2|}.\end{aligned} \tag{10}$$

Then the transition matrix elements $T_i(\pi_1,\mathbf{k}_1;\pi_2,\mathbf{k}_2;\theta)$ which are characterized by polarization angle θ, π_1 meson momentum $\mathbf{k}_1$, and π_2 meson momentum $\mathbf{k}_2$ are

$$\begin{aligned}T_1(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2,\theta)&=A_1\sin\theta+iB_1\cos\theta,\\ T_1(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2,\theta)&=-A_1\sin\theta+iB_1\cos\theta,\end{aligned} \tag{11}$$

$$\begin{aligned}T_2(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2,\theta)&=iA_2\cos\theta+B_2\sin\theta,\\ T_2(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2,\theta)&=-iA_2\cos\theta+B_2\sin\theta,\end{aligned} \tag{12}$$

$$T_1(\pi^0,\mathbf{k}_1;\pi^0,\mathbf{k}_2,\theta)=-C_1\cos\theta, \tag{13}$$

$$T_2(\pi^0,\mathbf{k}_1;\pi^0,\mathbf{k}_2,\theta)=-C_2\sin\theta, \tag{14}$$

where

$$\begin{aligned}B_i&=a_{0i}+a_{2i},\\ C_i&=a_{0i}-2a_{2i}.\end{aligned} \tag{15}$$

A_i, a_{0i}, and a_{2i} are functions of k_1^2, k_2^2 and $(\mathbf{k}_1\cdot\mathbf{k}_2)$ corresponding to the $I=1$, 0, and 2 $\pi-\pi$ states, respectively. The leading terms in the A_i and a_i are, respectively, proportional to unity and $(\mathbf{k}_1-\mathbf{k}_2)\cdot(\mathbf{k}_1+\mathbf{k}_2)$. We write

$$\begin{aligned}A_i&\equiv f_ie^{i\alpha_i}\\ B_i&\equiv g_ie^{i\beta_i}\\ C_i&\equiv h_ie^{i\gamma_i},\end{aligned} \tag{16}$$

where f_i, g_i, and h_i are taken as real and positive.

Using Eqs. (11)–(14) and (16), we obtain the decay probability for different final $\pi-\pi$ charge states as follows:

$$\begin{aligned}&\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)\\ &\propto\sin^2\theta\{f_1^2e^{-\lambda_1t}+g_2^2e^{-\lambda_2t}\\ &\qquad+2f_1g_2\sin[\alpha_1-\beta_2-(\Delta m)t]e^{-(\lambda_1+\lambda_2)t/2}\}\\ &\qquad+\cos^2\theta\{g_1^2e^{-\lambda_1t}+f_2^2e^{-\lambda_2t}\\ &\qquad+2g_1f_2\sin[\beta_1-\alpha_2-(\Delta m)t]e^{-(\lambda_1+\lambda_2)t/2}\}\\ &\qquad+2\sin\theta\cos\theta\{[f_1g_1\sin(\alpha_1-\beta_1)e^{-\lambda_1t}\\ &\qquad+f_2g_2\sin(\alpha_2-\beta_2)e^{-\lambda_2t}]\\ &\qquad-[f_1f_2\cos(\alpha_1-\alpha_2-(\Delta m)t)\\ &\qquad\ +g_1g_2\cos(\beta_1-\beta_2-(\Delta m)t)]e^{-(\lambda_1+\lambda_2)t/2}\},\end{aligned} \tag{17}$$

$$\begin{aligned}&\Gamma(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\ &\qquad\propto[f_i\to-f_i\quad\text{in}\quad\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)],\end{aligned} \tag{18}$$

$$\begin{aligned}&\Gamma(\pi^0,\mathbf{k}_1;\pi^0,\mathbf{k}_2;\theta;t)\\ &\propto\sin^2\theta h_2^2e^{-\lambda_2t}+\cos^2\theta h_1^2e^{-\lambda_1t}-2\sin\theta\cos\theta h_1h_2\\ &\qquad\times\cos[\gamma_1-\gamma_2-(\Delta m)t]e^{-(\lambda_1+\lambda_2)t/2}.\end{aligned} \tag{19}$$

From Eqs. (17) and (18) we can obtain simpler expressions:

$$\begin{aligned}&\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)+\Gamma(\pi^-,\mathbf{k}_1;\pi^+\mathbf{k}_2;\theta;t)\\ &\propto\sin^2\theta\{f_1^2e^{-\lambda_1t}+g_2^2e^{-\lambda_2t}\}+\cos^2\theta\{g_1^2e^{-\lambda_1t}+f_2^2e^{-\lambda_2t}\}\\ &\qquad-2\sin\theta\cos\theta\{f_1f_2\cos[\alpha_1-\alpha_2-(\Delta m)t]\\ &\qquad+g_1g_2\cos[\beta_1-\beta_2-(\Delta m)t]\}e^{-(\lambda_1+\lambda_2)t/2},\end{aligned} \tag{20}$$

$$\begin{aligned}&\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)-\Gamma(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\ &\propto2\sin^2\theta f_1g_2\sin[\alpha_1-\beta_2-(\Delta m)t]e^{-(\lambda_1+\lambda_2)t/2}\\ &\qquad+2\cos^2\theta g_1f_2\sin[\beta_1-\alpha_2-(\Delta m)t]e^{-(\lambda_1+\lambda_2)t/2}\\ &\qquad+2\sin\theta\cos\theta[f_1g_1\sin(\alpha_1-\beta_1)e^{-\lambda_1t}\\ &\qquad+f_2g_2\sin(\alpha_2-\beta_2)e^{-\lambda_2t}].\end{aligned} \tag{21}$$

The total probability of the transition in the time interval $(0,t)$ is defined by

$$P(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)=\int_0^t\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t). \tag{22}$$

Then we get

$$\begin{aligned}&P(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)+P(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\ &\propto\sin^2\theta\left(\frac{f_1^2}{\lambda_1}(1-e^{-\lambda_1t})+\frac{g_2^2}{\lambda_2}(1-e^{-\lambda_2t})\right)\\ &\quad+\cos^2\theta\left(\frac{g_1^2}{\lambda_1}(1-e^{-\lambda_1t})+\frac{f_2^2}{\lambda_2}(1-e^{-\lambda_2t})\right)\\ &\quad-2\sin\theta\cos\theta\frac{1}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}\\ &\quad\times\Bigg\{f_1f_2\Bigg[\frac{\lambda_1+\lambda_2}{2}\cos(\alpha_1-\alpha_2)-\Delta m\sin(\alpha_1-\alpha_2)\\ &\quad-e^{-(\lambda_1+\lambda_2)t/2}\left(\frac{\lambda_1+\lambda_2}{2}\cos(\alpha_1-\alpha_2-\Delta mt)\right.\\ &\quad\left.-\Delta m\sin(\alpha_1-\alpha_2-\Delta mt)\right)\Bigg]\\ &\quad+g_1g_2\Bigg[\frac{\lambda_1+\lambda_2}{2}\cos(\alpha_1-\alpha_2)-\Delta m\sin(\beta_1-\beta_2)\\ &\quad-e^{-(\lambda_1+\lambda_1)t/2}\left(\frac{\lambda_1+\lambda_2}{2}\cos(\beta_1-\beta_2-\Delta mt)\right.\\ &\qquad\qquad\left.-\Delta m\sin(\beta_1-\beta_2-\Delta mt)\right)\Bigg]\Bigg\},\end{aligned} \tag{23}$$

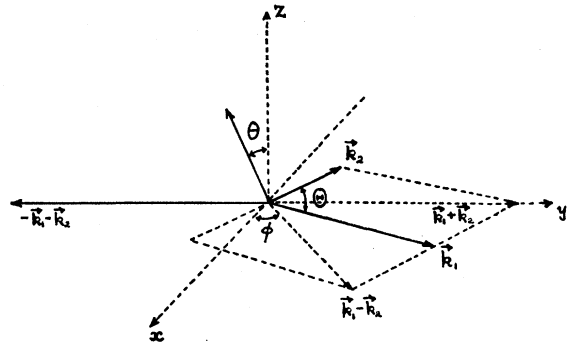

FIG. 1. Diagrammatic definition of the angles utilized in formulas (8)–(10) of the text.

$$
\begin{aligned}
&P(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)-P(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\
&\propto 2\sin^2\theta f_1g_2\frac{1}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}\\
&\quad\times\bigg[\frac{\lambda_1+\lambda_2}{2}\sin(\alpha_1-\beta_2)-\Delta m\cos(\alpha_1-\beta_2)\\
&\quad -e^{-(\lambda_1+\lambda_2)t/2}\bigg(\frac{\lambda_1+\lambda_2}{2}\sin(\alpha_1-\beta_2-\Delta mt)\\
&\quad -\Delta m\cos(\alpha_1-\beta_2-\Delta mt)\bigg)\bigg]\\
&\quad +2\cos^2\theta g_1f_2\frac{1}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}\\
&\quad\times\bigg[\frac{\lambda_1+\lambda_2}{2}\sin(\beta_1-\alpha_2)-\Delta m\cos(\beta_1-\alpha_2)\\
&\quad -e^{-(\lambda_1+\lambda_2)t/2}\bigg(\frac{\lambda_1+\lambda_2}{2}\sin(\beta_1-\alpha_2-\Delta mt)\\
&\quad -\Delta m\cos(\beta_1-\alpha_2-\Delta mt)\bigg)\bigg]\\
&\quad +2\sin\theta\cos\theta\bigg(f_1g_1\sin(\alpha_1-\beta_1)\frac{1}{\lambda_1}(1-e^{-\lambda_1t})\\
&\qquad +f_2g_2\sin(\alpha_2-\beta_2)\frac{1}{\lambda_2}(1-e^{-\lambda_2t})\bigg).\qquad(24)
\end{aligned}
$$

It is worth mentioning at this point the expressions (20), (21), (23), and (24) depend upon the *sign* as well as the magnitude of Δm. It is thus possible to determine the sign of Δm. However, since these equations contain many parameters, it is useful to simplify the expressions by relating the phases α_i, β_i, and γ_i to $\pi-\pi$ elastic scattering phase shifts in low angular-momentum states.

III. APPROXIMATE EXPRESSIONS

In the previous section we derived the general expressions for the decay probability at time t and the total probability in the time interval $(0-t)$. These expressions depend upon various amplitudes characterized by the isospin of the final $\pi-\pi$ system and the π-meson momenta. In this section we assume that only the $I=0$, D-wave and the $I=1$, P-wave elastic phase shifts for $\pi-\pi$ scattering may be large at the energies available to the $\pi-\pi$ system in this decay. We define the $\pi-\pi$ scattering phase shifts as follows:

$$
\begin{aligned}
\langle\pi\pi I=0,\,J=2|S|\pi\pi\rangle&=e^{2i\delta_{0,2}},\\
\langle\pi\pi I=1,\,J=1|S|\pi\pi\rangle&=e^{2i\delta_{1,1}}.
\end{aligned}\qquad(25)
$$

Utilizing the unitarity of the S matrix, we can represent α_i, β_i, and γ_i by these $\pi-\pi$ scattering phase shifts[6–8]:

$$
\begin{aligned}
\alpha_1&=\alpha_2=\delta_{1,1},\\
\beta_1&=\beta_2=\delta_{0,2},\\
\gamma_1&=\gamma_2=\delta_{0,2},\\
a_{2i}&=0,\\
h_i&=g_i
\end{aligned}\qquad(26)
$$

Then Eqs. (20), (21), (23), and (24) may be rewritten as follows:

$$
\begin{aligned}
&\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)+\Gamma(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\
&\quad\propto\sin^2\theta(f_1^2e^{-\lambda_1t}+g_2^2e^{-\lambda_2t})+\cos^2\theta(g_1^2e^{-\lambda_1t}+f_2^2e^{-\lambda_2t})\\
&\qquad -2\sin\theta\cos\theta(f_1f_2+g_1g_2)\cos(\Delta mt)e^{-(\lambda_1+\lambda_2)t/2},\qquad(27)
\end{aligned}
$$

$$
\begin{aligned}
&\Gamma(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)-\Gamma(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\
&\quad\propto 2\sin^2\theta f_1g_2\sin(\delta_{1,1}-\delta_{0,2}-\Delta mt)e^{-(\lambda_1+\lambda_2)t/2}\\
&\qquad -2\cos^2\theta g_1f_2\sin(\delta_{1,1}-\delta_{0,2}+\Delta mt)e^{-(\lambda_1+\lambda_2)t/2}\\
&\qquad +2\sin\theta\cos\theta\sin(\delta_{1,1}-\delta_{0,2})\\
&\qquad\qquad\times(f_1g_1e^{-\lambda_1t}+f_2g_2e^{-\lambda_2t}),\qquad(28)
\end{aligned}
$$

$$
\begin{aligned}
&P(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)+P(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)\\
&\quad\propto\sin^2\theta\bigg(\frac{f_1^2}{\lambda_1}(1-e^{-\lambda_1t})+\frac{g_2^2}{\lambda_2}(1-e^{-\lambda_2t})\bigg)\\
&\qquad +\cos^2\theta\bigg(\frac{g_1^2}{\lambda_1}(1-e^{-\lambda_1t})+\frac{f_2^2}{\lambda_2}(1-e^{-\lambda_2t})\bigg)\\
&\qquad -2\sin\theta\cos\theta\frac{f_1f_2+g_1g_2}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}\bigg[\frac{\lambda_1+\lambda_2}{2}\\
&\qquad -e^{-(\lambda_1+\lambda_2)t/2}\bigg(\frac{\lambda_1+\lambda_2}{2}\cos(\Delta mt)\\
&\qquad\qquad +(\Delta m)\sin(\Delta mt)\bigg)\bigg],\qquad(29)
\end{aligned}
$$

[6] See, for example, M. Kawaguchi and S. Minami, Progr. Theoret. Phys. (Kyoto) **12**, 789 (1954).
[7] S. Barshay, Phys. Rev. **120**, 267 (1960).
[8] F. E. Low (unpublished).

$$P(\pi^+,\mathbf{k}_1;\pi^-,\mathbf{k}_2;\theta;t)-P(\pi^-,\mathbf{k}_1;\pi^+,\mathbf{k}_2;\theta;t)$$
$$\propto 2\sin^2\theta f_1 g_2 \frac{1}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}$$
$$\times\left[\left(\frac{\lambda_1+\lambda_2}{2}\sin(\delta_{1,1}-\delta_{0,2})-\Delta m\cos(\delta_{1,1}-\delta_{0,2})\right)\right.$$
$$-e^{-(\lambda_1+\lambda_2)t/2}\left(\frac{\lambda_1+\lambda_2}{2}\sin(\delta_{1,1}-\delta_{0,2}-\Delta mt)\right.$$
$$\left.\left.-\Delta m\cos(\delta_{1,1}-\delta_{0,2}-\Delta mt)\right)\right]$$
$$-2\cos^2\theta g_1 f_2 \frac{1}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}$$
$$\times\left[\left(\frac{\lambda_1+\lambda_2}{2}\sin(\delta_{1,1}-\delta_{0,2})-\Delta m\cos(\delta_{1,1}-\delta_{0,2})\right)\right.$$
$$-e^{-(\lambda_1+\lambda_2)t/2}\left(\frac{\lambda_1+\lambda_2}{2}\sin(\delta_{1,1}-\delta_{0,2}+\Delta mt)\right.$$
$$\left.\left.+\Delta m\cos(\delta_{1,1}-\delta_{0,2}+\Delta mt)\right)\right]$$
$$+2\sin\theta\cos\theta\sin(\delta_{1,1}-\delta_{0,2})$$
$$\times\left(f_1 g_1\frac{1}{\lambda_1}(1-e^{-\lambda_1 t})+f_2 g_2\frac{1}{\lambda_2}(1-e^{-\lambda_2 t})\right). \quad (30)$$

Equation (19) can be approximated by

$$\Gamma(\pi^0,\mathbf{k}_1;\pi^0,\mathbf{k}_2;\theta;t)\propto\sin\theta\, g_2{}^2e^{-\lambda_2 t}+\cos^2\theta\, g_1{}^2e^{-\lambda_1 t}$$
$$-2\sin\theta\cos\theta\, g_1g_2\cos(\Delta mt)e^{-(\lambda_1+\lambda_2)t/}, \quad (31)$$

and also

$$P(\pi^0,\mathbf{k}_1;\pi^0,\mathbf{k}_2;\theta;t)$$
$$\propto\sin^2\theta\frac{g_2{}^2}{\lambda_2}(1-e^{-\lambda_2 t})+\cos^2\theta\frac{g_1{}^2}{\lambda_1}(1-e^{-\lambda_1 t})$$
$$-2\sin\theta\cos\theta\frac{g_1g_2}{(\Delta m)^2+[(\lambda_1+\lambda_2)/2]^2}$$
$$\times\left[\frac{\lambda_1+\lambda_2}{2}-e^{-(\lambda_1+\lambda_2)t/2}\right.$$
$$\left.\times\left(\frac{\lambda_1+\lambda_2}{2}\cos(\Delta mt)+\Delta m\sin(\Delta mt)\right)\right]. \quad (32)$$

Equations (28) and (30) depend on the sign as well as the magnitude of Δm, so it would be very useful to measure these probabilities. Also it might be possible to gain information about the $\pi-\pi$ scattering phase shift combination, $\delta_{1,1}-\delta_{0,2}$, by measuring these $K_1{}^0$ and $K_2{}^0$ decay interference phenomena. However, if we know neither $\delta_{1,1}-\delta_{0,2}$ nor Δm we cannot distinguish the following two kinds of solutions:

$$\begin{aligned}&\text{(i)}\quad \delta_{1,1}-\delta_{0,2}=\alpha, &&\Delta m=\pm|\Delta m|\\ &\text{(ii)}\quad \delta_{1,1}-\delta_{0,2}=\pi-\alpha, &&\Delta m=\mp|\Delta m|.\end{aligned} \quad (33)$$

In order to determine the sign of Δm by this method we must know at least whether $\delta_{1,1}-\delta_{0,2}$ is larger or smaller than $\pi/2$. Note that the interference effects in Eqs. (29) and (32) are completely independent of the $\pi-\pi$ phase shifts, they depend only on the magnitude of Δm.

In this section we have made two assumptions: (i) P and D wave (and, of course, S wave) $\pi-\pi$ elastic scattering phase shifts are dominant at total center-of-mass energies in the $\pi-\pi$ system $<3\,\mu$, where μ is the pion mass; (ii) $I=0$ and $I=1$, $\pi-\pi$ isospin states are dominant, i.e., we have neglected the D wave in the isospin 2 state. The first assumption would seem to be reliable because of the low energies involved. The second assumption also may be true, but it is not so sure. The second assumption can be checked by comparing $\pi^++\pi^-+\gamma$ decay and $\pi^0+\pi^0+\gamma$ decay, in other words, by comparing Eqs. (29) and (32).

IV. CONCLUSION

A recent estimate[9] of the frequency of the mode $K^+\to\pi^++\pi^0+\gamma$ puts this frequency at $\sim10^{-3}$ radiative decays per K^+ decay. This estimate attributes an appreciable contribution to the radiative decay to a direct emission term. Owing to the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule the internal bremsstrahlung contribution to K^+ radiative decay is severely suppressed.[10] The absence of this suppression in radiative K^0 decay together with an important direct emission might make an experimental study of the interference effect elucidated in this paper feasible. A measurement of the sign of the $K_1{}^0-K_2{}^0$ mass difference would then be possible.

---

[9] Monti, Quareni, and Vignudelli, Nuovo cimento **3**, 550 (1961).
[10] N. Cabibbo and R. Gatto, Phys. Rev. Letters **5**, 382 (1960).