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Conserved Current Hypothesis and the Beta Decay of RaE*

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It is shown that the conserved current theory of Feynman and Gell-Mann is consistent with knowledge of RaE beta decay. This assertion is made by extending the Siegert theorem to beta decay and comparing the theoretical prediction for $\langle\alpha\rangle$ with the experimental data, which are obtained from recent electron polarization and spectrum measurements. The Coulomb terms are evaluated using the Ahrens-Feenberg approximation. The conserved and nonconserved (conventional) theories are differentiated by the presence or absence of the exchange contributions. These are also evaluated by the Ahrens-Feenberg approximation. This re-analysis leads to an even larger disagreement between the shell-model prediction and the phenomenological nuclear matrix elements, $\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle/i\langle\hat{r}\rangle$. The origin of the disagreement is discussed. Several possible effects of the meson-cloud beta decay in a nucleus are also surveyed in the course of study.

1. INTRODUCTION

THE first-forbidden beta decay of RaE (Bi^{210}) has been a very controversial subject for a long time because the spectrum shape is quite different from the statistical one.[1] At present it is generally accepted that the peculiar shape originates from an accidental cancellation among the responsible nuclear matrix elements.[2–8] However, there still remain problems to be solved. For example, are the results of the current forbidden theory of beta decay consistent with our knowledge of nuclear structure? This problem originated with the attempt to determine the relative sign of the Gamow-Teller and Fermi coupling constants from the spectrum shapes of forbidden beta decays. One group used the beta spectra of odd-A nuclei[9,10] and another group used the spectrum[5] of RaE; these two groups were led to opposite conclusions. Since RaE consists of only one proton and one neutron outside of the doubly closed core, it is reasonable that our knowledge of RaE nuclear structure is fairly accurate. However, a direct experimental determination of the relative sign between the vector and axial vector coupling constants showed[11] that the interaction is V–A, opposite to the conclusion reached by the analysis of the RaE data.[5] Subsequently, the shell structure of RaE was studied more extensively because of the increase in experimental knowledge of nuclear levels.[12] The conclusion was embarrassing. The most plausible choice for the ground state of RaE was found to be $(h_{9/2},i_{11/2})$, confirming the result of reference 5. Newby and Konopinski[12] were led to the conjecture that the conventional forbidden theory is significantly altered by the inclusion of the meson-cloud beta decay that was suggested by Feynman and Gell-Mann.[13]

Recently, experimental knowledge of the electron longitudinal polarization in RaE has been obtained,[14] and several analyses have been performed, using the data of both spectrum and polarization.[15–17] If a cancellation between nuclear matrix elements is really responsible for the peculiar shape of RaE spectrum, according to the conventional forbidden theory of beta decay, the longitudinal polarization will be considerably reduced from the maximum value (v/c).[18–21] The recent polarization data show that the conventional forbidden theory is at least qualitatively correct in this respect.

The purpose of this paper is twofold. One purpose is to study the possible effects of meson-cloud beta decay in a nucleus. Another purpose is to reinvestigate the RaE problem taking into account the meson-cloud beta decay. Even if some aspects of the meson cloud are not free from the unreliability of pion theory, almost no ambiguities concerning the meson-cloud effect remain in the case of RaE decay, provided that the

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nuclear many-body problem is essentially nonrelativistic and the conserved current hypothesis[13] is valid. The conserved and nonconserved (conventional) theories are differentiated by the presence or absence of the exchange current contributions. Since the conserved current theory has not been established experimentally, our study of RaE is a test of the validity of the conserved current hypothesis and also is a check on the conventional beta-decay and nuclear structure theories. We conclude in this paper that the conserved current theory is consistent with the experimental information concerning $\langle\alpha\rangle$ of RaE, and the pure shell model gives incorrect nuclear matrix elements $\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle/i\langle\hat{r}\rangle$.

2. GENERAL CONSIDERATION ON THE MESON-CLOUD BETA DECAY

For simplicity we will restrict our discussion to ordinary beta decay (nucleus $A\rightarrow B$),

$$A\rightarrow B+e^-+\bar{\nu}.$$

First let us discuss the beta decay of a nucleon, $n\rightarrow p+e^-+\bar{\nu}$, then return to the nuclear problem later. We write the four momenta of A, B, e^-, $\bar{\nu}$ as P_A, P_B, p, p', respectively.

The S matrix for beta decay can be written as

$$S=-(2\pi)^4 i\delta(P_B+p+p'-P_A)\mathfrak{M}, \tag{2.1}$$

where

$$\mathfrak{M}=\mathfrak{M}_A+\mathfrak{M}_V=C_A\langle B|J_\lambda{}^A(0)|A\rangle L_\lambda{}^A+C_V\langle B|J_\lambda{}^V(0)|A\rangle L_\lambda{}^V, \tag{2.1a}$$

$$L_\lambda{}^A=\bar{u}_e(p)i\gamma_\lambda\gamma_5(1+\gamma_5)v_\nu(p'), \tag{2.1b}$$

$$L_\lambda{}^V=\bar{u}_e(p)\gamma_\lambda(1+\gamma_5)v_\nu(p'), \tag{2.1c}$$

and in the case of a free neutron ($A=n$ and $B=p$) the well-known invariance arguments[22,23] lead to the following four independent form factors,

$$\langle p|J_\lambda{}^A(0)|n\rangle=(M^2/P_{p0}P_{n0})^{\frac{1}{2}}\bar{u}_p(P_p)\{F_1{}^A(q^2)i\gamma_\lambda\gamma_5+F_2{}^A(q^2)\gamma_5 q_\lambda\}u_n(P_n), \tag{2.1d}$$

$$\langle p|J_\lambda{}^V(0)|n\rangle=(M^2/P_{p0}P_{n0})^{\frac{1}{2}}\bar{u}_p(P_p)\{F_1{}^V(q^2)\gamma_\lambda+F_2{}^V(q^2)\sigma_{\lambda\mu}q_\mu/2M\}u_n(P_n), \tag{2.1e}$$

where $q=p+p'$, and M is the nucleon mass.

If the conserved current theory $q_\mu J_\mu{}^V(0)=0$ is assumed, the vector part of beta decay can be related[13,24] to the well-known Stanford electron scattering experiment by Hofstadter *et al.*;

$$\begin{aligned}F_1{}^V(q^2)&=1-\tfrac{1}{6}q^2/(2m_\pi)^2+\cdots,\\ F_2{}^V(q^2)&=(\mu_p-\mu_n)\{1-\tfrac{1}{6}q^2/(2m_\pi)^2+\cdots\}\\ &=3.70\{1-\tfrac{1}{6}q^2/(2m_\pi)^2+\cdots\}.\end{aligned} \tag{2.2}$$

These relations show that we may legitimately assume $F^V(q^2)$ to be a constant, $F^V(0)$, for nuclear beta-decay $(|q^2|\ll(2m_\pi)^2)$.

The situation in the axial-vector part is a little more complicated.[25] If we assume the partially conserved current theory,[26] we can specify the induced scalar term $F_2{}^V(q^2)$. It is certain that a big induced scalar term would contradict with the experimental facts.[27,28] Fortunately, our RaE problem has nothing to do with the induced pseudoscalar term because of the selection rule, so that we shall not discuss this problem further. Let us assume that $F_1{}^A(q^2)$ can be taken to be one for nuclear beta decay. Also we know that $C_A/C_V=-1.2$.

2.1 Higher Forbidden Corrections

It is the ordinary approximation to handle a nuclear beta decay by assuming that the decaying nucleon in the nucleus is on the mass shell. First we will discuss the vector part more closely according to the conventional way. Using the Gordon equality,

$$\bar{u}(p_2)P_\mu u(p_1)=\bar{u}(p_2)(2M\gamma_\mu-\sigma_{\mu\nu}q_\nu)u(p_1), \tag{2.3}$$

where $P=p_1+p_2$, $q=p_1-p_2$, and $\sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2i$, we can obtain the following expression for the vector part,

$$\mathfrak{M}^V=C_V\bar{u}_P(P_p)\left\{\frac{P_\lambda}{2M}+\sigma_{\lambda\mu}q_\mu\frac{1+\mu_p-\mu_n}{2M}\right\}\times u_n(P_n)L_\lambda{}^V. \tag{2.4}$$

The second term in the right-hand side is known as weak magnetism[29];

$$\sigma_{\lambda\mu}q_\mu L_\lambda{}^V=-\boldsymbol{\sigma}\cdot(\mathbf{q}\times\mathbf{L}^V)+q_0\boldsymbol{\alpha}\cdot\mathbf{L}^V-(\boldsymbol{\alpha}\cdot\mathbf{q})L_0{}^V. \tag{2.5}$$

$(1+\mu_p-\mu_n)$ can be interpreted as the sum of Dirac and anomalous magnetic moments. We get the conventional theory if we set $(\mu_p-\mu_n)$ zero. It is easy to see what are the necessary modifications in order to include the anomalous magnetic moment. Let us write down the nonrelativistic form of the beta-decay operators,[30] then multiply the terms which originate from the second part of Eq. (2.4) by the factor $(1+\mu_p-\mu_n)$.

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Two examples are[29,31,32]:

$$\langle|\boldsymbol{\alpha}\times\mathbf{r}|\rangle \xrightarrow{\mathrm{NR}} (\langle\sigma\rangle-\langle|\mathbf{r}\times\mathbf{p}|\rangle)/M \xrightarrow{\mathrm{FG}} \{(1+\mu_p-\mu_n)\langle\sigma\rangle-\langle|\mathbf{r}\times\mathbf{p}|\rangle\}/M,$$

$$\langle A\rangle=\langle\alpha\hat{r}^2\rangle-3\langle(\boldsymbol{\alpha}\cdot\hat{r})\hat{r}\rangle \xrightarrow{\mathrm{NR}} \{\langle\hat{r}^2p\rangle-3\langle\hat{r}(\hat{r}\cdot\mathbf{p})\rangle + \tfrac{3}{8}\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle/R\}/M \xrightarrow{\mathrm{FG}} \{\langle\hat{r}^2p\rangle-3\langle\hat{r}(\hat{r}\cdot\mathbf{p})\rangle + \tfrac{3}{8}\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle(1+\mu_p-\mu_n)/R\}/M, \quad (2.6)$$

where NR and FG stand for the nonrelativistic representation and the conserved current theory, respectively.

The electric corrections which occur in Eq. (2.5) can be neglected in comparison with the main terms, the first term in the right-hand side of Eq. (2.4). The corrections given in (2.6) are higher forbidden corrections because, for instance, the $\langle A\rangle$ term is part of the third-forbidden correction.

2.2 Extension of Siegert Theorem to Beta Decay

It is well-known that the Siegert theorem[33,34] is valid in the radiative transitions of nuclei. For instance, in the electric-dipole transition the nuclear transition matrix element can be given by

$$\langle f|[H_N, \sum_{i=1}^{A}\tau_{zi}z_i]|i\rangle = i(E_f-E_i)\langle f|\sum_{i=1}^{A}\tau_{zi}z_i|i\rangle, \quad (2.7)$$

provided that the z axis is chosen to be parallel to the polarization vector of the incident photon. H_N is the nuclear Hamiltonian; E_i and E_f are the energies in the initial and final states, respectively. Of course, the expression Eq. (2.7) is equal to the classical one $\langle f|\sum_{i=1}{}^{A}\tau_{zi}p_{zi}|i\rangle/M$, when the exchange potential in H_N makes no contribution to the commutator on the left-hand side of (2.7). The theorem implies that the interactions between the electric field and the charged mesons intervening among nucleons have no influences on the explicit form of Eq. (2.7). This is not in contradiction with the statement that the exchange forces play an important role in determining nuclear wave functions and their energies. This theorem was proved by using the gauge invariance of interactions, the fact that nucleon velocities are much slower than the velocities of the exchanged mesons, and the assumption that the nuclear system can be described by nucleon coordinates only.

If the conserved current hypothesis is valid for beta decay, the Siegert theorem can be generalized to apply to beta decay. For instance, a correct expression for the electric-dipole beta moment, instead of the conventional expression $\langle\alpha\rangle=\langle p\rangle/M$, can be found by replacing τ_z in Eq. (2.7) by τ_+ (negatron emission). However, we must take account of the neutron-proton mass difference and Coulomb potentials, both of which destroy the charge independence of H_N. If H_N were rigorously charge-independent, $\sum_{i=1}{}^{A}\tau_{+i}$ would be exactly conserved quantity as is the electric charge $\sum_{i=1}{}^{A}\tau_{zi}$.[13] Therefore, the beta-decay formula corresponding to Eq. (2.7) requires a different interpretation. Divide the total nuclear Hamiltonian into two parts, the charge-independent part $H^{(1)}$ and the nonindependent part $H^{(2)}$.

$$H_N=H^{(1)}+H^{(2)}, \quad (2.8)$$

where

$$H^{(1)}=\sum_i\frac{P_i^2}{2M}+\sum_{ij}V_{ij}+AM \quad \left(M=\frac{M_p+M_n}{2}\right),$$

and

$$H^{(2)}=\sum\tau_{zi}\frac{M_p-M_n}{2}+\frac{1}{4}\sum_{ij}\frac{e^2(1+\tau_{zi})(1+\tau_{zj})}{r_{ij}}.$$

Suppose that a weak leptonic field is applied to the nucleus. The modified Hamiltonian in the presence of the external field $\mathbf{A}_\beta$ can be denoted by $H_N\{\mathbf{A}_\beta\}$. For the charge-independent part $H^{(1)}$ the usual gauge invariance relation is valid.[34]

$$\begin{aligned} H_N\{\mathbf{A}_\beta\} &= H^{(1)}\{\mathbf{A}_\beta\}+H^{(2)}\{\mathbf{A}_\beta\}, \\ H_N\{\mathrm{grad}G\} &= H^{(1)}\{\mathrm{grad}G\}+H^{(2)} \\ &= e^{iD}H^{(1)}\{0\}e^{-iD}+H^{(2)}. \end{aligned} \quad (2.9)$$

We used the fact that $H^{(2)}$ is not changed by the weak lepton field.[35] In Eq. (2.9) we wrote

$$D=\sum_i C_V G(\mathbf{x}_i)\tau_{+i}, \quad (2.10a)$$

and

$$G(\mathbf{x})=\mathbf{u}\cdot\mathbf{x}, \quad (2.10b)$$

where $\mathbf{u}$ is the polarization vector of lepton current, $(\bar{u}_e,\gamma v_\nu)(0)$. If we expand the $H_N\{\mathbf{A}_\beta\}$ in terms of $\mathbf{A}_\beta$, it is sufficient to determine the second term $H_1\{\mathbf{A}_\beta\}$, which is linear to $\mathbf{A}_\beta$:

$$H_N\{\mathbf{A}_\beta\}=H_0+H_1\{\mathbf{A}_\beta\}+\cdots. \quad (2.11)$$

Comparing (2.11) with (2.9) we obtain[36]

$$H_1\{\mathrm{grad}G\}=i[D,H^{(1)}]=i[D, H_N-H^{(2)}]. \quad (2.12)$$

[31] The correction to allowed beta decay was stated in M. Morita, Nuclear Phys. **14**, 106 (1959) and Phys. Rev. **113**, 1584 (1959).

[32] Notations here for nuclear matrix elements are the same as the textbook to be published by E. J. Konopinski. The relationship to the Konopinski-Uhlenbeck notations [Phys. Rev. **60**, 308 (1941)] is: $\langle\alpha\rangle=-\int\boldsymbol{\alpha}\xrightarrow{\mathrm{NR}}\langle P\rangle/M$; $\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle=-\int\boldsymbol{\sigma}\times\mathbf{r}/R$; $\langle\hat{r}\rangle=\int\mathbf{r}/R$ (where R nuclear radius); and $\langle\sigma\rangle=\int\boldsymbol{\sigma}$.

[33] A. F. J. Siegert, Phys. Rev. **52**, 787 (1937). More systematic proof for general electric transitions was given in reference 34.

[34] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953).

[35] The standard substitution rule $\mathbf{P}\to\mathbf{P}-C^V\tau_+\mathbf{A}$ has no effect on $H^{(2)}$, which includes no $\mathbf{P}$.

[36] For simplicity we have assumed that the nuclear potentials V_{ij} are charge-independent and H_N is nonrelativistic.

Therefore,

$$\langle f|H_1(\mathrm{grad}G)|i\rangle = (W_i-W_f)i\langle f|D|i\rangle - i\langle f|[D,H^{(2)}]|i\rangle. \quad (2.13)$$

Furthermore, we can reduce the Eq. (2.13) to the more practical form:

$$\langle f|H_1(\mathrm{grad}G)|i\rangle = (W_i-W_f-2.5m_e+1.2Z\alpha/R)i\langle f|D|i\rangle, \quad (2.14)$$

where Coulomb terms were estimated by assuming the uniform charge distribution and using the Ahrens-Feenberg approximation,[37]

$$\begin{aligned}\langle f|[D,H^{(2)}]|i\rangle &= \textstyle\sum_j(\langle f|D|j\rangle\langle j|H^{(2)}|i\rangle - \langle f|H^{(2)}|j\rangle\langle j|D|i\rangle) \\ &\cong(\langle i|H^{(2)}|i\rangle - \langle f|H^{(2)}|f\rangle)\langle f|D|i\rangle. \quad (2.15)\end{aligned}$$

Expression (2.14) shows that the part of the first-forbidden beta-decay interaction that corresponds to the electric-dipole interaction in gamma decay is given correctly by $-(W_i-W_f-2.5m_e+1.2Z\alpha/R)i\langle r\rangle$, instead of the conventional one, $\langle\alpha\rangle=\langle p\rangle/M$.

Of course Eqs. (2.9) and (2.10a) are equally valid for more general multipole fields. However, we must take account of the fact that the lepton current vector is not exactly transverse because of the electron-neutrino mass difference.

Let us derive the above expression in another way. The relationship between $\langle p\rangle$ and $\langle r\rangle$ has been discussed by Ahrens and Feenberg.[37,38] For e^- emission they obtain

$$\langle\alpha\rangle=\langle p\rangle/M=-(\alpha Z/2R)\Lambda' i\langle r\rangle, \quad (2.16)$$

where

$$\Lambda' = 2.4+\frac{W_i-W_f-2.5m_e}{m_e}\cdot\frac{A^{\frac{1}{3}}}{Z} + (\text{nuclear potential term}) \quad (2.16a)$$

and

$$(\text{nuclear potential term}) = (2R/\alpha Z)\langle|[\textstyle\sum V_{ij},i\sum \tau_{+k}\mathbf{r}_k]|\rangle/\langle ir\rangle.$$

The well-known Ahrens-Feenberg formula is obtained by inserting an estimate for the nuclear potential term, −1.4. If the mesons arising from exchange forces also perform beta decay, their contributions must be added to the nucleon beta-decay $\langle\alpha\rangle$. Especially in the case of the conserved current theory, just as in the electromagnetic interaction, the Siegert theorem suggests that the correct form of the relevant beta-decay interaction is obtained by omitting the (nuclear potential term) in Eq. (2.16a)[37]:

$$-\left(\frac{\alpha Z}{2R}\right)\Lambda\langle ir\rangle = -\left(\frac{\alpha Z}{2R}\right)\left\{2.4+\frac{W_i-W_f-2.5m_e}{m_e}\frac{A^{\frac{1}{3}}}{Z}\right\}\langle ir\rangle = -\left(1.2\frac{\alpha Z}{R}+W_i-W_f-2.5m_e\right)\langle ir\rangle, \quad (2.17)$$

where we have used the relation, $R=r_0A^{\frac{1}{3}}=(\alpha/2m_e)A^{\frac{1}{3}}$.[39] Equation (2.17) is the same as the expression in (2.14).

The formula (2.14) or (2.17) may be used to discriminate between the conventional bare-nucleon coupling theory and the conserved vector hypothesis, provided that the (nuclear potential term) in Eq. (2.16a) is not negligible. According to Ahrens-Feenberg,[37] it is known to be comparable with the Coulomb term. It should also be remembered that the validity of the approximation (2.15) was assumed in treating the Coulomb term. Therefore, it is desirable to investigate many first-forbidden transitions statistically.

2.3 Quenching Effect and Other Small Effects

The Siegert theorem does not apply to the magnetic operators.[34] The most important correction because of the existence of other nucleons is probably the quenching effect, which has been discussed in detail for nuclear magnetic moments.[40,41] The Pauli principle forbids transitions in which a recoiling nucleon jumps to an occupied nuclear level by emitting pions; this effect causes a quenching of the anomalous magnetic moment of a nucleon in nuclear matter. According to a recent estimate, this effect decreases the anomalous moment by 7%.[41]

Such an effect should naturally be expected in the case of beta decay. If the conserved current hypothesis is valid, the quenching effect should also be of order 7% for magnetic beta decay. This effect decreases the magnitude of the factor $(\mu_p-\mu_n)$ which was discussed in the subsection 2.1. Thus, weak magnetism may be modified in nuclear matter.

There also exist more complicated, but probably smaller, effects.[42] For instance, a nucleon in a nucleus is not on the mass shell. If the nuclear many-body problem were relativistic, this would be important because of the high probability of nucleon pairs production. These cannot be discussed reliably with current physical theories.

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[38] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).

[39] We used this relation in order to compare with the literature. The present data indicate that $r_0=1.2$ instead of $r_0=1.4$. The final expression is correct independent of the value of r_0.

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3. REINVESTIGATION OF RaE BETA DECAY

First we recapitulate the analyses using the conventional forbidden theory for both the RaE beta spectrum and the electron longitudinal polarization. As a method of including the finite size nuclear effect, we assume here the validity of the effective nuclear radii theory[43]; we derive the formulas in the small-αZ approximation for point charge[44] and include the nuclear size effect by modifying the nuclear radii appearing in the final formulas. Many other approximation methods have been proposed,[4,20] but they give similar answers,[8,17] if there are three nuclear matrix elements. Since it is awkward to distinguish each nuclear radius corresponding to a given nuclear matrix element, we use one nuclear radius R in this paper. Therefore, the nuclear matrix elements should be understood to have $+10\%$ errors.[43]

The formula for the beta spectrum ($1^- \to 0^+$) is given by[43]

$$C(W)=C_A{}^2R^2\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle^2\left[Y^2+\tfrac{2}{3}Y\left\{q(X+1)+\frac{p^2}{W}(X-1)\right\}+(X^2+\tfrac{1}{2})\frac{p^2+q^2}{3}+\frac{2}{9}\frac{qp^2}{W}(X^2-1)\right], \quad (3.1)$$

where

$$p^2=W^2-1,\quad q=W_0-W,\quad X=C_V\langle i\hat{r}\rangle/C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle,$$

and

$$YR=\{C_V\langle\alpha\rangle+(\alpha Z/2)(C_V\langle i\hat{r}\rangle-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle)\}/C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle.$$

On the other hand, the formula for electron polarization ($1^- \to 0^+$) is[13]

$$\mathcal{P}(W)=-\frac{p}{W}\left\{1-\frac{C(W)-D(W)}{C(W)}\right\},$$

$$C(W)-D(W)=C_A{}^2\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle^2\frac{2R^2}{3W}\{Y(1-X)+\frac{q}{3}(1-X^2)\}. \quad (3.2)$$

The numerical results using Eqs. (3.1) and (3.2) are shown in Fig. 1 ($W_0=3.29$). In deriving the results in Fig. 1 we equated the theoretical and experimental ratios of the correction factor for $W=1.2$ and 2.0 as well as for $W=2.0$ and 3.0 ($m_ec^2=1$). The experimental values are[45]

$$C(W)_{W=1.2}/C(W)_{W=2}=\left.\begin{matrix}1.57\\1.61\end{matrix}\right\},$$

$$C(W)_{W=3}/C(W)_{W=2}=\left.\begin{matrix}0.49\\0.55\end{matrix}\right\}.$$

For longitudinal polarization data, we used two energies[16];

$$-\mathcal{P}(W)W/p=0.74\pm0.04 \quad \text{at 120 kev.}$$
$$=0.73\pm0.06 \quad \text{at 390 kev.}$$

The electron polarization is ~ 0.75, independent of energy. More precise future experiments may reveal an energy dependence.

As is easily seen from Eq. (3.2), no reduction of electron polarization should be observed if $X=1$. If we fit our theory only with beta-spectrum data, we can obtain the long stripe of parameter region (the shaded area) in Fig. 1. If we add electron polarization data, we obtain the much smaller region (doubly shaded area) in Fig. 1.

It is important to observe that the doubly shaded area is consistent with the theoretical prediction Eq. (2.17). A straight line in Fig. 1 corresponds to Eq. (2.17). If we use the well-known expression by Ahrens and Feenberg[37] ($\Lambda=1+(W_i-W_f-2.5)A^{\frac{1}{3}}/Z$), agreement cannot be obtained. (It cannot even be plotted on the same diagram.) Since Eq. (2.17) was derived by an extension of Siegert theorem using the conserved current hypothesis, our results show the conserved current theory is consistent with RaE beta-decay data.

On the other hand, the predicted value for X is $+0.8$, if we assume the pure shell state ($1h_{9/2}, 1i_{11/2}$) for RaE.[46]

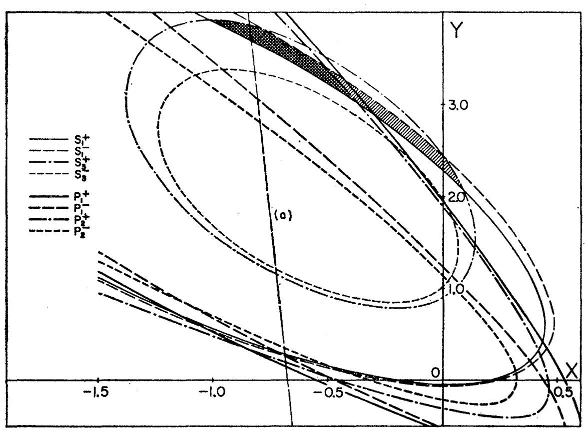

FIG. 1. This figure shows the parameter region which is consistent with the beta spectrum and electron polarization data of RaE. ($X=C_V\langle i\hat{r}\rangle/C_A\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle$ and $YR=C_V\langle\alpha\rangle/C_A\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle+(\alpha Z/2)(X-1)$.) The shaded area stands for the region obtained by using spectrum data, while the doubly shaded area is obtained by using both the spectrum and polarization data. We equated the theoretical and experimental ratios of the correction factor for $W=1.2$ and 2.0 ($S_1{}^\pm$) as well as for $W=2.0$ and 3.0 ($S_2{}^\pm$). For longitudinal polarization we used the data at 120 kev ($\mathcal{P}_1{}^\pm$) and 390 kev ($\mathcal{P}_2{}^\pm$). The straight line (a) represents the extended Siegert theorem (2.17); it is consistent with the RaE data. (We used $r_0=1.2$.)

[43] Z. Matumoto and M. Yamada, Progr. Theoret. Phys. (Kyoto) **19**, 285 (1958).

[44] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

[45] These values are based on reference 1 (Plassman and Langer) and are the same as Lee-Whiting used in their (ST) analysis (reference 5). It is easier to fit the experimental spectrum assuming (VA) rather than (ST). (See reference 8.)

[46] $X=C_V/C_A\epsilon$ and $\epsilon=-1$ for ($h_{9/2}, i_{11/2}$) and $\epsilon=10$ for ($h_{9/2}, g_{9/2}$) in our definition of nuclear matrix elements.[32]

This is the best choice as far as the pure shell model is concerned.[12] If we use the spectrum data only, Fig. 1 shows that the sign of X in the shaded area is consistent with $X=+0.8$. Another possible choice of shell model state $(1h_{9/2},2g_{9/2})^{5,12}$ leads to $X=-0.08$ which is included in the shaded area. However, the results of polarization experiments have removed these possibilities. This result indicates the complicated nuclear structure of RaE, although RaE has only a single neutron-proton pair outside the doubly closed core.

The analysis in this section was carried out without using the higher-forbidden correction discussed in 2.1. The effect of weak magnetism is usually of order $|\mathbf{q}|/M(\cong 10^{-3}$ for RaE), so that the effect is not observable, although in RaE the cancellation between nuclear matrix elements enhances the weak magnetism effect by approximately a factor of 10. The factor $(\mu_p-\mu_n)$ also enhances the weak magnetism effect. More detailed discussion is given in Appendix I.

4. NUCLEAR STRUCTURE OF RaE

The 127th neutron of Bi^{209} is known to be in the $(2g_{9/2})$ state. However, in order to explain the level structure of RaE we must assume[12] that the 127th neutron in Bi^{210} (RaE) is in the $(1i_{11/2})$ state. This can occur if the attractive potential between neutron and proton in $(1h_{9/2},1i_{11/2})$ state is larger than in the $(1h_{9/2},2g_{9/2})$ state.

The analysis in the last section showed that $\epsilon^{-1} \equiv\langle i\hat{r}\rangle/\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle\sim+1$ since $C_V/C_A=-0.8$. In the pure shell model,[5] ϵ is given by

$$\epsilon=-j_p(j_p+1)+j_n(j_n+1)+l_p(l_p+1)-l_n(l_n+1)$$

for the transition $(l_p(j_p);l_n(j_n))\rightarrow(l_p(j_p{}^2);0)$. If the wave function of RaE is $a|1h_{9/2},1i_{11/2}\rangle+b|1h_{9/2},2g_{9/2}\rangle$, ϵ can be given by[5] $\epsilon^{-1}=(1+t)/(10t-1)$, where

$$t=(1/3\sqrt{6})b\int_0^\infty R(1h)R(2g)r^3\,dr\bigg/a\int_0^\infty R(1h)R(1i)r^3\,dr.$$

Thus, if t is 2/9, we can obtain the value $\epsilon=1$.[47]

If Bi^{210} (RaE) is such a mixture of the $(1h_{9/2},1i_{11/2})$ and $(1h_{9/2},2g_{9/2})$, Pb^{210} must also be a mixture of $(\ \ ;1i^2{}_{11/2})$ and $(\ \ ;2g_{9/2}{}^2)$ states, since the transition Pb^{210} to Bi^{210} is not observed. This type of configuration mixing is likely from the viewpoint of the superconductivity model for nuclei.[48]

A sort of collective effect will be discussed in Appendix II.

[47] In reference 5, it was shown that, if $\int_0^\infty R(1h)R(2g)r^3dr$, $\int_0^\infty R(1h)R(1i)r^3dr$, and $\int_0^\infty R^2(1h)R(2g)R(1i)r^2dr$ have the same sign, t must be negative. However, these three may not have the same sign. For instance, if we use the solutions for square-well potentials that have the proton radius 20% smaller than the neutron radius, we find a positive t. [M. Yamada, A. Arima, and J. Fujita (unpublished).]

[48] A. Bohr, B. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958). S. T. Beliaev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **31**, Vol. 11 (1959).

5. CONCLUSIONS

In Sec. 2 we found that, assuming the validity of the conserved current theory, the theory of beta decay is quite similar to the theory of gamma transitions: To the electric-type matrix elements we can apply the Siegert theorem with only slight changes. Namely, the additional terms appear, among which the Coulomb terms are evaluated using the Ahrens-Feenberg approximation. For the magnetic-type matrix elements, complicated effects such as quenching may exist, and thus make dubious the application of the familiar nucleon beta-decay theory to nuclear beta decay. Fortunately, however, these effects are small, at least in the case of RaE.

In Sec. 3 we recapitulated the analyses of RaE using both the beta spectrum and electron polarization data. The final results are in good agreement with the extended Siegert theorem, showing the consistency between the conserved current hypothesis and RaE data. The conserved and nonconserved (conventional) theories are differentiated by the presence or absence of the exchange current contributions. It was also shown that the obtained value for $\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle/i\langle\hat{r}\rangle$ disagrees with the pure shell model prediction. In this section, we adopted the simplest conventional forbidden theory.[43] If one increases the number of unknown nuclear parameters, one might succeed in obtaining agreement with the shell model; however, the author feels that such a procedure merely shifts the anomaly in the RaE problem and is not fruitful.

Information concerning the first-forbidden beta-decay nuclear matrix elements is rapidly being increased.[49] It is desirable to analyze data from the viewpoint of the extended Siegert theorem as described in Sec. 2 and also experimentally investigate possible anomalies in the magnitude of $\langle r\rangle$. An anomaly in the magnitude of $\langle r\rangle$ is well-known to exist in gamma decay.

ACKNOWLEDGMENTS

The author would like to express his sincere thanks to Professor E. J. Konopinski, who suggested this problem, for his continual illuminating discussions. He also appreciates the helpful correspondence with Dr. M. Morita and Dr. T. Kotani. Finally, he thanks Dr. J. Bahcall for reading this manuscript.

APPENDIX I

Formulas for Spectrum and Polarization

In Sec. 2 we discussed the fact that the direct manifestation of the cloud beta decay is the nuclear matrix element $\langle A\rangle$ in the RaE case $(\langle A\rangle=\langle\alpha\hat{r}^2-3\hat{r}(\boldsymbol{\alpha}\cdot\hat{r})\rangle)$. The nuclear matrix element $\langle A\rangle$ is a so-called third-forbidden correction and represents the transverse retardation effect. The effect of $\langle A\rangle$ is negligibly small[3]

[49] D. D. Hoppes, E. Ambler, E. Hayward, and Kaeser, preprint in Ce^{141} experiment (to be published); R. Wilkinson (private communication).

insofar as $\langle A\rangle$ has the same order of magnitude as $\langle\alpha\rangle$. However, it is interesting to study the abnormal case, where $\langle A\rangle$ has a larger magnitude than expected from the usual order-of-magnitude theory; (a) $\langle A\rangle$ in conventional theory is much bigger than $\langle\alpha\rangle$; or (b) the magnetic cloud effect to be added to $\langle A\rangle$ is much larger than the one obtained from the conserved current hypothesis. It will be shown that, even if we adopt these unplausible assumptions, it is highly unlikely for the results of the conventional analyses to be altered significantly.

In the derivation below we will take into account the finite nuclear size effect in the sense of reference 43. The only new assumption is to consider the possibility of abnormally big effect of $\langle A\rangle$; namely, we assume the relationship

$$|\langle\alpha\rangle|\sim|\alpha Z\langle i\hat{r}\rangle|\sim|\alpha Z\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle|\sim|(\alpha Z)^2\langle A\rangle|.$$

Under these assumptions we can get the desired formulas by writing down the point charge formula and then using the small αZ approximation.[43] The nuclear finite size effect can be included by considering slight changes in nuclear radii. The results for beta spectrum of RaE are given by

$$C(W)=C(R^0)+C(R^1)+C(R^2), \tag{I1}$$

where

$$C(R^0)=y^2=\left(C_V\langle\alpha\rangle\left(\frac{1+\gamma}{2}\right)^{\frac{1}{2}}+C_V\langle i\hat{r}\rangle\left(\frac{1-\gamma}{2}\right)^{\frac{1}{2}}-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle\left(\frac{1-\gamma}{2}\right)^{\frac{1}{2}}\right)^2, \tag{I1a}$$

$$\gamma=[1-(\alpha Z)^2]^{\frac{1}{2}}.$$

In the small-αZ approximation we get

$$C(R^0)=\left\{C_V\langle\alpha\rangle+\frac{\alpha Z}{2}(C_V\langle i\hat{r}\rangle-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle)\right\}^2. \tag{I1b}$$

Next

$$\begin{aligned}C(R')=2R\Bigg\{&C_V\langle\alpha\rangle\left(\frac{1+\gamma}{2}\right)^{\frac{1}{2}}+\left(\frac{1-\gamma}{2}\right)^{\frac{1}{2}}(C_V\langle i\hat{r}\rangle-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle)\Bigg\}\Bigg\{C_V\langle A\rangle\frac{\alpha Z}{(1+\gamma)^{\frac{1}{2}}}\frac{\sqrt{2}}{9}q+C_V\left(\frac{1+\gamma}{2}\right)^{\frac{1}{2}}\langle\alpha\rangle\\&\times\left[\frac{-\alpha Z}{(1+\gamma)(2\gamma+1)}\left\{(2\gamma+3)W+\frac{\gamma}{W}\right\}+\frac{\alpha Z}{1+\gamma}\frac{q}{9}\right]\\&+C_V\langle i\hat{r}\rangle\left(\frac{1-\gamma}{2}\right)^{\frac{1}{2}}\left[\frac{\alpha Z}{(1-\gamma)(2\gamma+1)}\left\{(2\gamma-1)W-\frac{\gamma}{W}\right\}+\frac{\alpha Z}{1-\gamma}\frac{q}{3}\right]-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle\left(\frac{1-\gamma}{2}\right)^{\frac{1}{2}}\left[\frac{\alpha Z}{(1-\gamma)(2\gamma+1)}\right.\\&\left.\times\left\{(2\gamma-1)W-\frac{\gamma}{W}\right\}-\frac{\alpha Z}{1-\gamma}\frac{q}{3}\right]\Bigg\}.\end{aligned}$$

In the small αZ approximation we get

$$\begin{aligned}C(R')=2Ry\Bigg[&C_V\langle\alpha\rangle\frac{\alpha Z}{6}\left(\frac{q}{3}-5W-\frac{1}{W}\right)+C_V\langle i\hat{r}\rangle\frac{1}{3}\left(W-\frac{1}{W}+q\right)\\&-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle\frac{1}{3}\left(W-\frac{1}{W}-q\right)+C_V\langle A\rangle\frac{\alpha Z}{9}q\Bigg].\end{aligned} \tag{I1c}$$

The third term of (I1) in the small-αZ approximation is given by

$$\begin{aligned}C(R^2)=R^2\Bigg[&C_V^2\langle i\hat{r}\rangle^2\left(\frac{p^2+q^2}{3}+\frac{2}{9}\frac{qp^2}{W}\right)+C_A^2\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle^2\left(\frac{p^2+q^2}{6}-\frac{2}{9}\frac{p^2q}{W}\right)\\&+C_V^2(\alpha Z\langle A\rangle)^2\frac{1}{72}\left(q^2+\frac{p^2}{4}\right)+2C_V^2\langle i\hat{r}\rangle\alpha Z\langle A\rangle\left(\frac{2}{27}\frac{p^2q}{W}+\frac{q^2}{18}+\frac{p^2}{36}\right)\\&-2C_VC_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle\alpha Z\langle A\rangle\left(\frac{2}{27}\frac{p^2q}{W}-\frac{q^2}{36}-\frac{p^2}{72}\right)+2C_V^2\langle\alpha\rangle\langle A\rangle\frac{2}{27}\frac{p^2q}{W}\Bigg].\end{aligned} \tag{I1d}$$

To simplify these complicated formulas we make use of the following relation:

$$C_V\langle\alpha\rangle=y-\tfrac{1}{2}\alpha Z(C_V\langle i\hat{r}\rangle-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle), \tag{I2}$$

where y must be of the order R, since otherwise we cannot explain the remarkable energy dependence of experimental correction factor $C(W)$. After inserting Eq. (I2) into (I1)'s we introduce the transformations

$$\begin{aligned}C_V\langle i\hat{r}\rangle'&=C_V\langle i\hat{r}\rangle+\frac{\alpha Z}{6}C_V\langle A\rangle,\\C_A\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle'&=C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle+\frac{\alpha Z}{6}C_V\langle A\rangle.\end{aligned} \tag{I3}$$

Then we get the final result,

$$C(W)=y'^2+2Ry'\left[C_V\langle i\hat{r}\rangle'\frac{1}{3}\left(\frac{p^2}{W}+q^2\right)-C_A\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle'\frac{1}{3}\left(\frac{p^2}{W}-q\right)\right]$$
$$+R^2\left[C_V^2\langle i\hat{r}\rangle'^2\left(\frac{p^2+q^2}{3}+\frac{2}{9}\frac{qp^2}{W}\right)+C_A^2\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle'^2\left(\frac{p^2+q^2}{6}-\frac{2}{9}\frac{qp^2}{W}\right)\right]$$
$$+p^2R^2\frac{C_V\alpha Z\langle A\rangle}{72\times 7}\{C_V\alpha Z\langle A\rangle-16C_V\langle i\hat{r}\rangle'-8C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle'\},\quad \text{(I4)}$$

where

$$y'=C_V\langle\alpha\rangle+\tfrac{1}{2}\alpha Z(C_V\langle i\hat{r}\rangle'-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle')=y.\quad \text{(I4a)}$$

On the other hand, the formula for longitudinal polarization is expressed as

$$\mathcal{P}(W)=-\frac{p}{W}\left[1-\frac{C(W)-D(W)}{C(W)}\right],\quad \text{(I5)}$$

$$C(W)-D(W)=\frac{2R}{3W}\left[y'(C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle'-C_V\langle i\hat{r}\rangle')+\frac{R}{3}\{C_A^2\langle|\boldsymbol{\sigma}\times\hat{\mathbf{r}}|\rangle'^2-C_V^2\langle i\hat{r}\rangle'^2\}\right].\quad \text{(I5a)}$$

It is clear that we can get Eqs. (3.1) and (3.2) if we set $\langle A\rangle$ zero in Eqs. (I4) and (I5). It should be noted that appearance of Eq. (I5) is the same as (3.2). Therefore, the polarization analysis given by Fig. 1 is applied to the case of (I5) simply by replacing $\langle i\hat{r}\rangle$ and $\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle$ by $\langle i\hat{r}\rangle'$ and $\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle'$, respectively. On the other hand, the spectrum formula (I4) shows that, insofar as the last term on the right-hand side is not important, (I4) is the same as (3.1) except for primes on the quantities. If the last term is very big, we cannot explain the experimental spectrum because of the wrong energy dependence. If we assume the relation

$$16\langle\alpha\rangle+\tfrac{1}{2}\alpha Z(C_V\langle i\hat{r}\rangle'-C_A\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle')=0,$$

certainly we obtain another parameter region which can be derived from Fig. 1. In this case, the transformation (I3) implies that the doubly shaded region effectively lies closely to the axis $X=0$. However, $X=1$ can never be attained.

If we assume the validity of conserved current hypothesis, $\langle A\rangle_{\text{FG}}$ is given by (2.6):

$$\langle A\rangle_{\text{FG}}=\{\langle\hat{r}^2p\rangle-3\langle\hat{r}(\hat{r}\cdot\mathbf{p})\rangle+\tfrac{3}{8}\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle(1+\mu_p-\mu_n)/R\}/M.$$

This implies that $|\langle A\rangle|$ is much smaller than $|\langle|\boldsymbol{\sigma}\times\hat{r}|\rangle|$. Namely, the transformation (I3) has practically no effect.

APPENDIX II

Collective Model in Beta Decay

Suppose that there is a nucleus (spin 1^-) which consists of a neutron and a proton-hole besides the doubly closed core. If this nucleus performs a beta decay, the final state is the doubly closed shell (spin 0^+). Possible correlation effect among the neutron and proton hole may cause significant configuration mixing between the levels having the same angular momentum and parity. In gamma transition, the correlation effect between an excited nucleon and a hole has been closely studied. It was shown by Brown and Bolsterli[50] that, when repulsive particle-hole interactions dissolve the degeneracy of energy levels, one of the energy levels is pushed up; in the electric-dipole transition, the uppermost level corresponds to the well-known giant resonance, since it carries almost all of oscillator strength. Their schematic model[50] can be easily extended to the beta decay described above, and the uppermost level carries a big beta-moment $\langle ir\rangle$, provided that the neutron and proton-hole interaction is repulsive.

Let us write down the sum rule for beta decay. For the transitions, in which a nucleus in the nth level of A decays into the ground state of the nucleus B,[51]

$$\bar{W}_\beta=\frac{\sum_n(E_n-E_0)|\langle B_0|\sum\tau_{+i}z_i|A_n\rangle|^2}{\sum_n|\langle B_0|\sum\tau_{+i}z_i|A_n\rangle|^2}=\bar{W}_\gamma-1.2\frac{\alpha Z}{R}+2.5m_e,\quad \text{(II1)}$$

where we assumed, for simplicity, that total isospin of the nucleus B_0 is zero. The harmonic mean energy in gamma transitions[52] is

$$\bar{W}_\gamma=\frac{\sum_n(E_n-E_0)|\langle B_n|\sum\tau_{zi}z_i|B_0\rangle|^2}{\sum_n|\langle B_n|\sum\tau_{zi}z_i|B_0\rangle|^2}=\frac{NZ}{A}(1+0.8x)\Bigg/2M\left\langle\left(\frac{N}{A}\sum_{i=1}^{Z}z_i-\frac{Z}{A}\sum_{j=1}^{N}z_j\right)^2\right\rangle_{00}\quad \text{(II2)}$$

(x; fraction of Majorana force).

[50] G. E. Brown and M. Bolsteril, Phys. Rev. Letters **3**, 472 (1959). G. E. Brown, C. Castillejo, and J. A. Evans, Nuclear Phys. **22**, 1 (1960).

[51] The factor 1.2 on the right-hand side was obtained by the same approximation as (2.15.)

[52] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950). J. S. Levinger and D. C. Kent, *ibid.* **95**, 418 (1954). See also M. Goldhaber and E. Teller, *ibid.* **74**, 1046 (1948). M. Ferentz, M. Gell-Mann, and D. Pines, *ibid.* **92**, 836 (1958). J. Fujita, Progr. Theoret. Phys. (Kyoto) **16**, 112 (1956).

Equation (II1) shows that, because of electromagnetic corrections, the corresponding giant resonance level should have rather smaller energy in negatron decay.

Also in heavier nuclei the collective level as discussed above might have some influence on the relevant nuclear matrix element $\langle ir \rangle$. It is known that the observed $E1$ gamma transitions with small energies have extremely small probabilities in comparison with the shell-model values. Our analysis of RaE shows that the absolute magnitude of $\langle ir \rangle$ cannot be very small. While the $\log ft$ for RaE is 8.0, $\log f_c t \leqq 9.4$.[8] This represents that

$$|\langle ir \rangle| \cong |\langle \boldsymbol{\sigma} \times \mathbf{r} | \rangle| \geqq 0.09R,$$

which is not so much smaller than the shell-model prediction.

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Helium-Ion-Induced Fission of Bi, Pb, Tl, and Au†

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The fission cross sections have been measured with solid-state detectors for helium-ion-induced fission of bismuth, lead-206, thallium, and gold. The measurements were made at several helium-ion projectile energies between 30 and 43 Mev. The fission cross sections of bismuth, lead-206, thallium, and gold with 42.8-Mev helium ions are 7.3, 1.8, 0.65, and 0.28 mb, respectively, and the cross sections decrease rapidly with reduced-energy projectiles. The competition between fission and neutron emission as a function of excitation energy is compared with theoretical predictions of Γ_f/Γ_n and some comments are made on the effect of nuclear deformation on the Fermi gas level density parameter a. Fission thresholds for At^{213}, Po^{210}, $Bi^{207,209}$, and Tl^{201} of 15.8 ± 2.0, 18.6 ± 2.0, 20.6 ± 2.0, and 19.9 ± 2.0 Mev are derived. The saddle-point masses of these nuclei relative to Cameron's reference mass surface lie on a smooth curve with the heavy element data, indicating that the shell structure is completely destroyed during the distortion from equilibrium to saddle-point deformation. An empirical equation for fission thresholds is deduced from the saddle-point mass surface which is thought to be valid for nuclei with Z^2/A between 32 and 40.

I. INTRODUCTION

EXCITATION functions for heavy-element ($Z \geqslant 90$) fission have been measured[1] with a variety of projectiles. In these elements, fission accounts for a major share of the compound nucleus cross section. In addition, the competition between fission and neutron emission is rather independent of excitation energy.[2]

The fission of elements in the vicinity of lead shows quite different characteristics. The fission excitation functions have a strong energy dependence and the fission cross sections reach only a small fraction of the compound nucleus cross section even at excitation energies produced with 40-Mev helium-ion projectiles.

Preliminary measurements of fission excitation functions for target elements with $Z<88$ and at excitation energies less than 50 Mev were first made by Neuzil[3] using radiochemical techniques. Similar measurements were made recently by Nicholson[4] with proportional counters. The counting technique has several advantages over the radiochemical technique. Among the difficulties inherent in the radiochemical fission cross-section measurements are assumptions about the fission fragment mass and charge distributions, incomplete and erroneous decay scheme data, and the problems associated with absolute beta and gamma counting. Direct detection of the fission fragments with solid-state junction counters was employed in this research.

The fission fragment cross sections were measured for helium-ion-induced fission of bismuth, lead-206, thallium, and gold. From the fission cross-section measurements, the competition between fission and neutron emission (or the fission probability) was deduced as a function of the excitation energy. The fission probability is related to the height of the potential barrier which controls the fission process. The barrier arises from the forces involved in the large nuclear distortions which lead to fission. As the nucleus is distorted, the increase in the energy due to the nuclear forces (which act approximately as a surface tension) is initially greater than the decrease in Coulomb energy. However, at some distortion, usually designated as the saddle-point deformation, the decrease in Coulomb energy becomes equal to the increase in surface energy and the nucleus

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* On leave from Atomic Energy Establishment, Trombay, India, under sponsorship of the International Cooperation Administration.

1 References to some of these data can be found in recent review articles on fission (see references 2, 24, and 25).

2 R. Vandenbosch and J. R. Huizenga, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy* (United Nations, Geneva, 1958), Vol. 15, p. 284, Paper P688.

3 E. F. Neuzil, University of Washington, thesis, 1959 (unpublished).

4 W. J. Nicholson, Jr., University of Washington, thesis, 1960 (unpublished).