# Electromagnetic Properties of the Low-Lying States of ${}_6C_6^{12}$ in the Oscillating Drop Model*

J. D. WALECKA
*Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California*

(Received September 14, 1961)

The electromagnetic properties of the first four states of ${}_6C_6^{12}$ are discussed within the framework of the incompressible liquid-drop model. The radius of the drop is determined by the elastic electron scattering from the ground state. The surface tension and mass participating in the motion are treated as parameters and are fixed by the energy and lifetime of the first-excited state ($2^+$) at 4.43 Mev. This yields an inelastic-electron scattering cross section close to the experimental result. The $0^+$ state at 7.66 Mev is then identified with the two-surfon state and the inelastic electron scattering cross section and matrix element for pair emission to the ground state are found to be in quantitative agreement with the experimental results. The state at 9.63 Mev is studied as both the $2^+$, two-surfon state and $3^-$, one-surfon state which is also predicted at this energy, and it is shown that only the $3^-$ state has an inelastic electron-scattering cross section which is consistent with the experimental results. This total electromagnetic width of the $0^+$ state at 7.66 Mev, which is of astrophysical interest, is calculated to be $\Gamma_{el}=1.02\times10^{-2}$ ev. The other electromagnetic widths for the first four states are calculated and reasonable agreement is obtained with the experimental data where it exists. The other members of the $0^+$, $2^+$, $4^+$ triplet are unassigned. There are other levels in the vicinity of 10 Mev in $C^{12}$. It would be of interest to know what their spins are.

## I. INTRODUCTION

THE properties of the low-lying states of ${}_6C_6^{12}$ (illustrated in Fig. 1) have been studied by many workers and interpreted in terms of many different models. There is the $\alpha$-particle model of Glassgold and Galonsky,[1] the intermediate-coupling model of Kurath,[2] and the collective-shell model of Ferrell and Visscher.[3] It is therefore with some trepidation that the author wishes to propose that still another model appears to be useful in correlating the experimental data on the electromagnetic properties of the low-lying states. This is the oscillating, incompressible liquid-drop model of Rayleigh and Bohr.[4]

In general, one might say that one nucleus is not worth all this attention. However, there are reasons why ${}_6C_6^{12}$ is of particular interest. The first is that it has been studied extensively and a great deal is known about its excitations, including the cross sections for inelastic electron scattering to the first-three excited states.[5,6] The second, and more fundamental reason is that the $0^+$ state at 7.66 Mev is thought to be the crucial intermediary in the process proposed by Salpeter for the burning of hydrogen in red giant stars to form ${}_6C_6^{12}$ and to start the chain of building up the elements.[7–10] The process is supposed to proceed by

$$2\,\mathrm{He}^4 \rightleftharpoons \mathrm{Be}^8$$
$$\mathrm{Be}^8+\mathrm{He}^4 \rightleftharpoons \mathrm{C}^{12*}(7.6\text{ Mev}) \rightarrow \mathrm{C}^{12*}(4.4\text{ Mev})+\gamma$$
$$\searrow$$
$$\mathrm{C}^{12}(\text{ground})+e^++e^-.$$

It will go at an appreciable rate only if the electromagnetic rate for the decay of $C^{12*}$ (7.6 Mev), which is the essentially irreversible step in the chain, is large enough. It is very difficult experimentally to measure even the relative values of the electromagnetic and total widths, although such measurements have been and continue to be made,[11–13] and so it is perhaps not

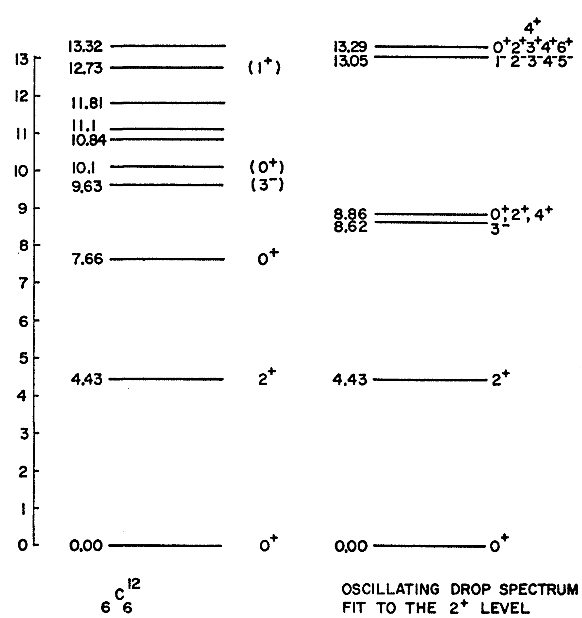


FIG. 1. The energy levels of ${}_6C_6^{12}$ and the oscillating-drop spectrum fit to the $2^+$ level.

* Supported in part by the U. S. Air Force through the Air Force Office of Scientific Research.

[1] A. E. Glassgold and A. Galonsky, Phys. Rev. **103**, 701 (1956).
[2] D. Kurath, Phys. Rev. **101**, 216 (1956); **106**, 975 (1957). [See also M. K. Pal and M. A. Nagarajan, *ibid.* **108**, 1577 (1957).]
[3] R. A. Ferrell and V. M. Visscher, Phys. Rev. **104**, 475 (1956).
[4] See J. D. Walecka, preceding paper [Phys. Rev. **126**, 000 (1962)]. This paper will be referred to as I and contains references to the earlier work.
[5] J. H. Fregeau and R. Hofstadter, Phys. Rev. **99**, 1503 (1955).
[6] J. H. Fregeau, Phys. Rev. **104**, 225 (1956).
[7] E. E. Salpeter, Phys. Rev. **107**, 516 (1957).
[8] C. W. Cook, W. A. Fowler, C. C. Lauritsen, and T. Lauritsen, Phys. Rev. **107**, 508 (1957). See also references 9 and 10.
[9] K. Nakagawa, T. Ohmura, H. Takebe, and S. Obi, Progr. Theoret. Phys. (Kyoto) **16**, 389 (1956).
[10] S. Hayakawa, C. Hayashi, M. Imoto, and K. Kikuchi, Progr. Theoret. Phys. (Kyoto) **16**, 507 (1956).
[11] R. W. Kavanaugh, Bull. Am. Phys. Soc. **3**, 316 (1958).
[12] S. F. Eccles and D. Bodansky, Phys. Rev. **113**, 608 (1959).
[13] D. E. Alburger, Phys. Rev. **118**, 235 (1960); **124**, 193 (1961).

without interest to have another theoretical prediction of what the electromagnetic widths should be. It should be noted that the greatest uncertainty in Salpeter's calculation of the rate of the above process was the electromagnetic width of the 7.66-Mev $0^+$ level, which he could only pin down to within a factor of $5\times10^2$.

The matrix element for the monopole transition with electron pair emission from the 7.66-Mev state to the ground state has also been the subject of much interest from a nuclear physics point of view. Schiff estimated this matrix element from the electron-scattering experiments of Fregeau and Hofstadter and tried to calculate its value theoretically in terms of several different models.[14] He found that the $\alpha$-particle model and a model of spherically symmetric compressional oscillations gave a *matrix element* which was too large by a factor of 5 and that if one attempted to interpret the level as a two-particle excitation in the $j$-$j$ coupling model, then the matrix element was too small by an order-of-magnitude. These results were corroborated by Sherman and Ravenhall.[15] Schiff concluded that one needed a model which was less collective than the collective models mentioned above but more collective than the shell model. It has since been suggested by Redmond[16] and Elliot[17] that other configurations, for example, a single-particle excitation $1s\rightarrow 2s$, could give a matrix element of the right order-of-magnitude. The oscillating-drop model, however, also attributes "semicollective" properties to this state if it is taken to be the state of two $l=2$ surfons coupled to $0^+$, for then the ground-state transitions are second order in the deformation parameter. The main result of this paper will be to show that by adjusting the parameters of the theory to describe the properties of the $2^+$ state at 4.43 Mev, one obtains quantitative agreement with the properties of the 7.66-Mev $0^+$ state.

There has also been considerable controversy over the properties of the 9.63-Mev state. Fregeau interpreted this state to be $2^+$ from the low-momentum-transfer behavior of the electron-scattering cross section.[6] However, Graue,[18] and later Maslin *et al.*[19] demonstrated quite convincingly through the stripping reaction $B^{11}(d,n)C^{12}$ that the proton went into an $l_p=2$ state and so the parity of the 9.63-Mev state should be odd. The situation appears to have been nicely clarified recently by Barker, Bradford, and Tassie who have reanalyzed all of the experimental data and found it to be consistent with a $3^-$ assignment.[20] A second conclusion of the present paper is that if the 9.63-Mev state is interpreted to be one of the excitations of the oscillating drop, it is most likely $3^-$ and not $2^+$ for, even though at low-momentum transfers, quantitative agreement is achieved with a $2^+$ excitation, the shape of the experimental electron-scattering cross section is only consistent with $3^-$. This is a further confirmation of the work of Barker *et al.*

[14] L. I. Schiff, Phys. Rev. **98**, 1281 (1955).
[15] B. F. Sherman and D. G. Ravenhall, Phys. Rev. **103**, 949 (1956).
[16] P. J. Redmond, Phys. Rev. **101**, 751 (1956).
[17] J. P. Elliott, Phys. Rev. **101**, 1212 (1956).
[18] A. Graue, Phil. Mag. **45**, 1205 (1954).
[19] E. E. Maslin, J. M. Calvert, and A. A. Jaffe, Proc. Phys. Soc. (London) **A69**, 754 (1956).
[20] F. Barker, E. Bradford, and L. Tassie, Nuclear Phys. **19**, 101 (1960).

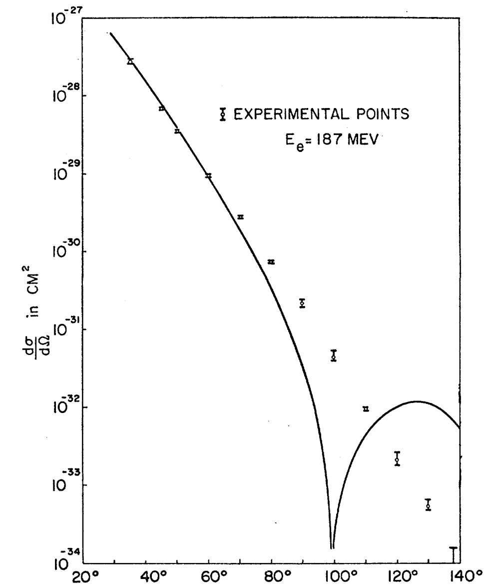


FIG. 2. The experimental and theoretical results for elastic-electron scattering from the ground state $(0^+)$ of $C^{12}$. The theoretical curve is drawn for a radius, $a=3.11\times10^{-13}$ cm. The electron energy is $E_e=187$ Mev.[24a]

## II. SUMMARY OF THE MODEL

In this section we merely give a brief summary of the formulas which we will need in applying the drop model. A full discussion and derivation together with references to earlier work can be found in I. The unperturbed Hamiltonian can be written[21]

$$H_0=\sum_{lm}\hbar\omega_l(a_{lm}{}^*a_{lm}),\tag{2.1}$$

where

$$\omega_l{}^2=C_l/B_l,\tag{2.2}$$

$$B_l=\mu a^5/l,\tag{2.3}$$

$$C_l=\sigma a^2(l-1)(l+2).\tag{2.4}$$

$\mu=$mass density; $a=$equilibrium radius; $\sigma=$surface tension. $a_{lm}{}^*$ and $a_{lm}$ are the creation and distruction operators for surfons of angular momentum $l$. The cross section for the Coulomb scattering of an electron by the oscillating drop is given by[22]

$$\frac{d\sigma}{d\Omega}(J_f\leftarrow J_i)=4\pi\sigma_M\sum_{L=0}^{\infty}\frac{1}{2J_i+1}\times|(J_f\|\mathfrak{M}_L(\Delta)\|J_i)|^2\tag{2.5}$$

[21] We have dropped the effect of the charge on the motion (the term $\gamma$ in $C_l$) since it is small for ${}_6C_6{}^{12}$ which is mostly surface.
[22] *Note.* The effect of distortion of the electron wave functions by the Coulomb potential is very small for a nucleus as light as $C^{12}$. R. Pratt and J. D. Walecka (to be published).

where

$$\sigma_M = (4\alpha^2/\Delta^4)k^2\cos^2(\theta/2);$$

$\alpha$=fine structure constant, $k$=electron wave number (we have assumed $k_1 \cong k_2 \equiv k$), $\mathbf{\Delta}^2 = (\mathbf{k}_1 - \mathbf{k}_2)^2$, and

$$\mathfrak{M}_{LM}(\Delta) = \int \rho_N(x) Y_{LM}(\Omega_x) j_L(\Delta x) d\mathbf{x}, \quad (2.6)$$

$\rho_N(x)$ being the nuclear charge density. Keeping terms through second order in the deformation parameter $q_{lm}$,

$$q_{lm} = [\hbar/2(B_l C_l)^{\frac{1}{2}}]^{\frac{1}{2}}[a_{lm}e^{-i\omega_l t} + (-1)^m a_{lm}^* e^{i\omega_l t}], \quad (2.7)$$

we find

$$\mathfrak{M}_{LM}(\Delta) = \frac{3Z}{(4\pi)^{\frac{1}{2}}}\delta_{L0}\delta_{M0}\left(\frac{j_1(\Delta a)}{\Delta a} - \frac{j_0(\Delta a)}{4\pi}\sum_{lm} :(|q_{lm}|^2):\right)$$
$$+ \frac{3Z}{4\pi} j_L(\Delta a) q_{LM}^* + \frac{3Z}{8\pi\Delta a}\left[\frac{\partial}{\partial\rho}\rho^2 j_L(\rho)\right]_{\rho=\Delta a}$$
$$\times \sum_{lm}\sum_{l'm'} :(q_{lm}q_{l'm'}):$$
$$\times \left[\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}\right]^{\frac{1}{2}}$$
$$\times \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l & l' & L \\ m & m' & M \end{pmatrix}.$$

This expression is normal ordered, that is, all annihilation operators stand to the right and the creation operators to the left (see I). The vacuum expectation value of this expression then gives us the elastic scattering from the ground state through which we can identify the radius $a$:

$$(d\sigma/d\Omega)_{\mathrm{el}} = Z^2\sigma_M[3j_1(\Delta a)/\Delta a]^2. \quad (2.8)$$

The transition rate for the emission of a real photon of multipolarity $J$ through an electric transition is obtained from the usual formula:

$$\omega(J_f \leftarrow J_i) = \frac{8\pi\alpha(Kc)}{[(2J+1)!!]^2}\left(\frac{J+1}{J}\right)K^{2J}\frac{1}{2J_i+1}$$
$$\times |(J_f\|Q_J\|J_i)|^2, \quad (2.9)$$

where the electric multipole operator $Q_{JM}$ can be obtained from $\mathfrak{M}_{JM}(\Delta)$ by

$$Q_{JM} = \lim_{\Delta\to 0}\frac{(2J+1)!!}{\Delta^J}\mathfrak{M}_{JM}(\Delta). \quad (2.10)$$

We shall also need the rate for electron-positron emission in a $0^+ \to 0$ (vacuum) transition, and this is

$$\omega(0 \leftarrow 0^+) = [\alpha^2(Kc)/135\pi](K^4|\mathrm{M.E.}|^2) \quad (2.11)$$

[provided $(m_e c^2/\hbar K c)^2 \ll 1$], with M.E. given by

$$\mathrm{M.E.} = \lim_{\Delta\to 0}\frac{-6(4\pi)^{\frac{1}{2}}}{\Delta^2}(0\|\mathfrak{M}_0(\Delta)\|0^+). \quad (2.12)$$

As usual, the electromagnetic width is defined in terms of the transition rate by

$$\Gamma = \hbar\omega_{fi} \quad (2.13)$$

and is related to the mean life and half-life by

$$\omega_{fi} = 1/\bar{\tau} = \ln(2)/\tau_{\frac{1}{2}} \quad (2.14)$$

provided there are no other channels open. (Internal conversion is neglible for the transitions under consideration.)

## III. APPLICATION OF THE MODEL

### A. Determination of the Radius

We shall determine the radius $a$, by trying to fit the elastic-scattering data of Hofstadter and Fregeau taken at $E_e$=187 Mev. The experimental points are indicated in Fig. 2. The theoretical curve, Eq. (2.8), given with the radius suggested by Hofstadter and Fregeau,[5]

$$a = 3.11\times 10^{-13}\ \mathrm{cm} \quad (3.1)$$

(corresponding to an $r_0$ in $a = r_0 A^{\frac{1}{3}}$ of $1.36\times 10^{-13}$ cm), is also plotted. The agreement is fine for low momentum transfers, but by the time the first zero of $j_1(\Delta a)$ is reached, the disagreement is appreciable. In general, we will not believe our results after the first zero of the cross section. The reason for this is that when first Born approximation is zero, the higher terms are obviously important, and when one goes to higher momentum transfers, the fact that the nucleus is not really a uniformly charged drop with a sharp surface begins to make itself evident.[23,24,24a]

We shall proceed to discuss the inelastic scattering using a radius given by Eq. (3.1).

### B. Choice of Parameters

The quantities $B_2$ and $C_2$ are now determined by

$$\hbar^2/B_2 = 8\pi\hbar^2/3AMa^2 = 3.00\ \mathrm{Mev}, \quad (3.2)$$

$$C_2 = 4\sigma a^2 = 29.6\ \mathrm{Mev}, \quad (3.3)$$

[23] L. J. Tassie, Australian J. Phys. **9**, 407 (1956).

[24] H. Crannel, R. Helm, H. Kendall, J. Oeser, and M. Yearian, Phys. Rev. **123**, 923 (1961). It is also possible to describe the smearing of the surface in terms of quantum fluctuations (I).

[24a] *Note added in proof.* If one tries to fit the elastic-scattering cross section by using Eq. (3.23) of I instead of Eq. (2.8), that is,

$$\left.\frac{d\sigma}{d\Omega}\right)_{\mathrm{el}} = Z^2\sigma_M\left[\frac{3j_1(\Delta a)}{\Delta a} + \frac{3}{8\pi}(\Delta a)j_3(\Delta a)(0|\sum_{l,m}^{l_{\max}}|q_{lm}|^2|0)\right]^2$$

then by using the same value of $a$, Eq. (3.1), and by letting $l_{\max}\cong 3$, a not unreasonable value for 12 particles, one finds a curve which passes through *all* of the experimental points indicated in Fig. 2.

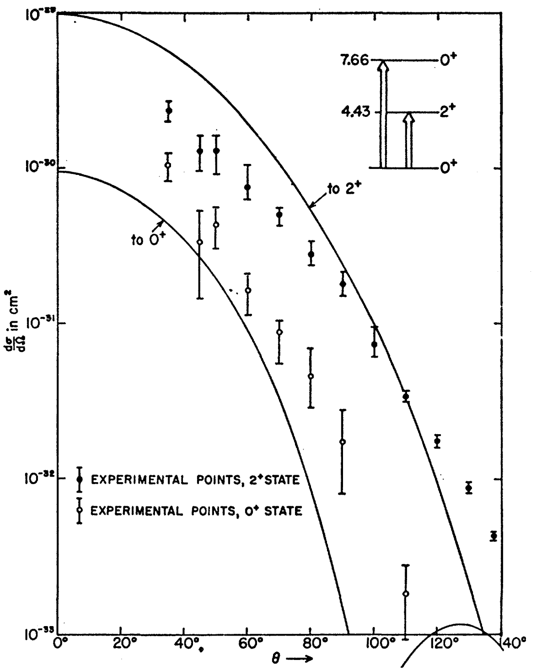


FIG. 3. Theoretical curves and experimental results for inelastic-electron scattering from the first-($2^+$) and second-($0^+$) excited states of $C^{12}$. The incident-electron energy is $E_e=187$ Mev.

where we have used[25]

$$4\pi\sigma a^2=17.8A^{\frac{2}{3}}\text{ Mev.} \tag{3.4}$$

This gives for the frequency

$$\hbar\omega_2=\hbar(C_2/B_2)^{\frac{1}{2}}=9.42\text{ Mev,} \tag{3.5}$$

and for the expansion parameter of the theory

$$\hbar/[2(B_2C_2)^{\frac{1}{2}}]=0.159. \tag{3.6}$$

$B_l$ and $C_l$ can be obtained from the above expressions by the use of Eqs. (2.3) and (2.4):

$$\begin{aligned} B_l/B_2&=2/l\\ C_l/C_2&=(l-1)(l+2)/4. \end{aligned} \tag{3.7}$$

We already face the major difficulty of this theory, and that is that it does not correctly give the energy spacings. For example, the first $2^+$ level is at 4.43 Mev, whereas Eq. (3.5) yields 9.42 Mev. We are therefore forced to make the standard compromise and treat $B_2$ and $C_2$ as parameters to be determined from experiment. $B_2$ becomes an "effective mass" and $C_2$ an "effective surface tension." Defining

$$8\pi B_2/3AMa^2\equiv b_2, \tag{3.8}$$

$$C_2/4\sigma a^2\equiv c_2, \tag{3.9}$$

we find that

$$\hbar\omega_2=9.42(c_2/b_2)^{\frac{1}{2}}\text{ Mev.} \tag{3.10}$$

$$\hbar/2(B_2C_2)^{\frac{1}{2}}=0.159/(b_2c_2)^{\frac{1}{2}}. \tag{3.11}$$

[25] A. E. S. Green, Phys. Rev. **95**, 1006 (1954).

$b_2$ and $c_2$ measure the ratio of $B_2$ and $C_2$ to the drop-model values and will be determined from the properties of the first-excited state.

### C. The 4.43-Mev State

The first-excited state of $C^{12}$ is well established to be $2^+$.[26,27] We shall therefore identify this state as one $l=2$ surfon. From Eq. (3.5), this implies

$$(c_2/b_2)^{\frac{1}{2}}=0.47. \tag{3.12}$$

The energy spectrum computed with this value is given in Fig. 1. The lifetime of the $E2$ transition to the ground state has been measured by several groups. The results are summarized in Table I.[28–30] The product $b_2c_2$ will be determined by arbitrarily fitting to the first value of the mean life given in Table I. The reason for choosing this value is that it will turn out to give the best over-all fit of the theory to experiment. This means we take

$$\omega_{\text{exp}}=3.84\times10^{13}\text{ sec}^{-1}. \tag{3.13}$$

The theoretical value of this $E2$ transition rate is given by

$$\begin{aligned}\omega_{\text{th}}(0\leftarrow 2^+)&=\frac{8\pi\alpha(Kc)}{[5!!]^2}\left(\frac{3}{2}\right)\left(\frac{9Z^2}{16\pi^2}(Ka)^4\frac{\hbar}{2(B_2C_2)^{\frac{1}{2}}}\right)\\ &=\frac{6.3\times10^{13}}{(b_2c_2)^{\frac{1}{2}}}\text{ sec}^{-1}.\end{aligned} \tag{3.14}$$

Equating these two expressions yields

$$(b_2c_2)^{\frac{1}{2}}=1.65. \tag{3.15}$$

The form factor for inelastic-electron scattering at 187 Mev has been measured and is given in Fig. 3. The theoretical curve given by

$$\begin{aligned}\frac{d\sigma}{d\Omega}(2^+\leftarrow 0)&=(4\pi\sigma_M)\left[5\times\frac{9Z^2}{16\pi^2}\frac{\hbar}{2(B_2C_2)^{\frac{1}{2}}}j_2^2(\Delta a)\right]\\ &=\frac{3.03\times10^{-30}}{(b_2c_2)^{\frac{1}{2}}}\cot^2(\theta/2)\csc^2(\theta/2)\\ &\qquad\times j_2^2(\Delta a)\text{ cm}^2,\end{aligned} \tag{3.16}$$

with

$$\Delta a=5.91\sin(\theta/2), \tag{3.17}$$

is also plotted in Fig. 3. The curve has the correct shape, but is somewhat too large at low-momentum transfers which is a reflection of the fact that the lifetime to which we have fit is somewhat shorter than that obtained

[26] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959). See also reference 27.

[27] E. Almqvist, D. A. Bromley, A. J. Ferguson, H. E. Gove, and A. E. Litherland, Phys. Rev. **114**, 1040 (1959).

[28] S. Devons, G. Manning, and J. H. Towle, Proc. Phys. Soc. (London) **A69**, 173 (1956).

[29] V. K. Rasmussen, F. R. Metzger, and C. P. Swann, Phys. Rev. **110**, 154 (1958).

[30] R. Helm, Phys. Rev. **104**, 1466 (1956).

from the electron-scattering experiments of Fregeau and Hofstadter.[30] There are also data available on the inelastic scattering from the $2^+$ state at 420 Mev and these are plotted in Fig. 4.[31] The curve obtained from Eq. (3.16) at this energy is also plotted. One cannot conclude much from the comparison since the points fall just over the first zero of the Bessel function.

### D. The 7.66-Mev State

This state is well-established as $0^+$.[26,27] From Fig. 1 we see that the drop-model predicts that there should be a $0^+$ state at about this energy. We will therefore identify this state as two $l=2$ surfons coupled to $J=0^+$. The properties of this state are then completely determined. The first thing we can compare with is the inelastic-scattering data at 187 Mev which is also given in Fig. 3. The cross section for this process is calculated to be

$$\frac{d\sigma}{d\Omega}(0^+\leftarrow 0)=(4\pi\sigma_M)\left[\frac{9Z^2}{16\pi^2}\frac{5}{8\pi}\left(\frac{\hbar}{2(B_2C_2)^{\frac{1}{2}}}\right)^2[\Delta a j_1(\Delta a)]^2\right]$$

$$=\frac{1.92\times10^{-32}}{b_2c_2}\cot^2(\theta/2)\csc^2(\theta/2)\times[\Delta a j_1(\Delta a)]^2\ \mathrm{cm}^2, \qquad (3.18)$$

where $\Delta a$ is given by Eq. (3.17). This curve is plotted in Fig. 3 and it is seen that the fit to both the shape and magnitude of the experimental data is quite satisfactory considering there are no new parameters involved.

The value for the M.E. for the electron-positron emission in the $0^+\rightarrow 0$ decay to the ground state is given theoretically by

$$\mathrm{M.E.}=(Za^2)\left(\frac{3\sqrt{10}}{4\pi}\right)\left(\frac{\hbar}{2(B_2C_2)^{\frac{1}{2}}}\right)$$

$$=\frac{6.95}{(b_2c_2)^{\frac{1}{2}}}\times10^{-26}\ \mathrm{cm}^2 \qquad (3.19)$$

$$=4.2\times10^{-26}\ \mathrm{cm}^2.$$

This is a little larger than the value estimated by

TABLE I. Mean-life for $\gamma$ decay of the 4.43-Mev state.

| $\bar{\tau}$(sec) | $\Gamma$(ev) | Source |
|---|---|---|
| $(2.6\pm0.9)\times10^{-14}$ | 0.039–0.019 | Devons, Manning, and Towle[a] |
| $(6.5\pm1.2)\times10^{-14}$ | 0.012–0.009 | Rasmussen, Metzger, and Swann[b] |
| $(5.3)\times10^{-14}$ | 0.012 | Helm[c] |

[a] Reference 28.
[b] Reference 29.
[c] Reference 30.

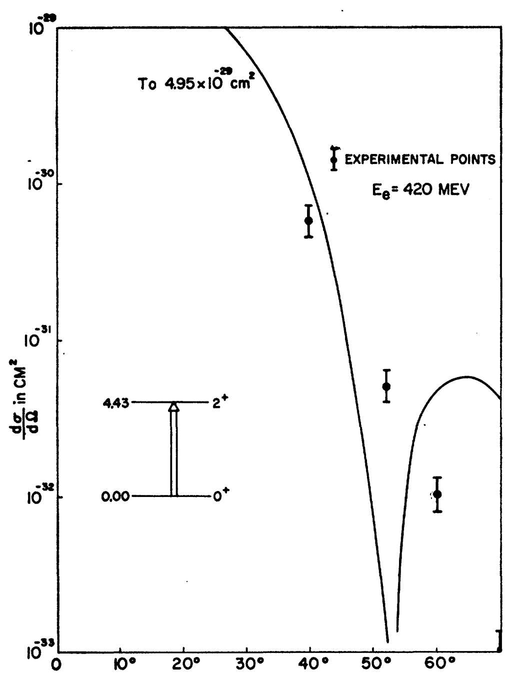


FIG. 4. Theoretical curve and experimental results for inelastic scattering from the first-excited state ($2^+$) of $C^{12}$. The incident-electron energy is $E_e=420$ Mev.

Schiff[14]:

$$\mathrm{M.E.}^{\mathrm{exp}}=3.8\times10^{-26}\ \mathrm{cm}^2, \qquad (3.20)$$

and a little less than the value estimated by Fregeau[6] from his data:

$$\mathrm{M.E.}^{\mathrm{exp}}=5\times10^{-26}\ \mathrm{cm}^2, \qquad (3.21)$$

but the agreement is good. This value of the M.E. (3.19) gives a width for pair emission to the ground state of [see (2.11)][23]

$$\Gamma_{e^\pm}=1.03\times10^{-4}/b_2c_2\ \mathrm{ev}$$
$$=3.78\times10^{-5}\ \mathrm{ev}. \qquad (3.22)$$

The best experimental value comes from the work of Alburger[13] and Ajzenberg-Selove[33]:

$$\Gamma_{e^\pm}{}^{\mathrm{exp}}=(6.6\pm2.2)\times10^{-6}\Gamma, \qquad (3.23)$$

where $\Gamma$ is the total width of the $0^+$ level (which decays primarily into $Be^8+He^4$). Since this is only the relative width, we cannot yet compare the theory with experiment.

[31] H. F. Ehrenberg, R. Hofstadter, U. Meyer-Berkhout, D. G. Ravenhall, and S. E. Sobottka, Phys. Rev. **113**, 666 (1959).

[32] In this and the following calculations of $\gamma$-ray transition probabilities, we use the experimental energy spacings to determine the wave number $K$. The electron-scattering cross sections (2.5) are independent of the level spacings.

[33] F. Ajzenberg-Selove and P. H. Stetson, Phys. Rev. **120**, 500 (1960).

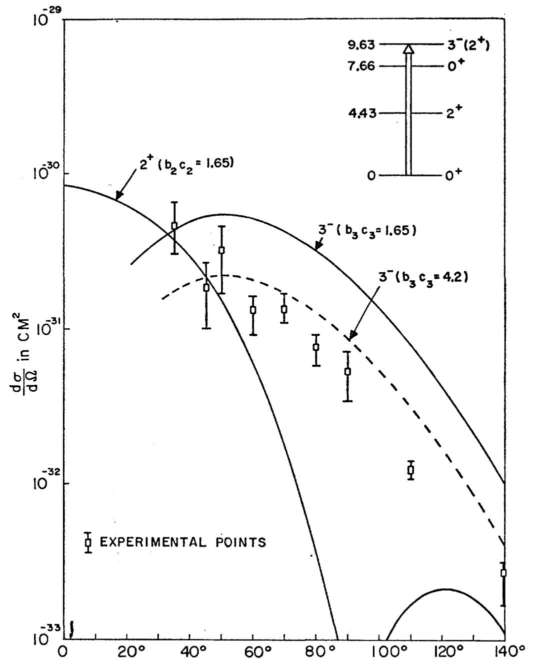


FIG. 5. Theoretical curves and experimental results for inelastic electron-scattering from the 9.63-Mev state of $C^{12}$. The incident-electron energy is $E_e=187$ Mev. The theoretical curves are drawn for both the $2^+$ two-surfon state and the $3^-$ one-surfon state.

The width for the $0^+\rightarrow 2^+$ $\gamma$-ray emission can be calculated to be

$$\Gamma_\gamma(2^+\leftarrow 0^+)=\frac{8\pi\alpha\hbar(Kc)}{[5!!]^2}\left(\frac{3}{2}\right)\left[2\times\frac{9Z^2}{16\pi^2}(Ka)^4\frac{\hbar}{2(B_2C_2)^{\frac{1}{2}}}\right]$$

$$=\frac{1.69}{(b_2c_2)^{\frac{1}{2}}}\times10^{-2}\text{ ev} \tag{3.24}$$

$$=1.02\times10^{-2}\text{ ev}.$$

The best experimental measurement of this is again a relative value recently reported by Alburger[13]:

$$\Gamma_\gamma^{\rm exp}=(3.3\pm0.9)\times10^{-4}\Gamma. \tag{3.25}$$

One way of comparing the theory with experiment would be to say that the theory gives a value of $\Gamma$, for choosing

$$\Gamma\cong 13\text{ ev} \tag{3.26}$$

yields

$$\Gamma_{e^\pm}{}^{\rm exp}=(8.6\pm2.9)\times10^{-5}\text{ ev}, \tag{3.27}$$

$$\Gamma_\gamma{}^{\rm exp}=(4.3\pm1.2)\times10^{-3}\text{ ev}, \tag{3.28}$$

which come within the theoretical estimates to within a factor of two in the square of the matrix element. We note that the theoretical estimate is too large for the $\gamma$ width and too small for the pair width just as the theoretical cross section for the inelastic electron scattering from the $2^+$ state is a little high in the forward direction while that from the $0^+$ state is somewhat low. We would therefore expect to get poorer agreement if we directly compare the *ratio*,

$$(\Gamma_{e^\pm}/\Gamma_\gamma)^{\rm th}=6.2\times10^{-3}/(b_2c_2)^{\frac{1}{2}}=3.7\times10^{-3}, \tag{3.29}$$

with

$$(\Gamma_{e^\pm}/\Gamma_\gamma)^{\rm exp}=1\times10^{-2}\quad\text{to}\quad 3.7\times10^{-2}. \tag{3.30}$$

The total theoretical electromagnetic decay width of the 7.66-Mev state is given by

$$\Gamma_{\rm el}=\Gamma_\gamma+\Gamma_{e^\pm}=1.02\times10^{-2}\text{ ev}. \tag{3.31}$$

This number can be compared with the number used by Salpeter in his calculation, which was[7]

$$\Gamma_{\rm el}\cong10^{-3}\text{ ev},$$

and lies just below the upper limit of this width $2\times10^{-2}$ ev, estimated by Salpeter.

### E. The 9.63-Mev State

The recent arguments of Barker *et al.* have fairly convincingly established this state as $3^-$.[20] Since there has always been some supposed opposition to this value from the inelastic electron-scattering experiments, which seemed to indicate $2^+$ according to Fregeau,[6] we have calculated the inelastic cross section by considering it to be first, the state of two $l=2$ surfons coupled to $J=2^+$ and second, the state of one $l=3$ surfon ($J=3^-$) lying at approximately the same energy in our model (see Fig. 1). The inelastic cross sections of 187 Mev are

$$\frac{d\sigma}{d\Omega}[(2)_{2^+}^2\leftarrow 0]=(4\pi\sigma_M)\left[\frac{9Z^2}{16\pi^2}\left(\frac{100}{7\pi}\right)\left(\frac{\hbar}{2(B_2C_2)^{\frac{1}{2}}}\right)^2\times\left(j_2(\Delta a)-\frac{\Delta a}{4}j_3(\Delta a)\right)^2\right]$$

$$=\frac{4.38}{b_2c_2}\times10^{-31}\cot^2(\theta/2)\csc^2(\theta/2)\times\left[j_2(\Delta a)-\frac{\Delta a}{4}j_3(\Delta a)\right]^2\text{ cm}^2 \tag{3.32}$$

$$\frac{d\sigma}{d\Omega}(3^-\leftarrow 0)=(4\pi\sigma_M)\left[\frac{9Z^2}{16\pi^2}\times7\frac{\hbar}{2(B_3C_3)^{\frac{1}{2}}}[j_3(\Delta a)]^2\right]$$

$$=\frac{3.28}{(b_3c_3)^{\frac{1}{2}}}\times10^{-30}\cot^2(\theta/2)\csc^2(\theta/2)\times[j_3(\Delta a)]^2\text{ cm}^2. \tag{3.33}$$

We have plotted these curves, along with the experimental points, on Fig. 5. One word is necessary about the quantity $b_3c_3$. Within the framework of the drop theory, all of the $b_lc_l$ should be one, of course. So far we have treated $b_2$ and $c_2$, or equivalently $AM$ and $\sigma$, as parameters and calculated all of the other $B_lC_l$ from Eq. (3.7). This yields, for example, a $3^-$ state at about

the correct energy. We might expect, however, that if we have to treat $(b_2c_2)$ as a parameter, we will also have to treat the $(b_lc_l)$ as separate parameters. From Eq. (3.33), this gives us the leeway of scaling the cross sections (but not changing their shape). We have therefore also plotted in Fig. 5 the cross section obtained by fitting to

$$(b_3c_3)^{\frac{1}{2}}=4.2. \tag{3.34}$$

Note that within our model, there is no room for tampering with the $2^+$ cross section.

The conclusion is that while the $2^+$ cross section fits quantitatively at low-momentum transfer, it certainly does not have the correct shape, while the $3^-$ cross section gives the correct shape and by treating $(b_3c_3)^{\frac{1}{2}}$ as a parameter, can be scaled to fit the data. Our conclusion therefore supports the results of Barker *et al.*, that the state appears to be $3^-$.

The $\gamma$ width for the ground-state decay is given by

$$\begin{aligned}\Gamma(0\leftarrow 3^-)&=\frac{8\pi\alpha(Kc)}{[7!!]^2}\left(\frac{4}{3}\right)\left[\frac{9Z^2}{16\pi^2}(Ka)^6\frac{\hbar}{2(B_3C_3)^{\frac{1}{2}}}\right]\\ &=\frac{6.45}{(b_3c_3)^{\frac{1}{2}}}\times 10^{-4}\text{ ev.}\quad (E3\text{ trans})\end{aligned} \tag{3.35}$$

The $\gamma$ width for the two-surfon $3^-\rightarrow 2^+$ transition is not as easily calculated. If the charge distribution is the same as the mass distribution, as has been assumed in this model, then the fact that we must demand that the center-of-mass remain at rest says that the electric dipole operator vanishes identically:

$$\int \mathbf{r}\rho(\mathbf{r})d\mathbf{r}\equiv 0. \tag{3.36}$$

(This is also the origin of the isotopic spin selection rule that there can be no $E1$ transitions with $\Delta T=0$, if[34] $T_z=0$.) There will be a small center-of-mass effect on other transitions but it is crucial for $E1$ transitions. The transition probability will then involve forbidden $E1$, $M2$ [which we cannot calculate (I)], and $E3$. These should all be of the same order-of-magnitude. The electromagnetic decays of the $3^-$ state are very difficult to see because the predominant decay of the 9.63-Mev state is again into $Be^8+\alpha$ (with a total width of[26] $\sim$30 kev). $\gamma$ transitions to the $0^+$ level at 7.66 Mev are expected to be down by a factor of $2\times 10^{-5}$ from the ground-state transition from energy considerations alone.

### F. Other Levels

We are now forced to stop with the applications of the model since there is no electron-scattering data to the higher levels and the spins of these levels are not established. We note that we are left with two levels, $(2^2)_{2^+}$ and $(2^2)_{4^+}$, the other two members of the degenerate triplet, which we are not able to assign. There are several other levels around 10 Mev, though, which could correspond to these levels. The tentatively assigned $0^+$ at 10.1 Mev would have to be described as $(2^3)_{0^+}$ which is lowered from 13.29 Mev. (Note: The $0^+$ at 7.66 Mev has also been lowered from its original value.) The interpretation, at any rate, begins to be unclear.

We note with regard to the missing $2^+$ and $4^+$, that Kurath's calculations also indicate there should be such levels in this energy region,[2,27] while the $\alpha$-particle model[1] says there should also be a $2^+$ in this region with a $4^+$ somewhat higher.

## IV. DISCUSSION

The main conclusion is that with the simple drop-model, one can correlate the electromagnetic properties of the first-three excited states of ${}_6C_6{}^{12}$. By adjusting the 2 parameters of the theory to fit the energy and lifetime of the $2^+$ state, the electron-scattering cross sections, and M.E. for the $0^+\rightarrow 0$ pair emission to the ground state are well described. The level at 9.63 Mev appears to be a $3^-$ state in this model upon comparison with the electron-scattering data.

Solving Eqs. (3.12) and (3.15) gives

$$b_2=3.51, \tag{4.1}$$

$$c_2=0.78. \tag{4.2}$$

This value of $b_2$ is much closer to 1 than for any other vibrational nuclei[35] and the effective surface tension is not much different from the value obtained from the semiempirical mass formula.[25]

From Eq. (3.34) and

$$(c_3/b_3)^{\frac{1}{2}}=0.53, \tag{4.3}$$

which gives the $3^-$ state the exact energy, we have

$$b_3=8.0, \tag{4.4}$$

$$c_3=2.2. \tag{4.5}$$

These values are consistent with those obtained by Lane and Pendlebury in their general analysis of octupole vibrations throughout the periodic table.[36]

One of the difficulties of the interpretation of the excitations of $C^{12}$ presented here is that one would expect, on the basis of this model, a similar spectrum in ${}_8O_8{}^{16}$. There, however, the $2^+$ state has been pushed way up leaving the $0^+$ and $3^-$ as the first-two excited states. Since this is also true in ${}_{20}Ca_{20}{}^{40}$ and ${}_{82}Pb_{126}{}^{208}$, it may be a property of the doubly magic closed shells. It would be interesting to have more electron-scattering data on other light nuclei to see if this model really has

[34] M. Gell-Mann and V. L. Telegdi, Phys. Rev. **91**, 169 (1953).

[35] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **104**, 967 (1956).

[36] A. M. Lane and E. D. Pendlebury, Nuclear Phys. **15**, 39 (1960).

any validity. It is perhaps a good idea to emphasize those results which have been obtained that are particularly model dependent. In this catagory we have the $0^+ \rightarrow 0^+$ matrix element for pair emission to the ground state[14] and the $0^+ \rightarrow 0^+$ electron-scattering cross section [see (I)]. The $0^+ \rightarrow 2^+$ $\gamma$-ray width and hence the entire electromagnetic width of the $0^+$ state are also model sensitive.[7] In general, the angular distributions for the elastic electron-scattering cross section and the ground state to first $2^+$ excited-state scattering are not particularly sensitive to the model used, however, they are fairly sensitive to the radius chosen and it is a feature of this model that the same radius is to be used for all of the transitions. This model does predict quite a difference in shape and magnitude between an "allowed" $0^+ \rightarrow 2^+$ and "forbidden" $0^+ \rightarrow 2^+$ electron-scattering cross section, for example. The general shape of the $3^-$ cross section is relatively model independent.

Finally, it should be mentioned that the approach taken here certainly throws no light on the explanation of the excitation spectrum in terms of the fundamental particle-particle interactions inside the nucleus which must give rise to them. In this sense, the paper must be considered just phenomenology. It is, however, interesting that such a simple model can have any connection with reality.


## ACKNOWLEDGMENTS

The author is very grateful to R. Hofstadter, W. Meyerhof, L. Schiff, and I. Talmi for many interesting discussions.