Electromagnetic Production of Charged Vector Mesons*†

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The Coulomb scattering, Compton scattering, bremsstrahlung, and pair production of charged particles of spin one is investigated, in comparison with spin zero and one-half. The singular role of the longitudinal meson degree of freedom in high-energy processes is emphasized. The pair production of charged vector mesons of unit and zero magnetic moment is calculated in Weizsäcker-Williams approximation and compared with the cross section for their production by neutrinos.

I. INTRODUCTION

CHARGED vector mesons have recently been hypothesized as possible intermediary quanta in the weak and strong interactions. In this paper we wish to consider the standard electromagnetic processes—Coulomb scattering, Compton scattering, bremsstrahlung, and pair production—as applied to mesons of spin one. We are specifically interested in the possibility of photoproduction of $B^{\pm}$ pairs in the Coulomb field of some nucleus of charge eZ. The cross section, σ_P, for this second-order electromagnetic process is of order $\alpha Z^2(e^2/M_Bc^2)^2$, or about a thousand times larger than the cross section $\sigma_\nu \sim \alpha^2 Z^2 G$ for the semi-weak process in which weakly coupled vector mesons are supposed to be produced by high-energy neutrinos in the nuclear Coulomb field, eZ.

The most interesting feature of this cross section, σ_P, is that for the production of high-energy vector mesons, when the momentum transfer to the Coulomb field is small, σ_P is (in Born approximation) expected to increase linearly with photon energy. This increase of the cross section with energy is well-known for the bremsstrahlung by charged $S=1$ mesons, and is the basis of Christy and Kusaka's conclusion from the size of cosmic-ray bursts that the spin of cosmic-ray mesons had to be less than one.[1] Nevertheless this increase in cross section at high energies does not obtain for particles with $S=0, \frac{1}{2}$ and cannot be expected to continue indefinitely with increasing energy. Therefore, in the next two sections we will compare the Coulomb and Compton scattering of $S=1$ particles with that of $S=0, \frac{1}{2}$ particles. We will find that the increasing cross section is associated with the longitudinal polarization state that does not exist for $S=0, \frac{1}{2}$. Application of the unitarity limit to the Compton cross section will enable us to obtain a theoretical limit for the applicability of our formulae.

In Sec. IV we will then use the Compton cross sections obtained, in order to calculate by the Weizsäcker-Williams method[2] the cross sections for bremsstrahlung and pair production in the low-momentum-transfer limit. In the concluding section, we will discuss qualitatively some of the experimental difficulties associated with electromagnetic—as compared with neutrino—production of B mesons.

II. COULOMB SCATTERING

The cross sections for vector-meson processes turn out to have a much stronger energy dependence than those for $S=0, \frac{1}{2}$ particles and, in the Born approximation, increase indefinitely with energy. In this section we investigate the Coulomb scattering of vector mesons and show that this singular behavior is associated with the extra longitudinal-spin degree of freedom that $S=1$ particles possess.

We begin with the plane-wave expansion of the free vector-meson field,

$$U_\mu(x) = (2\pi)^{-\frac{3}{2}} \sum_{r=1}^{3} \int \frac{d^3p}{(2E)^{\frac{1}{2}}} \times[\epsilon_\mu{}^r a^r e^{-ip\cdot x} + \epsilon_\mu{}^{r+} b^{r+} e^{ip\cdot x}], \quad (2.1)$$

where E is the meson energy, and a^r, b^r are respectively destruction and creation operators for particles and antiparticles of spin polarization $\epsilon_\mu{}^r$ $(r=1, 2, 3)$. Because of the subsidiary condition, $\partial_\mu U_\mu = 0$, we have

$$p\cdot e^r = \mathbf{p}\cdot\boldsymbol{\epsilon}^r - E\epsilon_0{}^r = 0.$$

If we choose the z axis in the direction of propagation so that

$$\mathbf{p} = (0,0,p),$$

then in the two transverse polarization states $(r=1, 2)$, we have $\epsilon_3{}^r = \epsilon_0{}^r = 0$, and ϵ^1 and ϵ^2 are each unit vectors in the xy plane. For the longitudinal polarization state we can write

$$\boldsymbol{\epsilon}^3 = (0,0,E/M), \quad \epsilon_0{}^3 = p/M. \quad (2.2)$$

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[1] R. F. Christy and S. Kusaka, Phys. Rev. 59, 414 (1941).

[2] C. F. v. Weizsäcker, Z. Physik 88, 612 (1934); E. J. Williams, Phys. Rev. 45, 729 (1934); W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed. Appendix.

In the plane-wave expansion (2.1), the amplitude of the longitudinally polarized state therefore exceeds that of the transverse polarized states by the factor E/M, which can be large for a fast-moving vector meson. In the rest frame there is, of course, no distinction among the three possible polarization states.

Because $\boldsymbol{\varepsilon}^1$, $\boldsymbol{\varepsilon}^2$, and $\mathbf{p}/|p|$ are orthogonal unit vectors, we have

$$\sum_{r=1}^{2} \epsilon_i{}^r \epsilon_j{}^r = \delta_{ij} - p_i p_j / p^2, \tag{2.3}$$

for i, $j=1$, 2, 3. The covariant polarization sum is given by

$$\sum_{r=1}^{3} \epsilon_\mu{}^r \epsilon_\nu{}^r = \delta_{\mu\nu} + p_\mu p_\nu / M^2, \tag{2.4}$$

for μ, $\nu=1$, 2, 3, 4.

For the interacting Lagrangian we have

$$\begin{aligned}\mathcal{L} = {}& \tfrac{1}{2} U_{\mu\nu}{}^+ (\pi_\mu U_\nu - \pi_\nu U_\mu) + \tfrac{1}{2}(\pi_\mu{}^+ U_\nu{}^+ - \pi_\nu{}^+ U_\mu{}^+) U_{\mu\nu} \\ & - \tfrac{1}{2} U_{\mu\nu}{}^+ U_{\mu\nu} + M^2 U_\mu{}^+ U_\mu \\ & + (ie\gamma/2)(U_\mu{}^+ U_\nu - U_\nu{}^+ U_\mu) F_{\mu\nu} \\ & + (ieq/4M^2)[U_{\mu\nu}{}^+ U_\lambda - U_\lambda{}^+ U_{\mu\nu}] \partial_\lambda F_{\mu\nu},\end{aligned} \tag{2.5}$$

where $\pi_\mu = \partial_\mu - ieA_\mu$, and γ and q are specific magnetic-moment and electric-quadrupole moment factors. In this paper we will assume $q=0$ and consider only $\gamma=0$ (or, in Sec. III and IV, $\gamma=-1$), i.e., vector mesons of unit magnetic moment $e\hbar/2Mc$ (or zero magnetic moment). The matrix element of the vector-meson current operator between free-particle states of momentum p and p' is then

$$\begin{aligned}\langle p'|J_\mu|p\rangle = {}& -\tfrac{1}{2} e (2\pi)^{-3} (EE')^{-\frac{1}{2}} [(p_\mu + p_\mu')\epsilon\cdot\epsilon' \\ & + (1+\gamma)(p_\nu - p_\nu')(e_\mu{}'\epsilon_\nu - \epsilon_\nu{}'\epsilon_\mu)].\end{aligned} \tag{2.6}$$

For the differential cross section for Coulomb scattering we find (for $\gamma=0$)

$$\begin{aligned}d\sigma_{\text{Coul}}/d\Omega = {}& \sigma_R (1/4E^2)[4(p\cdot n)^2(\epsilon\cdot\epsilon')^2 \\ & + (p\cdot\epsilon')^2(n\cdot\epsilon)^2 + (p'\cdot\epsilon)^2(n\cdot\epsilon')^2 \\ & + 2(p\cdot\epsilon')(p'\cdot\epsilon)(n\cdot\epsilon)(n\cdot\epsilon') \\ & - 4(p\cdot n)(p\cdot\epsilon')(n\cdot\epsilon)(\epsilon\cdot\epsilon') \\ & - 4(p'\cdot\epsilon)(\epsilon'\cdot n)(p\cdot n)(\epsilon\cdot\epsilon')],\end{aligned} \tag{2.7}$$

where n_μ is the polarization of the virtual photon and

$$\sigma_R = \left[\frac{\alpha Z}{2pv \sin^2(\theta/2)}\right]^2 = \frac{1}{4} Z^2 r_0^2 \left(\frac{1}{\beta^2\gamma}\right)^2 \frac{1}{\sin^4(\theta/2)} \tag{2.8}$$

is the relativistic Rutherford cross section for scattering through the angle θ ($\gamma = E/Mc^2$, $\beta = p/E$, $r_0 = e^2/Mc^2$).

a. Transverse-transverse spin transitions. When both the initial and final mesons are transverse polarized, Eq. (2.7) gives

$$d\sigma/d\Omega = \sigma_R(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}')^2,$$

or, if Eq. (2.3) is used to sum over the transverse polarizations,

$$d\sigma/d\Omega = \sigma_R(1+\cos^2\theta) \quad \text{(trans.-trans.).} \tag{2.9}$$

b. Longitudinal-longitudinal spin transitions. When both the initial and final mesons are longitudinally polarized, Eqs. (2.2) and (2.7) give

$$d\sigma/d\Omega = \sigma_R \cos^2\theta \quad \text{(long.-long.).} \tag{2.10}$$

c. Transverse-longitudinal spin transitions. Finally, when the initial meson is transversly polarized and the longitudinal meson longitudinally polarized (or vice versa), we have

$$d\sigma/d\Omega = \sigma_R[(E^2+M^2)/2ME]^2(\mathbf{p}'\cdot\boldsymbol{\varepsilon}/|\mathbf{p}|)^2.$$

By summing over the transverse polarizations, we obtain

$$d\sigma_{\text{Coul}}/d\Omega = \sigma_R((1+\gamma^2)/2\gamma)^2 \quad \text{(trans.-long.).} \tag{2.11}$$

We sum over final polarizations and average over initial polarizations by adding Eqs. (2.9), (2.10), and twice (2.11) (to account for both transverse-longitudinal-transverse transitions) and then dividing by three, the statistical weight of the initial state. We then obtain,[3] for $S=1$,

$$d\sigma_{\text{Coul}}/d\Omega = \sigma_R(1+\tfrac{1}{6}\beta^4\gamma^2\sin^2\theta). \tag{2.12}$$

For comparison, the cross sections for the Coulomb scattering of particles with $S=0$ and $\frac{1}{2}$ are respectively,

$$d\sigma_{\text{Coul}}/d\Omega = \sigma_R, \tag{2.13}$$

and

$$d\sigma_{\text{Coul}}/d\Omega = \sigma_R[1-\beta^2\sin^2(\theta/2)]. \tag{2.14}$$

Of course, in the nonrelativistic limit ($\beta\rightarrow 0$), the Coulomb cross section is given in all cases by the classical Rutherford formula. The cross section (2.12) increases with increasing energy γMc^2 of the vector meson. By reference to Eq. (2.11), we see that this increase of the cross section with increasing energy is due to the increase with energy of the matrix element for spin-flip transitions. In the next section, we will see that transitions involving longitudinal vector mesons also lead to Compton, bremsstrahlung and pair production cross sections that (in Born approximation) increase with energy.

III. COMPTON EFFECT

The scattering of photons off vector mesons of unit magnetic moment has been calculated by Booth and Wilson,[4] who obtain in the rest frame of the initial meson

$$\begin{aligned}d\sigma_{\text{Com}}/d\Omega = \sigma_C\Big[& 1+\cos^2\theta + \frac{kk'}{12m^2}(7-16\cos\theta+3\cos^2\theta) \\ & + \frac{k^2+k'^2}{48m^2}(29-16\cos\theta+\cos^2\theta)\Big], \quad (g=1),\end{aligned} \tag{3.1}$$

[3] H. S. W. Massey and H. C. Corben, Proc. Cambridge Phil. Soc. **35**, 463 (1939).

[4] F. Booth and A. H. Wilson, Proc. Roy. Soc. (London) **175**, 483 (1940).

where

$$\sigma_C = \tfrac{1}{2} r_0^2 (k'/k)^2. \tag{3.2}$$

Here k and k' are the momenta of the incident and scattered photon, and θ is the scattering angle, so that

$$\frac{k'}{k} = \frac{M}{M+k(1-\cos\theta)} = \frac{M-k'(1-\cos\theta)}{M}. \tag{3.3}$$

This cross section also increases for increasing k^2.

We have recalculated the Compton cross section for vector mesons of zero magnetic moment (gyromagnetic ratio $g=0$, or $\gamma=-1$) and obtain

$$d\sigma_{\mathrm{Com}}/d\Omega = \sigma_C\left[1+\cos^2\theta - \frac{4}{3}\frac{kk'}{m^2}\cos\theta(1-\cos\theta) + \frac{k^2+k'^2}{3m^2}(5+\cos^2\theta)\right], \quad (g=0). \tag{3.4}$$

That Eq. (3.4) shows the same increase with energy as Eq. (3.1), suggests that this effect is not associated with the precise value of the magnetic moment but is again associated with the third kinematic degree of freedom.

For comparison, the Compton cross sections for particles with $S=0, \frac{1}{2}$ are

$$d\sigma_{\mathrm{Com}}/d\Omega = \sigma_C[1+\cos^2\theta] \quad (S=0), \tag{3.5}$$

and

$$d\sigma_{\mathrm{Com}}/d\Omega = \sigma_C[1+\cos^2\theta + (k'/k) + (k/k') - 2] \quad (S=\tfrac{1}{2}). \tag{3.6}$$

In the long-wavelength limit ($k \to 0$), these cross sections all reduce, of course, to the Thomson cross section (3.5). In the forward direction ($k'=k$) the Klein-Nishina formula (3.6) agrees with the Thomson formula (3.5), but the $S=1$ cross sections (3.1) and (3.4) do not. This difference between the $S=1$ forward scattering and the classical result is again due to longitudinal-transverse vector-meson ($\Delta M=1$) transitions at the absorption and emission of the electromagnetic quantum. This over-all $\Delta M=2$ transition leads to forward scattering of the photon with spin flip ($\Delta m=2$); in the scattering of $S=\frac{1}{2}$ particles, on the other hand, $\Delta m=2$ is impossible.

The Compton cross sections (3.1) and (3.4) can not increase indefinitely with energy. It is interesting to impose unitarity as a limit on the validity of these formulae. For this purpose, one must express the Compton cross section in the photon-particle center-of-mass (c.m.) system (designated with subscript c).[5] Now $d\sigma$ is invariant. Introducing the invariants

$$\begin{aligned} \bar{s} &= (p+k)^2 - M^2 = 2p_c(E_c+p_c), \\ t &= (p-p')^2 = -2p_c^2(1-\cos\theta_c), \\ \bar{u} &= (p-k')^2 - M^2 = -2p_c(E_c+p_c\cos\theta_c), \end{aligned} \tag{3.7}$$

so that

$$\bar{s}+t+\bar{u}=0,$$

we obtain

$$\sigma_C d\Omega_{\mathrm{lab}} = \frac{1}{2} r_0^2 \frac{M^2}{\bar{s}+M^2} d\Omega_c, \tag{3.8}$$

and

$$1-\cos\theta = (1-\cos\theta_c)\frac{1-\beta}{1+\beta\cos\theta_c}, \tag{3.9}$$

where $\beta=(p/E)_c$ is the velocity of the center of mass relative to the meson rest frame. Equation (3.9) is the relativistic angular-aberration formula. The right-hand side of Eq. (3.8) contains no dependence on the c.m. scattering angle θ_c. The angular dependence of $(d\sigma/d\Omega)_c$ is therefore contained in the square brackets of Eqs. (3.1), (3.4), (3.5), and (3.6). In terms of the invariants (3.7), we have:

$$\begin{aligned} \cos\theta &= 1+2M^2 t/\bar{s}(\bar{s}+t), \\ (k/k')+(k'/k)-2 &= t^2/\bar{s}(\bar{s}+t), \\ kk'/M^2 &= \bar{s}(\bar{s}+t)/(2M^2)^2, \\ (k^2+k'^2)/M^2 &= [\bar{s}^2+(\bar{s}+t)^2]/(2M^2)^2, \end{aligned} \tag{3.10}$$

so that, particularly when $\bar{s}$ (the energy available in the center of mass) is large, none of the square-bracketed terms is very sensitive to t, which contains the dependence on θ_c. The angular distribution $(d\sigma_{\mathrm{Com}}/d\Omega)_c$ is therefore relatively flat, which suggests that, in the c.m. system, only a few partial waves contribute to the Compton scattering. These cross sections will therefore be limited by unitarity to some few multiples of π/p^2 or,

$$\text{unitarity limit} \sim N/\bar{s},\ N\sim 10. \tag{3.11}$$

Referring to Eqs. (3.1) and (3.4), we have, for the Compton scattering by vector mesons

$$d\sigma_{\mathrm{Com}}/d\Omega_c \lesssim r_0^2\bar{s}/8M^2. \tag{3.12}$$

The requirement that Eq. (3.12) not exceed (3.11), the unitarity limit, restricts the validity of Eqs. (3.1) and (3.4) to

$$\bar{s}/M^2 \leqslant (8N)^{\frac{1}{2}}(137). \tag{3.13}$$

Since, in the laboratory frame, we have $\bar{s}/M^2=2k/M$, the vector-meson Compton-scattering formulas (3.1) and (3.4) will not violate unitarity for photon energies

$$k < 500M. \tag{3.14}$$

This result allows us to confidently apply, in the next section, these Compton scattering formulas to the calculation of pair production.[6]

[5] We are indebted to Dr. S. Frautschi for helpful discussions of this point.

[6] The same limit to the Born approximation has been derived by J. R. Oppenheimer, Phys. Rev. **59**, 462 (1941) and by L. Landau, J. Phys. U.S.S.R. **2**, 483 (1940). Oppenheimer requires that the interaction energy in the center of mass be small compared with the total energy. Landau requires that the cross section for all competing processes be small compared with the cross section in question.

IV. BREMSSTRAHLUNG AND PAIR PRODUCTION

A. Coherent and Incoherent Pair Production

Our principal purpose is to arrive at a cross section for the electromagnetic production of vector-meson pairs in the Coulomb field of a nucleus of charge eZ and radius d. [We define d as the radius of the equivalent uniform charge distribution so that for heavy nuclei[7]

$$d=(1.2)A^{\frac{1}{3}}\text{ fermi}, \tag{4.1}$$

and

$$q_{\max}=\hbar/d \tag{4.2}$$

is the maximum momentum value occurring in the analysis of the nuclear momentum distribution. Thus $\hbar/Mcd\approx(m_\pi/M)A^{-\frac{1}{3}}$.] For an individual nucleon we have[9]

$$d=1.4\text{ fermi}, \tag{4.3}$$

and

$$q_{\max}\approx 500\text{ Mev}/c. \tag{4.4}$$

In the production of charged particles of mass M by photons of momentum k,

$$q_{\min}=M^2/2k \tag{4.5}$$

is the minimum possible momentum transfer to the nucleus. For

$$k<M^2/2q_{\max}, \tag{4.6}$$

the pair production will be off individual nucleons rather than the nucleus as a whole. The cross section for the pair production coherently off the nucleus as a whole is proportional to $Z^2F^2(q)$, where $F(q)$ is the nuclear form factor. In the high-momentum-transfer limit, this factor is replaced by $ZF_0{}^2(q)$, where $F_0(q)$ is the nucleon form factor. According to Eq. (4.6), for B mesons with the mass of the K meson produced off lead, coherent production is to be expected for photon energies $k>16$ Bev. It thus appears that for existing or presently envisaged electron synchrotrons or linear accelerators, any B mesons produced will be produced incoherently off individual nucleons, and that for B mesons produced in really high-energy accelerators, the coherent production off heavy nuclei will be more important.

For low photon energies (high momentum transfer), the meson-spin degrees of freedom cannot be excited, and the cross section for the production of pairs of $S=1$ mesons will be similar to that of $S=0$, $\frac{1}{2}$ particles. (This is clear for bremsstrahlung, where a threshold theorem applies; pair production and bremsstrahlung are, of course, related by the substitution rule.) We will therefore devote ourselves to the calculation of vector-meson pair production in the opposite limit of high energies or low momentum transfer. In this limit, features specifically characteristic of vector mesons do appear. The most interesting of these features is that the cross section (4.38) or (4.39) is expected to increase with increasing photon energy. This means that the coherent pair production by photons of energy k is ultimately expected to exceed the incoherent production by the factor

$$Z^2\frac{k}{M}\frac{\hbar}{Mcd}\bigg/ZF^2(q)\approx Z^{\frac{2}{3}}\frac{k}{M}\frac{1}{F^2(q)}\left(\frac{M_\pi}{M}\right). \tag{4.7}$$

B. Weizsäcker-Williams Approximation

In the low-momentum-transfer limit we can calculate pair production from the Compton cross-section formulas in Sec. III using the method of Weizsäcker and Williams.[2] We first calculate bremsstrahlung in the low-momentum-transfer limit and obtain the pair-production formulas by the usual substitution rule. The bremsstrahlung from vector mesons of unit magnetic moment was calculated by Christy and Kusaka in this way.[1]

In the Weizsäcker-Williams method, the bremsstrahlung from a meson moving rapidly past a nucleus at rest is calculated by going to the opposite Lorentz frame in which the meson is at rest and the heavy nucleus is passing by rapidly. In this frame, the bremsstrahlung of photons off the meson is viewed as the Compton scattering of virtual photons of initial energy k^* (from the electromagnetic field of the fast moving nucleus) to give (real) photons of energy k'^*. (Unstarred and starred quantities are respectively in the laboratory frame, where the nucleus is at rest, and in the meson rest frame. We recall that the Compton cross sections (3.1) (3.4) (3.5), and (3.6),

$$d\sigma_{\text{Com}}=d\Omega^*\sigma_C[\ ], \tag{4.8}$$

were all calculated in the particle rest frame. Therefore the Ω, θ, k, and k' appearing in these formulas will, in this section, all carry stars.)

If, by using Eq. (3.3), we express the angle of scattering in terms of the scattered quantum energy k'^*, then Eq. (4.8) becomes

$$d\sigma_{\text{Com}}=\pi r_0{}^2(M/k^{*2})dk'^*[\ ]. \tag{4.9}$$

For a fast-moving meson, the Lorentz transformation from the nuclear rest frame to the meson rest frame gives

$$k^*=(2E/M)k, \tag{4.10}$$

$$k'^*=(2E'/M)k, \tag{4.11}$$

where $E'\equiv E-k'$, and we have assumed $E\gg k$. From Eqs. (4.10) and (4.11) we have

$$dk'^*/k^{*2}=dk'/Ek^*, \tag{4.12}$$

so that in terms of the bremsstrahlung quantum energy k' in the laboratory frame, we can write

$$d\sigma_{\text{Com}}=\pi r_0{}^2(Mdk'/Ek^*)[\ ]\equiv\phi(k',k^*)dk'. \tag{4.12}$$

In the bracket, $\cos\theta^*$ is also to be expressed in terms of

[7] R. Hofstadter, Ann. Rev. Nuclear Sci. 7, 231 (1957).

k^* and k'; we have

$$\tfrac{1}{2}(1-\cos\theta^*)=k_{\min}^*/k^*, \tag{4.13}$$

where

$$k_{\min}^*=(M/2)(k'/E') \tag{4.14}$$

is, by Eqs. (3.3) and (4.11), the minimum momentum transfer permitted by the kinematics.

It is useful to define

$$y\equiv k_{\min}^*/k^*=\tfrac{1}{2}(1-\cos\theta^*),$$

which runs between the limits

$$B\leqslant y\leqslant 1,$$

where

$$B=k_{\min}^*/k_{\max}^*=(k'/2E')(Mcd/\hbar)\ll 1.$$

Then, in the four cases considered we have:

$$[(S=0)]=2-4y+4y^2, \tag{4.15}$$

$$[(S=\tfrac{1}{2})]=E/E'+E'/E-4y+4y^2, \tag{4.16}$$

$$\begin{aligned}[(S=1),\,(g=1)]=1&+(1-2y)^2+y^{-2}(k'^2/48EE')\\ &\times[7-16(1-2y)+3(1-2y)^2\\ &+y^{-2}(k'^2/192)(1/E'^2+1/E^2)\\ &\times[29-16(1-2y)+(1-2y)^2],\end{aligned} \tag{4.17}$$

$$\begin{aligned}[(S=1),\,(g=0)]=1&+(1-2y)^2\\ &-y^{-2}(k'^2/3EE')(1-2y)2y\\ &+y^{-2}(k'^2/12)(1/E'^2+1/E^2)\\ &\times[5+(1-2y)^2].\end{aligned} \tag{4.18}$$

The cross section for the bremsstrahlung of a photon of energy k' is thus given by

$$d\sigma_B=\int_{k_{\min}^*}^{k_{\max}^*} q(k^*)dk^*\phi(k',k^*)dk'. \tag{4.19}$$

Here $q(k^*)dk^*$ is the equivalent number of virtual quanta with energies between k^* and k^*+dk^* that is contained in the Coulomb field of the nucleus. The integration over virtual quantum energies in Eq. (4.19) extends from a $k_{\min}^*$ determined by the kinematics, to a $k_{\max}^*$ determined by the spatial extension of the nucleus.

The number of equivalent quanta of momentum k^* is determined by integrating over impact parameters the quantity $p(\nu)d\nu$, which is the number of equivalent photons of frequency $\nu^*=k^*/\hbar$ appearing at impact parameter b in the electromagnetic field of the fast moving nucleus. Thus we have

$$q(k^*)dk^*=\int_{b_{\min}}^{b_{\max}} p(\nu)d\nu 2\pi b db. \tag{4.20}$$

By the condition that

$$\int_0^\infty p(\nu^*)h\nu^*d\nu^*,$$

gives the Poynting flux at distance b, one obtains

$$p(\nu^*)\approx\frac{\alpha Z^2}{\pi^2}\frac{1}{\nu^*}\frac{1}{b^2}. \tag{4.21}$$

Equation (4.21) is restricted by an approximation involved in estimating the Poynting flux, to values

$$b<b_{\max}=(E/k^*)(\hbar/Mc), \tag{4.22}$$

where E is the meson energy in the nuclear rest frame. (We are neglecting screening, i.e., assuming $b_{\max}<137(\hbar/Mc)Z^{-\frac{1}{3}}$, the atomic radius on the Thomas-Fermi model.)

The lower limit in the integral (4.20) is determined by the nuclear size,

$$b_{\min}=d. \tag{4.23}$$

(For a point nucleus, $b_{\min}$ is determined by the requirement that the impact parameter be considerably larger than the wave packet size in order for the Weizsäcker-Williams classical picture to apply; then we have

$$b_{\min}=\hbar/Mc.)$$

Thus, we can write

$$q(k^*)dk^*=\frac{2\alpha Z^2}{\pi}\frac{dk^*}{k^*}\ln\frac{b_{\max}}{b_{\min}}, \tag{4.24}$$

where $b_{\max}/b_{\min}=(E/k^*)(\hbar/Mcd)$ for an extended nucleus, and $b_{\max}/b_{\min}=E/k^*$ for a point nucleus.

From Eqs. (4.19) and (4.12) we have,

$$d\sigma_B=2\bar{\phi}dk'\left[\int_{k_{\min}^*}^{k_{\max}^*}\frac{dk^*}{k^{*2}}[\ ]\frac{M}{E}\ln\left(\frac{E}{k^*}\frac{\hbar}{Mcd}\right)\right], \tag{4.25}$$

where

$$\bar{\phi}=\alpha Z^2r_0^2\approx Z^2(6\times10^{-34}\text{ cm}^2)(M_K/M_B)^2.$$

For a point nucleus the logarithm should be replaced by $\ln E/k^*$. Finally, we have

$$d\sigma_B=4\bar{\phi}\frac{dk'}{k'}\frac{E'}{E}\left[\int_B^1 dx[\ ]\ln Ax\right], \tag{4.26}$$

where $A=(2EE'/Mk')(\hbar/Mcd)$ for an extended nucleus, and $A=2EE'/Mk$ for a point nucleus, and the expression in the brackets is given by Eqs. (4.15) through (4.18) for the four cases being considered.

C. Bremsstrahlung Cross Sections

Carrying out the integration (4.26), we obtain in the no-screening, relativistic limit (E, $E'\gg M$):

$$d\sigma_B=4\bar{\phi}\ln A\frac{4E'}{3E}\frac{dk'}{k'},\quad (S=0), \tag{4.27}$$

$$d\sigma_B=4\bar{\phi}\ln A\frac{E^2-\frac{2}{3}EE'+E'^2}{E^2}\frac{dk'}{k'},\quad (S=\tfrac{1}{2}),\tag{4.28}$$

$$\begin{aligned}d\sigma_B=4\bar{\phi}\biggl\{&\frac{7E^2-12EE'+7E'^2}{48EM^2d}\\&+\frac{7E^2+20EE'+7E'^2}{96E^2}\frac{E-E'}{E'}\ln^2 A\\&+\left[\frac{4}{3}\frac{E'}{E-E'}-\frac{E-E'}{E'}\frac{5E^2-36EE'+5E'^2}{96E^2}\right]\ln A\\&-\frac{13}{3}\frac{E'}{E-E'}-\frac{E^2+12EE'+E'^2}{48E^2}\frac{E-E'}{E'}\biggr\}\frac{dk'}{E},\end{aligned}$$
$$(S=1,\quad g=1),\tag{4.29}$$

$$d\sigma_B=4\bar{\phi}\frac{E^2+E'^2}{2E^2}\left(\frac{\hbar}{Mcd}\right)\frac{dk'}{Mc^2},\quad (S=1,\quad g=0).\tag{4.30}$$

In obtaining Eqs. (4.28) and (4.30), we have retained only the leading terms in k^* or y^{-1}. The remarkable difference between the two $S=1$ cases and the $S=0, \frac{1}{2}$ cases is due to the singular energy dependence of the vector-meson electrodynamics expressed in the energy-increasing Compton cross sections (4.1) and (4.4).

Cross sections (4.27) through (4.29) are the same as those quoted by Pauli (for an extended nucleus),[8] together with original references. Cross section (4.29) is qualitatively no different from (4.28), that obtained by Christy and Kusaka[1] by the same method. Our result merely serves to suggest that energy-increasing cross sections obtained in vector-meson electrodynamics are not peculiar to any particular magnetic-moment value.

D. Pair Production

To go from bremsstrahlung to pair production we merely change k' to k, change the sign of E relative to E' (except in the logarithm) and change the phase-space factors

$$\frac{p'}{p}\frac{dk'}{k'}\rightarrow(2S+1)\frac{1}{2}\frac{pp'dE}{k^3}.\tag{4.31}$$

On the right side, p and p' refer to the momenta of the two charged particles produced and $k=E+E'$. The spin phase-space factors $(2S+1)$ and $\frac{1}{2}$ are present because, while in bremsstrahlung we average over spins of the incident particle and sum over the two photon polarization states, in pair production we sum over spins of the emergent antiparticle and average over the photon polarization states.

[8] W. Pauli, Revs. Modern Phys. **13**, 203 (1941).

The cross sections for the production of charged particles of energy E and $E'=k-E$ that are obtained in this way are

$$d\sigma_P=\bar{\phi}(\ln B)\frac{8EE'}{3k^3},\quad (S=0)\tag{4.32}$$

$$d\sigma_P=\bar{\phi}(\ln B)\frac{4(E^2+E'^2+\frac{2}{3}EE')}{k^3}dE,\quad (S=\tfrac{1}{2})\tag{4.33}$$

$$\begin{aligned}\sigma_P=\bar{\phi}\biggl\{&\frac{7E^2+12EE'+7E'^2}{8M^2dk}-\frac{7k^2-34EE'}{16EE'}\ln^2 B\\&+\left[\frac{26EE'+5k^2}{16EE'}-\frac{8EE'}{k^2}\right]\ln B\\&+\frac{26}{3}\frac{EE'}{k^2}+\frac{k^2-14EE'}{8EE'}\biggr\}\frac{dE}{k},\quad (S=1,\quad g=1),\end{aligned}\tag{4.34}$$

$$d\sigma_P=\bar{\phi}\left(\frac{\hbar}{Mcd}\right)\cdot\frac{3(E^2+E'^2)}{Mc^2k^2}dE,\quad (S=1,\quad g=0),\tag{4.35}$$

where $B=(2EE'/Mk)(h/Mcd)$ for an extended nucleus, and $B=2EE'/kM$ for a point nucleus. In these formulas we have assumed E, $E'\ll Mc^2$, but screening has been neglected, i.e., $2EE'/Mk\ll 137Z^{-\frac{1}{3}}$. Equation (4.33) is a standard result.[9] Equation (4.32) differs, as Drell has already noted,[10] by a factor of two from the result quoted by Pauli for $S=0$.[8]

Integrating Eqs. (4.32) through (4.35) from $E=M$ to $k-M$, we obtain the total cross sections for the production of pairs by quanta of energy k (assumed large compared with M):

$$\sigma_T=\bar{\phi}(\ln\xi)(4/9),\quad (S=0),\tag{4.36}$$

$$\sigma_T=\bar{\phi}(\ln\xi)(28/9),\quad (S=\tfrac{1}{2}),\tag{4.37}$$

$$\sigma_T=\bar{\phi}\left[\frac{5}{12}\xi+\frac{5}{12}\ln^3\xi+\frac{39}{16}\ln^2\xi+\frac{13}{24}\ln\xi\right],$$
$$(S=1,g=1),\tag{4.38}$$

$$\sigma_T=\bar{\phi}\xi,\quad (S=1,\quad g=0),\tag{4.39}$$

where $\xi=(2k/M)(h/Mcd)$ for an extended nucleus and $\xi=2k/M$ for a point nucleus. Dividing Eqs. (4.32) through (4.35) by the corresponding quantities σ_T in Eqs. (4.36) through (4.39), we obtain the normalized probabilities of producing a pair with energies E and E'. In units of the photon energy $[E\equiv kx,\ E'\equiv k(1-x)]$,

[9] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed.
[10] S. D. Drell, Phys. Rev. Letters **5**, 278 (1960).

this distribution is given by

$$d\sigma_B/\sigma_T = 6x(1-x), \quad (S=0) \tag{4.40}$$

$$= (3/7)(4x^2-4x+3)dx, \quad (S=\tfrac{1}{2}) \tag{4.41}$$

$$= (3/20)(2x^2-2x+7)dx, \quad (S=1, g=1) \tag{4.42}$$

$$= \tfrac{3}{2}(2x^2-2x+1)dx, \quad (S=1, g=0). \tag{4.43}$$

The probability of producing a pair of spinless mesons is thus a maximum for $E=E'=k/2$ and falls to zero for E or $E'=0$. On the other hand, for a Dirac particle and for the two vector-meson cases considered, the probability that one member of the pair will take all of the photon energy is respectively 3/2, 21/20, and 2 times the probability that the photon energy will be divided equally. The energy distribution of vector mesons produced is thus rather flatter or steeper than the energy distribution for relativistic spin one-half particles, according to whether the meson magnetic moment is zero or one meson magneton.

V. CONCLUSIONS

The total cross section for the production of single B mesons in the Coulomb field of a nucleus by neutrinos of momentum k is[11]

$$\sigma_\nu = \alpha Z^2(G/6\pi\sqrt{2})\{(g-2)(\ln\xi)^3 \times[-(7/2)(g-2)^2+24(g-1)](\ln\xi)^2+\cdots\}, \tag{5.1}$$

in the same low-momentum-transfer approximation that was used to calculate Eqs. (4.38) and (4.39). This cross section increases only logarithmically with k because the greatest contribution to the neutrino production is at relatively large impact parameters in the Coulomb field. The ratio of Eq. (4.38) or (4.39) to (5.1) is about

$$\alpha(h/Mcd)(k/Mc^2)6\pi\sqrt{2}/QM^2 \approx 2000(m_\pi/M)A^{-\frac{1}{3}}(k/Mc^2). \tag{5.2}$$

The cross section for the electromagnetic production of vector mesons is thus large compared with that for the neutrino production. The probability of competing electromagnetic processes is also extremely large. This "background" will consist principally of photoproduced pions which decay into muons and electrons, and of pairs of electron and of muons (for which $\bar{\phi}$ is at least 10^6 or 25 times larger, respectively, than for B mesons).

The B meson is to be distinguished from this large background by its large mass and prompt decay. On both these accounts, the B-meson decay products will tend to appear at relatively large angles compared with directly produced particles. Two interesting B-meson signatures would seem to be wide-angle μ^+, μ^-, or $\mu^\pm e_\pm$ coincidences. Each of these leptons will typically have one-quarter the original photon energy, while with directly produced pairs each of the particles obtains on the average one half the photon energy. The lepton products of the semi-weak B-meson decay will also be partially polarized.

It would seem that the neutrino and electromagnetic production of B mesons may constitute parts of two different programs: one a study of weak neutrino interactions, the other a study of the electromagnetic creation of new charged particles.

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[11] T. D. Lee, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1961), p. 567