Remarks on the Possibility of Quantum Electrodynamics without Potentials

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(Received November 10, 1961)

In this article, we reply to a suggestion of DeWitt for formulating a nonlocal quantum electrodynamics without potentials, showing that his proposal is not a real elimination of potentials, but only a substitution with the aid of which the essential role of potentials is somewhat obscured. Thus, we are not led to change our conclusion that potentials are significant in the formulation of quantum electrodynamics. We do, however, discuss some of the conditions that would have to be satisfied by an essentially nonlocal electrodynamics, and show that such a theory, if it could be developed, would very probably bring about profound modifications in the forms of the laws of quantum mechanics.

IN a recent article,[1] De Witt has objected to a certain conclusion that we drew in a paper on the role of electromagnetic potentials in the quantum theory,[2] aimed at treating in more detail some questions that we raised in an earlier paper.[3] Briefly, the main point at issue is that we have argued that in the fundamental formulation of quantum electrodynamics, the field quantities, $F_{\mu\nu}$, are not by themselves adequate, and the potentials, A_μ, play an essential part, which they do not play in the corresponding classical theory. We based this conclusion on the fact that the current form of electrodynamics is determined in part by the requirement that the theory must be a *local* one (i.e., one in which the basic field quantities operate only where the charge is), and that such a local formulation is possible only with the aid of the potentials.

DeWitt argues against the above requirement of locality, however, by presenting what at first sight appears to be a counter-example of a nonlocal theory in which electrodynamics is formulated on terms of the field strengths, $F_{\mu\nu}$, alone without the use of the potentials, A_μ. Briefly, what he does is to suggest a gauge transformation,

$$\psi'=e^{\Lambda}\psi, \tag{1}$$

with

$$\Lambda=\int_{\infty}^{0} A_\mu(z^\nu)\frac{\partial z^\mu}{\partial \xi^\nu}d\xi, \tag{2}$$

where ξ is a parameter going from $-\infty$ to 0, while $z^\nu=z^\nu(x^\mu,\xi)$ is a continuous function of ξ, varying from x^ν at $\xi=0$ to an infinitely distant vector as $\xi\rightarrow-\infty$. He then finds that in terms of the new wave function, ψ', the gauge invariant operator $\partial/\partial x^\mu - ieA_\mu$ takes the form

$$\frac{\partial}{\partial x^\mu}-ie\int_{-\infty}^{0} F_{\nu\sigma}\frac{\partial z^\nu}{\partial \xi}\frac{\partial z^\sigma}{\partial x^\mu}d\xi, \tag{3}$$

which evidently involves the field strengths alone, but in terms of nonlocal line integrals.

In response to the suggestion of DeWitt, we wish to point out that Eqs. (1)–(3) lead only in a purely nominal way to a nonlocal set of equations not involving a vector potential. For the theory is still determined in fact by the requirement that the theory shall be transformable into a local theory which must be expressed in terms of potentials. Thus, the potentials have been eliminated only in a trivial sense (as we might substitute $y=z^2$ in a linear equation and then assert that the equation is now nonlinear).

In order that a theory of the kind suggested by DeWitt should be *essentially nonlocal*, it would be necessary firstly that the theory have meaning even when nonlocal operators more general than (3) are involved, and secondly that the particular form (3) defining quantum electrodynamics shall be obtainable (at least in a suitable approximation) as a special case of such a more general theory, further limited by the adjunction of appropriate principles and conditions. The effort to obtain a systematic over-all formulation of covariant nonlocal theories of this kind has led, however, to a wide variety of as yet unsolved problems. These problems do not seem to be purely technical, but appear to involve the basic principles of the quantum mechanics itself.

Among these problems of principle, there is a particularly important one that does not seem to have been given adequate attention; viz., that of defining what Dirac[4] has called a complete commuting set of observables. If we denote the eigenvalues of such a set symbolically by α and the corresponding eigenfunctions by ψ_α, then an arbitrary wave function can be written as $\psi=\sum_\alpha C_\alpha\psi_\alpha$ and the average value of an operator, $O_{\alpha\alpha'}$, by $\bar{O}=\sum_{\alpha,\alpha'} C_\alpha{}^*O_{\alpha\alpha'}C_{\alpha'}$ (where α may represent any set of parameters, continuous or discrete). It is clear that unless such a set exists, there will be no way even to write a wave function, or to express the mean

[1] B. DeWitt, preceding paper [Phys. Rev. **125**, 2189 (1962)].
[2] Y. Aharanov and D. Bohm, Phys. Rev. **123**, 1511 (1961).
[3] Y. Aharanov and D. Bohm, Phys. Rev. **115**, 485 (1959).

[4] P. A. M. Dirac, *The Principles of Quantum Theory* (Clarendon Press, Oxford, 1947).

value of a physical quantity. Thus, the assumption of such a complete set is an integral part of the basic postulates of the quantum theory.

Now, in current local theories, there is always a complete commuting set of observables; viz., the appropriate field operators on a space-like hypersurface. In covariant forms of nonlocal theories, however, these field operators will generally not commute, so that it will not be possible to write wave functions and matrix elements as functions of the eigenvalues of such operators.[5] To the authors' knowledge, no example of a consistent over-all formulation of a nonlocal theory in terms of *some* complete commuting set of observables has yet been given, nor is it even clear that such a set exists for any nonlocal theory at all. The actual situation is then that the only electrodynamical theories that have thus far received any kind of over-all formulation in terms of a complete set of operators are these which are *essentially local* and which must involve potentials when their local character is clearly exhibited. We are therefore led to conclude that potentials are in fact playing a basic and indispensible part in the formulation of the equations of quantum electrodynamics, and that the transformation proposed by DeWitt does not eliminate the need to determine the mathematical form of the theory and to give its physical meaning with the aid of the potentials.

Of course, we certainly did not wish to suggest in our articles[2,3] that no consistent over-all formulation of a nonlocal theory can ever be developed at all. (Indeed, in reference 3 we explicitly pointed out that it might be worth seeking to develop a gauge-invariant nonlocal formulation not involving the potentials.) It is quite evident that theoretical physics is now in a state of flux, in which new techniques are being explored (e.g., those involving S matrices and dispersion relations), some of which may well lead to an expression of the laws of physics not requiring the assumption of local observables. If this is to happen, however, it is necessary either that in the use of these techniques, one will solve the problem of obtaining some new kind of complete commuting set of variables, or else that one will find a way to give up altogether the requirement of expressing the state of the system as a function (or functional) of such a set. It must be emphasized that either of these alternatives would lead to *fundamentally new* features in the formulation of the laws of physics. Thus, the discovery of a complete commuting set of observables that was not definable on a spacelike surface would in all probability imply a basic change in our ideas concerning what is usually called "causality" in S-matrix theory, while if we were to give up the need for a complete commuting set altogether, then the whole of the statistical interpretation of the quantum theory (e.g., probabilities, uncertainty relations, etc.) might have to be changed. Moreover, in any such nonlocal theory, one would of course have to explain why the theory is approximately local, why it takes just the form that it does, etc.

Meanwhile, however, it must be remembered that none of the methods described above has actually led, as yet, to the suggestion of a new kind of over-all formulation of a degree of generality comparable, for example, to that given by Dirac[4] (even though a great many specific effects can be calculated with their aid). And as we have shown, current theories of electrodynamics are still *essentially local.* It is quite possible, moreover, that future theories, whether nonlocal or of some other still unknown type, may continue the present characteristic of involving the vector potential in an essential way. It was with this possibility in mind that we suggested in our articles that the vector potential may have a new kind of significance in quantum theory.[6]

It is evident, then, that the question of the real role of the vector potential has not yet been settled. In the interests of clarity of thought concerning this problem, it seems important, however, to keep all the possibilities in mind.

ACKNOWLEDGMENT

We wish to thank B. DeWitt for sending us a preprint of his article, and also for very helpful correspondence concerning the problems discussed here.

[5] A similar situation arises with the angular momentum operators, L_x, L_y, L_z, for which a complete commuting set are L^2 and L_z. The wave function is then expressed as a function of L^2 and L_z; all matrix indices involve L^2 and L_z; all probabilities, uncertainties, etc., are calculated in terms of such indices, etc.

[6] In reference 2, we suggested in effect that the potentials are related to the topological properties of the electromagnetic field. In a future article by one of us (D.B.), this possibility will be discussed further.