Electric Dipole Ground-State Transition Width Strength Function and 7-Mev Photon Interactions*†

PETER AXEL
Physics Department, University of Illinois, Urbana, Illinois
(Received December 6, 1961)

Formulas are given which describe some significant effects that a Porter-Thomas distribution of ground-state transition widths would have on the interpretation of the nuclear interaction of photons which reach closely spaced, but separated energy levels. When these formulas are used to reinterpret existing data, the parameters implied by photon interaction become consistent with those resulting from neutron capture data.

These compatible parameters are further shown to be consistent with a crude generalized extrapolation of the giant dipole resonance. At energies near 7 Mev, the average photon absorption cross section can be written approximately as $\langle\sigma_a\rangle=5.2$ mb $(E/7\text{ Mev})^3$ $(A/100)^{8/3}$. This extrapolation also implies a ground-state transition width strength function which does not have the $E^3A^{\frac{2}{3}}$ dependence usually used because of single-particle model predictions. Near 7 Mev, $\langle\Gamma_0\rangle/D=2.2\times10^{-5}$ $(E/7\text{ Mev})^5$ $(A/100)^{8/3}$; below 3 Mev, $\langle\Gamma_0\rangle/D=6.7\times10^{-9}$ $(E/1\text{ Mev})^4$ $(A/100)^{7/3}$. These estimates, while subject to refinements, are in better accord with experiments than are the more popular single-particle estimates.

I. INTRODUCTION

THIS paper correlates the electromagnetic transition probabilities implied by (a) the radiative decay of states formed during slow neutron capture, (b) the nuclear absorption and scattering of gamma rays of intermediate energy, and (c) the giant dipole resonance of photonuclear reactions. In the past, these three related experiments were interpreted with the aid of different, somewhat inconsistent analyses. Single-particle shell model estimates have dominated the analysis of any radiation width found in neutron capture even though this model cannot explain the relative values of total radiation widths and partial widths to the ground state. At higher energies, the single-particle model has been modified drastically to explain the giant dipole resonance; unfortunately, the altered theory has not been developed enough to predict the transition probabilities at the lower energies of interest in neutron capture. The scattering and absorption of the 7-Mev photons, which might form a bridge between the other two experiments, temporarily hindered a unified approach because the interpretations of these data disagreed with the conclusions drawn from neutron capture studies.

One important aim of this paper is to decrease inferred average widths and level spacings by reinterpreting the photon experiments in terms of a group of neighboring levels which have, instead of a single transition width to the ground state, a distribution of widths. Using the Porter-Thomas distribution, it is possible to explain the 7-Mev photon experiments with a set of nuclear parameters compatible with neutron capture data. Furthermore, these parameters are consistent with an extrapolation of the photon absorption cross section to lower energy from the giant resonance region.

The formulas necessary in the analysis are developed in Sec. II which is subdivided to cover (A) the absorption and scattering of photons by a single narrow level, (B) the average values obtained for many levels of one spin, and (C) the implications of a Porter-Thomas distribution of partial radiation widths for transitions to the ground state. These are followed by (D) single-particle model estimates of widths adjusted to neutron capture data, and (E) width and cross section estimates obtained by extrapolating the giant dipole resonance. The final part of Sec. II discusses, in (F), the effect of averaging over levels with different spins.

The analysis of data on 7 Mev photons is given in Sec. III which includes (A) elastic scattering, (B) resonant absorption, (C) inelastic scattering, and (D) total interaction cross sections. The conclusions of the paper are summarized in Sec. IV.

II. GENERAL FORMULAS

A. Absorption or Scattering by a Single Nuclear Level[1]

In the absence of Doppler broadening, the cross section for the absorption of a photon of energy $E=\hbar c/\lambda\!\!\!^{-}$ by a nuclear ground state of spin, I_g, to a single isolated level of spin, I_e, and excitation energy, E_R, is

$$\sigma_{an}(E)=2\pi\lambda\!\!\!^{-2}\left(\frac{2I_e+1}{2I_g+1}\right)\frac{\Gamma_0}{\Gamma}\frac{1}{[(2/\Gamma)(E-E_R)]^2+1},\quad(1)$$

where Γ is the full width of the excited state, and where Γ_0 is its partial width for de-excitation to the ground state. (Γ is assumed to be negligibly small compared with E or E_R.) The subscript n is used to denote the natural line shape energy dependence of Eq. (1). It will be convenient to use the notation

$$\omega\equiv(2I_e+1)/(2I_g+1).\quad(2)$$

* The original formulation of some of the ideas of this paper were presented in informal seminars conducted at the Physics Department, University of Washington, Seattle during the summer of 1961.
† This research was supported in part by the U. S. Office of Naval Research.

[1] H. A. Bethe and G. Placzek, Phys. Rev. 51, 150 (1937).

Many authors use $g=\omega/2$ as an abbreviation for the statistical factor.

The maximum value of the cross section, denoted by the subscript, m, is

$$\sigma_{anm}=2\pi\lambda^2\omega\Gamma_0/\Gamma=50\text{ b }(7\text{ Mev}/E)^2\omega\Gamma_0/\Gamma. \quad (3)$$

When the thermal motion of the absorbing nuclei produces a significant energy shift in the center-of-mass coordinate system, the Doppler width, Δ, becomes important:

$$\Delta=(E_\gamma/c)(2kT/M)^{\frac{1}{2}}$$
$$=3.6\text{ ev}\left(\frac{E_\gamma}{7\text{ Mev}}\right)\left(\frac{200}{A}\right)^{\frac{1}{2}}\left(\frac{T}{300}\right)^{\frac{1}{2}}, \quad (4)$$

where A is the total mass number of the molecule in which the absorbing nucleus is bound. T is an equivalent temperature, equal to the absolute temperature, T', only if T' is sufficiently above the Debye temperature; for lower values of T', T is slightly larger than T'.[2]

If there is Doppler broadening, the cross section is a rather complicated function of energy[1,3]

$$\sigma_{a\Delta}(E)=\frac{\sigma_{anm}}{\pi^{\frac{1}{2}}\Delta}\int_0^\infty dE'\frac{\exp[-(E-E')^2/\Delta^2]}{[(2/\Gamma)(E'-E_R)]^2+1}. \quad (5)$$

Extensive numerical results have been obtained[4–7] of the integral in Eq. (5) in conjunction with the analysis of slow neutron resonances. Convenient graphical summaries exist[8] which include the effects of finite sample thicknesses. For the purposes of this paper, however, it is adequate to concentrate on the limiting form of Eq. (5) valid for $\Delta\gg\Gamma$

$$\sigma_{a\Delta}(E)=\sigma_{anm}(\pi)^{\frac{1}{2}}/2(\Gamma/\Delta)\exp[-(E-E_R)^2/\Delta^2]; \quad (6)$$

$$\sigma_{a\Delta m}=\sigma_{anm}\Gamma/1.13\Delta=44\text{ b }(7\text{ Mev}/E)^2\omega\Gamma_0/\Delta. \quad (7)$$

If the absorption cross section is integrated over an energy range which includes the entire resonance, one obtains (independent of Δ/Γ)

$$\int\sigma_a(E)dE\equiv(\text{Int})_a=\pi\sigma_{anm}\Gamma/2=\pi^2\lambda^2\omega\Gamma_0$$
$$=78\text{ b }(7\text{ Mev}/E)^2\omega\Gamma_0. \quad (8)$$

The corresponding formulas appropriate to elastic scattering rather than absorption can be obtained simply from Eqs. (1), (3), (5), (6), and (8) by multiplying by Γ_0/Γ

$$\sigma_s(E)=(\Gamma_0/\Gamma)\sigma_a(E); \quad (9)$$

$$(\text{Int})_s=(\Gamma_0/\Gamma)\,(\text{Int})_a. \quad (9a)$$

The factor Γ_i/Γ could be introduced instead of Γ_0/Γ to describe inelastic scattering with Eq. (9).

The above equations do not include atomic interactions which must be treated together with nuclear interactions to calculate the self-absorption in thick scattering samples. These atomic effects have been included approximately[9] for situations in which the nuclear absorption is represented adequately by the extreme Doppler limit Eq. (6). However, the errors introduced by treating the nuclear and atomic absorption separately in a thick scatterer would not influence the qualitative conclusions of this paper. Therefore, in the remainder of this paper atomic absorption will not be mentioned; it will be assumed that atomic effects are taken into account adequately by using average atomic absorption coefficients.

B. Poor Resolution Experiments Involving Many Levels of One Spin

We shall be interested mainly in experiments performed with incident photons spread uniformly over an energy interval, ΔE, which includes n energy levels. The average level spacing, D, is equal to $\Delta E/n$. Under these conditions, the experimentally observed average cross section $\langle\sigma\rangle$ is

$$\langle\sigma\rangle=\frac{1}{\Delta E}\sum_{i=1}^{n}(\text{Int})_i=\langle\text{Int}_{I_e}\rangle/D_{I_e}. \quad (10)$$

The value of Γ_0 for each level determines its contribution to the average; but the relative contributions of levels in different experiments depend on different powers of Γ_0.

If the absorption cross section were measured directly,

$$\langle\sigma_a\rangle=\pi^2\lambda^2\omega(\langle\Gamma_0\rangle/D)_{I_e}. \quad (11)$$

The corresponding equations for elastic and inelastic scattering are

$$\langle\sigma_s\rangle=\pi^2\lambda^2\omega(\langle\Gamma_0^2/\Gamma\rangle/D)_{I_e}; \quad (11a)$$

$$\langle\sigma_i\rangle=\pi^2\lambda^2\omega(\langle\Gamma_0\Gamma_i/\Gamma\rangle/D)_{I_e}. \quad (11b)$$

Absorption experiments which use a resonant detector can give information about the average-peak-absorption cross section. Consider the measurement of the number of elastically-scattered gamma rays with and without an absorber between the original gamma ray source and the scatterer. The number of detected counts with no absorber, C_{NA}, can be written as

$$C_{NA}=F\int\sigma_s(E)dE=F\langle\text{Int}_s\rangle=F\langle\sigma_s(E)\rangle\Delta E, \quad (12)$$

[2] W. E. Lamb, Jr., Phys. Rev. **55**, 190 (1938).
[3] H. A. Bethe, Revs. Modern Phys. **9**, 69 (1937).
[4] W. W. Havens, Jr. and J. Rainwater, Phys. Rev. **83**, 1123 (1951); **92**, 702 (1953); the latter paper contains references to earlier papers which included relevant graphs.
[5] G. V. Dardel and R. Persson, Nature **170**, 117 (1952).
[6] V. L. Sailor, Brookhaven National Laboratory Report 257 T-40, 1953 (unpublished).
[7] M. E. Rose, W. Miranber, P. Leak, L. Rosenthal, J. K. Hendrickson, and D. Schiff, Westinghouse Electric Corporation Report WAPD-SR-506, 1954 (unpublished).
[8] For example, see either D. J. Hughes, J. Nuclear Energy **1**, 237 (1955); or J. Rainwater, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XL, p. 373.
[9] K. Reibel, thesis, University of Pennsylvania (unpublished).

where F contains all factors such as monitor normalization, scatterer thickness, and detection efficiency. (To keep the analysis simple, we restrict the discussion to scatterers thin enough to have negligible self absorption.) When an absorber containing $\mathfrak{N}_A$ atoms/cm^2 of the absorbing isotope is inserted, the number of detected counts (corrected for atomic absorption) is

$$C_A = F\int \sigma_s(E)\exp[-\mathfrak{N}_A\sigma_a(E)]dE. \tag{13}$$

The exponential in Eq. (13) can be expanded in an infinite series; using the notation $T=T(E)=\mathfrak{N}_A\sigma_a(E)$, the fraction of the original counts absorbed is

$$R_{\mathrm{el}s}=\left(\frac{C_{NA}-C_A}{C_{NA}}\right)_{\mathrm{el}s} = \frac{\int dE\,\sigma_s(E)[T-T^2/2!+T^3/3!-\cdots]}{\int dE\,\sigma_s(E)}. \tag{14}$$

Independent of Γ/Δ, the leading term for small thicknesses is proportional to $\langle\Gamma_0^3\rangle/\langle\Gamma_0^2\rangle$. For inelastic scattering σ_i would replace σ_s in Eq. (14); if Γ_i and Γ_0 were uncorrelated, the result for thin absorbers would depend on $\langle\Gamma_0^2\rangle/\langle\Gamma_0\rangle$.

The integration in Eq. (14) can be performed simply if the energy dependence of the cross section is given by either the extreme Doppler approximation or the Breit-Wigner formula. For the extreme Doppler approximation, introduce the thickness parameter, t:

$$t=\mathfrak{N}_A\sigma_{a\Delta m}=\mathfrak{N}_A\pi^{\frac{3}{2}}\lambda\!\!\!^{-2}\omega\Gamma_0/\Delta. \tag{15}$$

Term-by-term integration of Eq. (14) gives:

$$R_{\mathrm{el}s\Delta}=\left\langle\Gamma_0^2\left[\frac{t}{\sqrt{2}}-\frac{t^2}{2!}\frac{1}{\sqrt{3}}+\frac{t^3}{3!\sqrt{4}}-\cdots\right]\right\rangle\bigg/\langle\Gamma_0^2\rangle. \tag{16}$$

The analogous expression for inelastic scattering would have $\Gamma_0\Gamma_i$ in place of Γ_0^2 in both numerator and denominator. If the left-hand side of Eq. (16) is found experimentally, and if the distribution function of Γ_0 were known, $\langle t\rangle$ could be determined. One could then find $\langle\sigma_{a\Delta m}\rangle$ and $\langle\Gamma_0\rangle$ from

$$\langle\sigma_{a\Delta m}\rangle=\langle t\rangle/\mathfrak{N}_A=\pi^{\frac{3}{2}}\lambda\!\!\!^{-2}\omega\langle\Gamma_0\rangle/\Delta. \tag{17}$$

If the cross section has a Breit-Wigner shape, Eq. (14) can be integrated before the series expansion is made,[3]

$$R_{\mathrm{el}n}=\langle\Gamma_0^2[1-e^{-t'/2}J_0(it'/2)]\rangle/\langle\Gamma_0^2\rangle, \tag{18}$$

where $J_0(iy)$ is the Bessel function of imaginary argument, often written as $I_0(y)$. In this case, the thickness parameter, t', is

$$t'=\mathfrak{N}_A\sigma_{anm}=\mathfrak{N}_A 2\pi\lambda\!\!\!^{-2}\omega\Gamma_0/\Gamma. \tag{19}$$

The first few terms of the series expansion of Eq. (18) are

$$R_{\mathrm{el}n}=\left\langle\Gamma_0^2\frac{t'}{2}\left[1-\frac{3}{4}\left(\frac{t'}{2}\right)+\left(\frac{7}{12}\right)\left(\frac{t'}{2}\right)^2-\frac{35}{192}\left(\frac{t'}{2}\right)^3+\frac{63}{320}\left(\frac{t'}{2}\right)^4\cdots\right]\right\rangle\bigg/\langle\Gamma_0^2\rangle. \tag{20}$$

C. Porter-Thomas Distribution

A series of measurements could, in principle, determine the various moments of the distribution function which describes the widths, Γ_0, of the different levels. However, since photon scattering and absorption measurements will probably not provide sufficiently detailed information in the near future, it will remain necessary to use fewer measurements together with an assumed distribution function to find parameters such as $\langle\Gamma_0\rangle$, $\langle\Gamma_0/\Gamma\rangle$, and $\langle\Gamma_0/D\rangle$. A very special assumption that has been used[9,10] is that all levels have equal values of Γ_0; this implies the equally special result that $\langle\Gamma_0^q\rangle=(\langle\Gamma_0\rangle)^q$. This paper will give some of the consequences of the more reasonable Porter-Thomas distribution[11] which successfully described the variation of reduced neutron widths,[12,13] and which seems to fit the ground state radiation widths, Γ_0, for the few nuclei whose capture gamma rays from different resonances have been studied adequately.[14,15]

According to the Porter-Thomas distribution, the probability of finding a width Γ_0 is

$$P(\Gamma_0)d\Gamma_0=\frac{1}{(2\pi)^{\frac{1}{2}}}\left(\frac{\bar{\Gamma}_0}{\Gamma_0}\right)^{\frac{1}{2}}\exp(-\Gamma_0/2\bar{\Gamma}_0)d\Gamma_0, \tag{21}$$

where $\bar{\Gamma}_0$ is the average value. This distribution implies an average value of the qth power of Γ_0 given by

$$\langle\Gamma_0^q\rangle=(2^q/\sqrt{\pi})(q-\tfrac{1}{2})!(\langle\Gamma_0\rangle)^q=(2q-1)!!(\bar{\Gamma}_0)^q, \tag{22}$$

where $(2q-1)!!=(2q-1)(2q-3)\cdots(3)(1)$ and where the gamma function of $(q+\frac{1}{2})$, is written as $(q-\frac{1}{2})!$ (Note that $\bar{\Gamma}_0$ and $\langle\Gamma_0\rangle$ are used interchangeably to denote average value.)

The use of the Porter-Thomas distribution introduces a factor of 3 into the elastic scattering cross section if

[10] K. Reibel and A. K. Mann, Phys. Rev. **118**, 701 (1960).

[11] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

[12] D. J. Hughes and R. L. Zimmerman, *Nuclear Reactions*, edited by P. E. Endt and M. DeMeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I.

[13] L. M. Bollinger, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part A.

[14] L. M. Bollinger, R. E. Coté, and T. J. Kennett, Phys. Rev. Letters **3**, 376 (1959).

[15] R. E. Coté and L. M. Bollinger, Phys. Rev. Letters **6**, 695 (1961).

$\Gamma_0 \ll \Gamma$; substituting Eq. (22) in Eq. (11a) gives

$$\langle\sigma_s\rangle_{\text{P-T}} = \pi^2 \lambda\!\!\!^{-2} \omega 3 (\langle\Gamma_0\rangle_{I_e})^2 / D_{I_e}\Gamma. \tag{23}$$

(Note that the additional assumption has been made that the full width, Γ, is the same for all levels. This assumption is strongly supported by experimental data on slow neutron resonances when $\Gamma_0 \ll \Gamma$. On the other hand, the assumption must break down if Γ_0 varies and becomes comparable with Γ. For example, if Γ and Γ_0/D did not vary as rapidly with energy as does D, Γ_0 might be expected to tend toward Γ as the excitation energy decreased and therefore D increased. As Γ_0 approaches Γ, $\langle\Gamma_0^2/\Gamma\rangle$ approaches $\langle\Gamma_0\rangle$, and the factor of 3 in Eq. (23) disappears.)

We shall need the numerical value of Eq. (23) for the particular case $\omega = 3$, (i.e., $I_g = 0$, $I_e = 1$), and $E_\gamma = 7$ Mev:

$$\langle\sigma_s \text{ (7 Mev)}\rangle_{\text{P-T}} = 705 \text{ b } (\bar{\Gamma}_{01})^2 / D_1\Gamma, \tag{24}$$

where the subscript 1 is a reminder that the $\bar{\Gamma}_0$ and D values appropriate to levels of spin 1 should be used, and the subscript P-T emphasizes that the factor of 3 originates from the assumed distribution.

Because both the average absorption and inelastic scattering cross sections involve only the first power of Γ_0 [see Eqs. (11) and (11b)], the Porter-Thomas distribution does not affect them. For 7-Mev photons, and $\omega = 3$, the numerical values are

$$\langle\sigma_a \text{ (7 Mev)}\rangle = 235 \text{ b } \langle\Gamma_{01}\rangle / D_1; \tag{25}$$

$$\langle\sigma_i \text{ (7 Mev)}\rangle = 235 \text{ b } \langle\Gamma_{01}\Gamma_{i1}\rangle / \Gamma D_1. \tag{25a}$$

If the Porter-Thomas distribution is used in the analysis of resonant absorption data when $\Delta \gg \Gamma$ [i.e., if Eq. (22) is substituted in Eq. (16)], a satisfactory thin absorber expression can be obtained using only three terms

$$R_{\text{el}\Delta\text{ P-T}} = (5t/\sqrt{2})[1 - 0.75(5t/\sqrt{2}) + 0.3(5t/\sqrt{2})^2]. \tag{26}$$

Equation (26) is accurate to within 1% for $t \leq 0.25$; this upper limit of t corresponds to absorption which reduces the counting rate by a factor of 2. Note that the parameter t is now the average value; $\bar{\Gamma}_0$ has replaced Γ_0 in Eq. (15). The corresponding expression for a single level or for constant Γ_0 rather than a distribution is

$$R_{\Delta\text{ single level}} = (t/\sqrt{2})[1 - 0.57(t/\sqrt{2}) + 0.14(t/\sqrt{2})^2]. \tag{27}$$

For inelastic scattering, the analog to Eq. (26) is

$$R_{\text{inel}\Delta\text{ P-T}} = (3t/\sqrt{2})[1 - 0.9(3t/\sqrt{2}) + 0.45(3t/\sqrt{2})^2], \tag{28}$$

which is accurate to about 1% for $t \leq 0.4$; in this case, $t = 0.4$ corresponds to a reduction of about 43% in the inelastic counting rate. If only a single level is involved, Eq. (27) applies to inelastic as well as elastic scattering.

Comparing Eq. (26) with Eq. (27) shows that for a given Γ_0, the existence of a Porter-Thomas distribution enhances the resonant absorption by a factor of 5 for elastic scattering; the corresponding enhancement is 3 if inelastic scattering is used to measure the resonant absorption. These enhancements would remain unchanged for thin absorbers for cross sections with smaller values of Δ/Γ [as can be seen from Eq. (14)].

D. Single Particle Shell Model Estimates

Although many photon transition probabilities encountered in radioactive decays were treated successfully with the aid of the single-particle shell model,[16–18] it has had only limited success with $E1$ transitions. In very light nuclei, it seems possible to explain $E1$ transition rates with a single-particle model using more refined estimates of the model wave functions.[18] This partial success implies that the model gives an adequate description of some excited states in light nuclei, but it should not be misconstrued as supporting the adequacy of the model for all $E1$ transitions. In heavy nuclei, the single particle spherical shell model predicts that states which can decay to the ground state by $E1$ radiation occur only at relatively high energies. The few low-lying states known to decay by $E1$ radiation have transition rates much below the single-particle estimates; special interpretations have been invoked to explain these states of opposite parity and their transition rates.[19–21]

At higher energies (i.e., about 7 Mev) in medium and heavy nuclei, there is a much higher level density than is implied by the single-particle model. These higher level densities are consistent with an independent particle model which includes states involving the excitation of more than one nucleon.[22–24] If there were no residual interactions one might expect some highly-excited nuclear states to correspond to pure single-particle excitation, others to involve particular two particle excitations, etc. However, due to residual interactions, the wave function of any actual state would be a mixture of wave functions of pure independent-particle states. The number of states which are mixed, or equivalently, the energy interval over which states are mixed, depends on the strength of the residual interactions. If the interaction energy is large compared with the single-particle level spacing, each actual state might be expected to have in its wave function a part corresponding to the single-particle model wave function. Under these conditions, if the single-particle model predicts a width Γ_{0M} and a spacing D_M the actual strength function Γ_0/D

[16] S. A. Moszkowski, Phys. Rev. **83**, 1071 (1951); **89**, 474 (1953).
[17] V. F. Weisskopf, Phys. Rev. **83**, 1073 (1951).
[18] D. H. Wilkinson, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B.
[19] D. Strominger and J. O. Rasmussen, Nuclear Phys. **3**, 197 (1957).
[20] A. Bohr and B. R. Mottelson, Nuclear Phys. **4**, 529 (1957).
[21] G. E. Brown, J. A. Evans, and D. J. Thouless, Nuclear Phys. **24**, 1 (1961).
[22] C. Bloch, Phys. Rev. **93**, 1094 (1954).
[23] T. Ericson, Nuclear Phys. **6**, 62 (1958); **8**, 265 (1958); **9**, 697 (1958–1959).
[24] T. Ericson, Suppl. Phil. Mag. **9**, 425 (1960).

would be given by[25]

$$\Gamma_0/D=\Gamma_{0M}/D_M. \quad (29)$$

The above reasoning makes it clear that Eq. (29) would not be appropriate (or, at least, would require a new justification) if D_M were much larger than the energy interval over which residual interactions could be expected to mix states. Blatt and Weisskopf used the estimate $D_M=0.5$ Mev, and somewhat higher values would not threaten Eq. (29). However, an analysis which used Eq. (29) to find D_M is not internally consistent if the inferred value of D_M is 10 Mev or higher. The fact that single particle shell spacings are of the order of 10 Mev is irrelevant. In an actual nucleus, many single particle dipole excitations can be expected. If these were spread out in energy, D_M could be small enough to make Eq. (29) seem reasonable. On the other hand, if these were all concentrated in energy (e.g., at the giant dipole resonance) the transition probability to that energy region would be greater than estimated for a single particle, and some new mechanism would have to be invoked for explaining both why and how much transition probability was transferred to a different energy region (e.g., at 7 Mev). Of course, D_M can be treated as an arbitrary adjustable parameter, temporarily without significance. If an excessive D_M value is obtained using Eq. (29) and a particular estimate of the transition width Γ_{0M} one can cling to the hope that another model M' would give a much smaller $\Gamma_{0M'}$ so that the commensurately reduced value $D_{M'}$ would be small enough to justify Eq. (29). However, without a clue about M' it is surely questionable that $\Gamma_{0M'}$ will have the energy and mass dependence characteristic of Γ_{0M}.

According to the single-particle shell model, Γ_{0M} is proportional to $E^3A^{\frac{4}{3}}$; Eq. (29) can therefore be rewritten as

$$\Gamma_0/D\equiv C(E/7\text{ Mev})^3(A/100)^{\frac{4}{3}}. \quad (30)$$

If one uses the formula of Moszkowski[16] or Weisskopf,[17] and sets $R=1.2\times10^{-13}A^{\frac{1}{3}}$, $C=500$ ev$/D_M$ (ev). Introducing an effective nucleon charge of $\frac{1}{2}$ (to approximate N/A for protons and Z/A for neutrons), reduces the 500 ev in C by a factor of 4.

When experimental data are used to evaluate C in Eq. (30), two different values are called for by two different types of experiment. An analysis by Kinsey[26] of ground-state capture gamma rays, similar to earlier summaries,[27] gives $C=2.5\times10^{-5}$. (Note that this analysis did not include the A dependence; a typical A value was used only when comparing the data with theory.) In contrast, the analysis of total radiation widths (which depend primarily on lower gamma energies) has led to considerably lower C values. Hughes and Levin[28] gave $C=3\times10^{-6}$; Cameron[29] deduced $C=10^{-5}$ and later[30] $C=2.5\times10^{-6}$. The fact that C as inferred from total radiation widths is about a factor of ten below that obtained from ground state transitions, implies a more rapid energy dependence of Γ_0/D than is given by Eq. (30). Furthermore, all of these C values give values of D_M too large to justify the uniform mixing of single particle strength among all levels as required to obtain Eq. (29). The simple single particle model with uniformly distributed Γ_0/D fails even more dramatically when its predictions are applied to the energy region of the giant dipole resonance.

E. Estimates of Γ_0/D from Giant Dipole Resonance

The giant dipole resonance can be explained on the basis of the single-particle model only if Γ_{0M}/D_M exhibits resonance behavior. Wilkinson[31] has argued that the single particle model could explain the resonance if the model states with large values of Γ_0 are concentrated strongly in energy; this, of course, implies that $\Gamma_{0M}/D_M=E^3f(E)$ where $f(E)$ is by no means constant as in Eq. (30). Elliot and Flowers[32] showed that when particle-hole interactions are included in calculating the electric dipole photon absorption by O^{16}, the single-particle model states are mixed, are shifted in energy, and have their transition probability shifted so as to be concentrated in energy. Brown and his collaborators,[33,34] by showing that the particle-hole interaction can be expected to have a similar effect in all nuclei, have left little doubt that Eq. (30) is changed drastically for the main dipole absorption.

The question which remains open is whether the photon interactions at energies well below the giant resonance are intimately related to this resonance. (For example, it is conceivable that the giant resonance involves only dipole excitation of closed shells, and that excitations of a single valence nucleon contribute to the low-energy region more or less in accordance with the naive single particle model.) Even if the lower energy photon interactions are related to the giant dipole resonance, one is faced with the problem of predicting these interactions in the absence of any significant theoretical guidance as to the energy dependence.

In order to check on the existence of a connection between the dipole resonance and lower energies, and to discover a recipe for predicting lower energy photon interactions, we need a relatively simple expression with

[25] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).
[26] B. B. Kinsey, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XL, (a) p. 309, ff; (b) p. 314, ff.
[27] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **93**, 1260 (1954).
[28] J. S. Levin and D. J. Hughes, Phys. Rev. **101**, 1328 (1956).
[29] A. G. W. Cameron, Can. J. Phys. **35**, 666 (1957).
[30] A. G. W. Cameron, Can. J. Phys. **37**, 322 (1959).
[31] D. H. Wilkinson, Ann. Rev. Nuclear Sci. **9**, 1 (1959).
[32] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).
[33] G. E. Brown and M. Bolsterli, Phys. Rev. Letters **3**, 472 (1959).
[34] G. E. Brown, L. Castillejo, and J. A. Evans, Nuclear Phys. **22**, 1 (1961).

few adjustable parameters. Even though the photon absorption cross sections of different elements (and even different isotopes of the same element) have different energy dependences,[35] there is considerable value in starting with a caricature of the dipole resonance. If this caricature comes close, refinements can be added. (Furthermore, it is conceivable, to an optimist, that the connection between the giant dipole and lower energies is profound enough to be insensitive to whatever causes many of the observed wiggles and bumps.)

An approximation which reproduces many of the important systematic features of the giant dipole resonance is the classical Lorentz line

$$\sigma_a(E)=\frac{(1.3A/100\Gamma_g)E^2\Gamma_g{}^2}{(E_R{}^2-E^2)^2+E^2\Gamma_g{}^2}\text{ b}, \tag{31}$$

where Γ_g, E, and E_R are given in Mev. The term in parentheses in the numerator is the peak cross section of a Lorentz line which has a constant full width at half maximum Γ_g and an integrated cross section equal to $2.1(A/100)$ Mev-b (which corresponds to the sum rule predictions[31,35,36]). Equation (31) was suggested earlier by Brink,[37] but it has not yet been used widely.

Approximate values of $\sigma(E)$ below the giant resonance can be obtained from Eq. (31) by using the empirical value $E_R=80A^{-\frac{1}{3}}$, and the approximate value $\Gamma_g=5$ Mev as a universal estimate for the fluctuating widths.[35] These simple parameters give values of σ (7 Mev) which are close to those predicted by empirically adjusted Lorentz-lines in the nine cases collected in reference 35; the worst deviations occur in Ta and Au for which the more detailed extrapolation is only 30% below the cross section predicted by these suggested parameters.

For many purposes it is useful to obtain a simpler energy dependence which approximates Eq. (31) in a limited energy region. Near $E=7$ Mev, one can find, numerically

$$\sigma_a(E\approx 7\text{ Mev})=5.2\times10^{-3}\left(\frac{E}{7\text{ Mev}}\right)^3 \times\left(\frac{A}{100}\right)^{8/3}\left(\frac{\Gamma_g}{5\text{ Mev}}\right)\text{ b}. \tag{32}$$

Below $E=3$ Mev, one obtains

$$\sigma_a(E\leq 3\text{ Mev})=3.8\times10^{-3}\left(\frac{E}{7\text{ Mev}}\right)^2 \times\left(\frac{A}{100}\right)^{7/3}\left(\frac{\Gamma_g}{5\text{ Mev}}\right)\text{ b}. \tag{33}$$

[35] E. G. Fuller and Evans Hayward in Nuclear Reactions' edited by Endt, Demeur, and Smith (North-Holland Publishing Company, Amsterdam), Vol. II (to be published).

[36] J. S. Levinger, *Nuclear Photodisintegration* (Oxford University Press, New York, 1960).

[37] D. M. Brink, thesis, Oxford University, 1955; [(unpublished) reported in reference 26b].

If one interprets these extrapolated cross sections as the expected average values, $\langle\Gamma_0/D\rangle$ can be calculated using Eq. (11); assuming $\omega=3$ (i.e., $I_e=1$, $I_g=0$) one obtains

$$\langle\Gamma_0/D\rangle_{I_e=1,\ E\approx 7\text{ Mev}} =2.2\times10^{-5}\left(\frac{E}{7\text{ Mev}}\right)^5\left(\frac{A}{100}\right)^{8/3}\left(\frac{\Gamma_g}{5\text{ Mev}}\right); \tag{34}$$

$$\langle\Gamma_0/D\rangle_{I_e=1,\ E\leq 3\text{ Mev}} =1.6\times10^{-5}\left(\frac{E}{7\text{ Mev}}\right)^4\left(\frac{A}{100}\right)^{7/3}\left(\frac{\Gamma_g}{5\text{ Mev}}\right). \tag{35}$$

If $\langle\Gamma_0\rangle/D$ has the same value for the different spins which can be reached by a dipole transition from the ground state $I_g\neq 0$, (as will be made plausible in the next section), Eqs. (34) and (35) apply to any values of spin, I_e, where $I_e=I_g-1$, I_g, or I_g+1.

These equations for the radiative strength function are consistent with neutron capture data. The experimental ground state transition data[26] (corrected for more recent values[13] of D) can be analyzed in terms of the E and A dependences given in Eq. (34); this analysis yields an empirical coefficient of 3.1×10^{-5} in place of the predicted value, 2.2×10^{-5}. So small a difference would not be significant for this early stage of attempted correlation. However, before taking this empirical constant seriously, it must be remembered that the experimental data are limited; if only one element V^{52} is omitted from the analysis, the empirical coefficient drops to 1.7×10^{-5}.) The observed total radiation widths Γ_γ can also be explained if Eq. (35) is used. For dipole radiation, one expects

$$\Gamma_{\gamma I}=3\int_0^{E_B}\frac{\langle\Gamma_0/D\rangle_I D_I(E_B)}{D_I(E_B-E)}dE. \tag{36}$$

The factor of 3 takes into account the transitions to levels of spin $I-1$, I, and $I+1$; this factor of 3, and therefore Γ_γ, is independent of I if $(2I+1)D_I=(2I'+1)D_{I'}$, and if the transition probability does not have any spin dependence. Equations (35) and (36) can explain the observed total radiation widths if acceptable parameters are chosen in any of the usual energy-level density formulas.[24] Inasmuch as Γ_γ is rather sensitive to these parameters, improved values obtained from other experiments could test Eq. (35) more critically. For example, if $D(E)\propto e^{-E/T}$, $\Gamma_\gamma(\text{ev})=0.48(A/100)^{7/3}(T/1\text{ Mev})^5$; reasonable T values can easily be chosen to match observed values of Γ_γ. The suitability of Eq. (35) for explaining Γ_γ can be understood by noticing that Eq. (35) predicts the same value of Γ_0/D for $A=100$ and $E=1.1$ Mev that Eq. (30) predicts if C is adjusted (to 2.5×10^{-6}) to fit Γ_γ values.[30]

Lane and Lynn[38] used a modified Breit-Wigner energy

[38] A. M. Lane and J. E. Lynn, Nuclear Phys. 11, 646 (1959); see also 11, 625 (1959).

dependence for $\sigma_a(E)$ instead of the Lorentz dependence of Eq. (31). Their suggestion corresponds to changing Eq. (31) by setting $E_R+E=2E$, and by multiplying by $(E/E_R)\exp[0.3(E-E_R)]$; the exponential with the factor of 0.3 was chosen empirically to fit some neutron capture data. Equation (31) seems preferable because it fits at least as much data even though it has one less adjustable parameter. [Note that inasmuch as Lane and Lynn already use a stronger energy dependence than given by Eq. (30), their discussion of direct neutron capture at high neutron energies[38] would not be modified strongly by the giant resonance extrapolation suggested here. On the other hand, the same authors probably underestimated the high-energy photons expected from neutron capture in the resonant region[39] by using the lower energy dependence of Eq. (30).]

F. Many Levels with Different Spin Values

Additional ambiguities enter the analysis if levels of different spin contribute significantly to the photon interaction in a given energy region. Of course, only levels with a single spin and parity can be excited due to the absorption of a single multipole by a nucleus with ground-state spin, $I_g=0$. It is also conceivable that excited states of a single spin would dominate due to nuclear model-dependent selection rules; for example, if the single-particle model adequately represented highly excited states, $I_e=I_g+1$ might dominate the electric dipole absorption due to a single valence nucleon.[31,18] On the other hand, it may be more reasonable to expect several values of I_e to participate if $I_g\neq 0$.

When more than one value of I_e plays a role, Eq. (10) should be rewritten as

$$\langle\sigma\rangle=\sum_{I_e}\langle \mathrm{Int}_{I_e}\rangle/D_{I_e}\propto\sum_{I_e}\left(\frac{2I_e+1}{2I_g+1}\right)\frac{\langle\Gamma_0{}^q\rangle_{I_e}}{\Gamma D_{I_e}}. \quad (37)$$

Additional assumptions about spin dependences are needed before Eq. (37) can be simplified. There are data which guide the assignments of relative values of Γ and D for levels of different spin but about the same energy in a single nucleus. (We consider only levels for which Γ is essentially equal to the radiation width.) Slow neutron experiments indicate that Γ, the total radiation width, is relatively constant; compared with other uncertainties in this analysis, it seems quite safe to assume that for a single isotope $\Gamma_I=\Gamma_{I'}$. (This assumption would break down if Γ_0 were comparable with Γ, and if Γ_0 were spin dependent.) Reasonable nuclear models[3,22–24] and relevant experimental data[40] indicate that the density of levels with spin I is proportional to $2I+1$; therefore we write

$$(2I+1)D_I=(2I'+1)D_{I'}. \quad (38)$$

Since there are few data bearing on spin dependence of Γ_0, we shall use the attractive, simplifying assumption:

Case A: $$\langle\Gamma_{0I}\rangle/D_I=\langle\Gamma_{0I'}\rangle/D_{I'}. \quad (39)$$

This assumption is attractive because: (a) it implies that states of spin I account for a fraction of the absorption proportional to the statistical weight $(2I+1)$, (b) it predicts that the total radiation widths observed in slow neutron experiments would be independent of spin [see Eq. (36)], and (c) it gives predictions which relate $\langle\Gamma_0/D\rangle$ to experimental observables independent of spin.

To show the changes which might occur if Case A does not apply, we also tabulate the relations obtained for another case:

Case B: $$\Gamma_{0I}=\Gamma_{0I'}. \quad (40)$$

Case B corresponds to a strength function $\langle\Gamma_{0I}\rangle/D_I$ whose value is proportional to $(2I+1)$, and therefore to states of spin I contributing a fraction proportional to $(2I+1)^2$ to the cross section. Case B was used by Reibel and Mann[9,10] in interpreting 7-Mev photon data. If states of each possible spin had the same fractional contribution to the cross section, $(\Gamma_{0I}/\Gamma_{0I'})=(2I'+1)^2/(2I+1)^2$; this case will not be treated.

Table I gives various dipole cross sections for different I_g, and for both Case A and Case B; all values in Table I correspond to a Porter-Thomas distribution. The average elastic and inelastic scattering cross sections (using thin scatterers) are independent of Δ/Γ. The results of resonant absorption experiments are given in terms of R, the fraction of the original counts absorbed by a thin absorber [as in Eqs. (26), (28), and (15)]; as shown in Eq. (17), these entries are related to the peak cross sections. The inelastic cross sections are based on the additional assumption that $\Gamma_{iI}=\Gamma_{iI'}$. The values given for $I_g=0$ do not depend on whether Case A or Case B is assumed provided the derived values are expressed in terms of Γ_{01} and D_1. All of the widths listed in Table I should be understood as average widths; when no ambiguity can arise, the average sign is omitted.

III. ANALYSIS OF 7-MEV PHOTON DATA

Most of the data to be used comes from the two systematic studies of the nuclear interaction of photons whose energy was near 7 Mev. Reibel and Mann[9,10] studied the elastic scattering by 32 elements using mixtures of the 7.12-, 6.92-, and 6.14-Mev gamma rays emitted during the $F^{19}(p,\alpha)O^{16}$ reaction. Their most extensive results, which will be used below, were obtained with relative intensities 8:2:1; the two higher-energy gamma rays had an energy spread of 130 kev. Reibel and Mann measured resonant absorption for 6 of the elements they studied; they also obtained crude data on inelastic scattering.

---

[39] A. M. Lane and J. E. Lynn, Nuclear Phys. **17**, 586 (1960); see also **17**, 563 (1960).

[40] See for example J. R. Huizenga and R. Vandenbosch, Phys. Rev. **120**, 1305 (1960); R. Vandenbosch and J. R. Huizenga, *ibid.* **120**, 1313 (1960); note that we neglect the spin cutoff factor $\exp[-(J+\frac{1}{2})^2/2\sigma^2]$ which according to these references would be close to one for the low spins of prime interest in this paper.

TABLE I. Relations between cross sections and widths including factors due to the Porter-Thomas distribution. The absorption cross sections refer to resonant experiments described in text [thin absorber approximations of Eq. (26) and Eq. (28)].

Measured quantity	Ground-state spin	Case A	Case B
		Elastic scattering	
$\langle\sigma_s\rangle/3\pi^2\lambda\!\!\!^{-2}$	0	$(\langle\Gamma_{00}\rangle)^2/\Gamma D_0$	$3(\langle\Gamma_{01}\rangle)^2/\Gamma D_1=9(\langle\Gamma_{00}\rangle)^2/\Gamma D_0$
	$\frac{1}{2}$	$2(\langle\Gamma_{0\frac{1}{2}}\rangle)^2/\Gamma D_{\frac{1}{2}}$	$5(\langle\Gamma_{0\frac{1}{2}}\rangle)^2/\Gamma D_{\frac{1}{2}}$
	$\geq\frac{3}{2}$	$3(\langle\Gamma_{0I_g}\rangle)^2/\Gamma D_{I_g}$	$\dfrac{3(\langle\Gamma_{0I_g}\rangle)^2}{\Gamma D_{I_g}}\left[1+\dfrac{8}{3(2I_g+1)^2}\right]$
		Inelastic scattering	
$\langle\sigma_i\rangle/\pi^2\lambda\!\!\!^{-2}$	0	$3\Gamma_{01}\Gamma_{i1}/\Gamma D_1$	$3\Gamma_{01}\Gamma_{i1}/\Gamma D_1$
	$\frac{1}{2}$	$3\Gamma_{0\frac{1}{2}}\Gamma_{i\frac{1}{2}}/\Gamma D_{\frac{1}{2}}$	$5\Gamma_{0\frac{1}{2}}\Gamma_{i\frac{1}{2}}/\Gamma D_{\frac{1}{2}}$
	$\geq\frac{3}{2}$	$3\Gamma_{0I_g}\Gamma_{iI_g}/\Gamma D_{I_g}$	$\dfrac{3\Gamma_{0I_g}\Gamma_{iI_g}}{\Gamma D_{I_g}}\left[1+\dfrac{8}{3(2I_g+1)^2}\right]$
		Resonant absorption (thin absorber)	
$\sqrt{2}R_{\mathrm{el}\Delta\ \mathrm{P\text{-}T}}/5\pi^{\frac{3}{2}}\lambda\!\!\!^{-2}\mathfrak{N}_A$ or $\sqrt{2}R_{\mathrm{inel}\Delta\ \mathrm{P\text{-}T}}/3\pi^{\frac{3}{2}}\lambda\!\!\!^{-2}\mathfrak{N}_A$	0	$3\Gamma_{01}/\Delta=\Gamma_{00}/\Delta$	$3\Gamma_{01}/\Delta=3\Gamma_{00}/\Delta$
	$\frac{1}{2}$	$\Gamma_{0\frac{1}{2}}/\Delta$	$9\Gamma_{0\frac{1}{2}}/5\Delta$
	$\geq\frac{3}{2}$	Γ_{0I_g}/Δ	$\dfrac{\Gamma_{0I_g}}{\Delta}\left[1+\dfrac{16}{3(2I_g+1)^2+8}\right]$

Fuller and Hayward[41] used the top 10% of a bremsstrahlung spectrum to find the poor resolution energy dependence of the elastic-scattering cross section for fewer elements over a wider energy range; only their 7-Mev data will be used.

A. Elastic Scattering Cross Sections

Table II lists the scattering cross sections and the inferred values of $(\langle\Gamma_0\rangle)^2/\Gamma D$ for the elements whose level spacing at 7 Mev may not be excessive. (The lightest elements studied and Pb were omitted because they probably have excessive level spacing; some of the listed elements may well have level spacings that are too large to warrant the analysis in terms of average values.) The dominant value of the ground-state spins of the different isotopes is listed in column 2. Column 3 gives the total elastic-scattering cross sections at 7 Mev. The values taken from the work of Reibel and Mann[9,10] were calculated from the measured differential cross sections at 90° by assuming dipole absorption; their calculation involves an averaging procedure when $I_g\neq 0$, but negligible error would be introduced by incorrect averaging. The values attributed to Fuller and Hayward were corrected slightly from values read from graphs in their published work.[41] (Possible errors made in obtaining these values from the graphs or possible ambiguities in the corrections are small compared to the assigned statistical errors. One correction merely follows the instructions of the authors[42,35] and reduces the originally published values by the factor 0.866. The other correction was the small one appropriate if the angular distribution of elastically scattered 7-Mev photons is isotropic rather than $1+\cos^2\theta$. This correction was made for $I_g\neq 0$ because dipole radiation would be almost isotropic for non-overlapping levels.)

The data listed in Table II do not include any corrections for resonant self-absorption in the scattering sample; such absorption is probably small enough so that the values listed underestimate the cross section only slightly due to this effect. Both experiments would have mistaken photons actually due to high-energy inelastic scattering as due to elastic scattering; insofar as high-energy inelastic scattering is important, the values given in column 3 are too large. All entries assume that all isotopes contribute equally; there might well be some isotopes which give larger values than those listed while others give smaller values.

Column 4 lists the total absorption cross section obtained from the extrapolation of the giant dipole resonance as given in Eq. (32). In most cases, the observed elastic-scattering cross section is comfortably below the predicted total cross section, but some values are high enough to threaten the theoretical interpretation. It would be very worthwhile to obtain better experimental values which are not subject to the factor of 2 uncertainty which seems to exist in the values in column 3. [As Reibel and Mann pointed out, their values for the elastic-scattering cross sections were consistently below those of Fuller and Hayward by a factor of more than 2; the factors are 3.9 (Mn), 3.1 (Ni), 2.1 (Cu); 2.4 (Sn), and 2.1 (I). The two groups agreed only for Pb and Bi; Pb and possibly Bi have important resonant structures which cast doubt on agreement obtained with such different resolutions. Until further experimental clarification becomes available, the best experimental estimates may be taken as 1.5 times the values given by Reibel and Mann, or 0.75 times the values given by Fuller and Hayward.]

Column 5 lists the values of $(\langle\Gamma_0\rangle)^2/\Gamma D$ for spin 1

[41] E. G. Fuller and Evans Hayward, Phys. Rev. **101**, 692 (1956).
[42] Evans Hayward and E. G. Fuller, Phys. Rev. **106**, 991 (1957).

TABLE II. Elastic scattering of 7-Mev photons and ratios of widths and level spacings.

(1) Element	(2) Dominant spin	(3) $\langle\sigma_s\rangle$ (mb)	(4) $\langle\sigma_a\rangle$ extrapolated (mb)	(5) $10^5\frac{\langle\Gamma_{01}\rangle^2}{\Gamma D_1}$	(6) $10^5\frac{\langle\Gamma_{0I_g}\rangle^2}{\Gamma D_{I_g}}$	(7) $10^5\frac{\Gamma_0}{D}$ predicted	(8) $\frac{\langle\Gamma_0\rangle}{\Gamma}$
V	$\frac{7}{2}$	0.65±0.10[a]	0.9		0.092[c]	0.36	0.26[e]
Cr	0	0.30±0.05[a]	0.9	0.042[c]		0.38	0.11
Mn	$\frac{5}{2}$	0.31±0.05[a]	1.1		0.044[c]	0.45	0.10
Mn	$\frac{5}{2}$	1.2 ±0.4[b]	1.1		0.17	0.45	0.38[e]
Fe	0	0.40±0.06[a]	1.1	0.057[c,d]		0.47	0.12
Co	$\frac{7}{2}$	0.29±0.04[a]	1.3		0.041[c]	0.54	0.076
Ni	0	0.20±0.02[a]	1.2	0.028[c]		0.52	0.054
Ni	0	0.62±0.13[b]	1.2	0.088		0.52	0.17[e]
Cu	$\frac{3}{2}$	0.57±0.06[a]	1.6		0.081[c,d]	0.67	0.12[e]
Cu	$\frac{3}{2}$	1.2 ±0.16[b]	1.6		0.17[d]	0.67	0.25[e]
Zn	0	0.86±0.13[a]	1.6	0.12[c]		0.69	0.17[e]
Zr	0	1.0 ±0.1[a]	4.1	0.14		1.7	0.082
Mo	0	1.4 ±0.2[a]	4.8	0.20		2	0.10
Ag	$\frac{1}{2}$	0.65±0.10[a]	6.4		0.14	2.7	0.052
Cd	0	1.4 ±0.2[a]	7.0	0.20		3.0	0.067
Sn	0	4.1 ±0.5[a]	8.3	0.58		3.5	0.17[e]
Sn	0	10 ±0.9[b]	8.3	1.4		3.5	0.40[e]
Sb	$\frac{5}{2}$	0.89±0.13[a]	8.8		0.13	3.7	0.035
Te	0	5.8 ±0.8[a]	10	0.83		4.2	0.20
I	$\frac{5}{2}$	0.71±0.11[a]	9.9		0.10	4.2	0.024
I	$\frac{5}{2}$	1.5 ±0.9[b]	9.9		0.21	4.2	0.05
Ba	0	4.9 ±0.7[a]	12	0.7		5.0	0.14[e]
Sm	0	<0.67[a]	16	<0.95		6.6	<0.14
W	0	1.8 ±0.4[a]	26	0.25		11	0.023
Au	$\frac{3}{2}$	<2.7[a]	32		<0.38	13	<0.029
Au	$\frac{3}{2}$	2.3 ±1.2[b]	32		0.33	13	0.025
Hg	0	3.5 ±0.4[a]	33	0.50		14	0.036
Tl	$\frac{1}{2}$	3.9 ±0.6[a]	35		0.83	15	0.055
Pb^{206}	0	10.5 ±0.8[a]	36	1.5		15	0.10[e]
Bi	$\frac{9}{2}$	17.5 ±1.3[a]	37		2.5[c]	16	0.16[e]
Bi	$\frac{9}{2}$	15 ±3.5[b]	37		2.1	16	0.13[e]
Th	0	0.86±0.13[a]	49	0.12		21	0.006[f]
U	0	2.8 ±1.1[b]	53	0.40		22	0.018[f]

[a] Values taken from reference 10. Photons were predominantly in a 130-kev interval about 7.12 Mev.
[b] Values taken from reference 41 (with minor corrections described in text). Gamma rays are in an interval of about 700 kev near 7 Mev.
[c] Significance of average values and level spacing is questionable because apparently similar neighboring nuclei have rather large level spacings (as given in reference 13).
[d] Significance uncertain in view of the rapid energy dependence of the cross section reported in references 9 and 10. Whether the energy dependence is due to very few strong levels or to a sharply peaked strength function, the average has little meaning.
[e] This high value of $\langle\Gamma_0\rangle/\Gamma$ shows that many values of Γ_0 in a Porter-Thomas distribution would be comparable with $\langle\Gamma\rangle$. In this case, Γ should not be considered constant, and the true value of $\langle\Gamma_0\rangle/\langle\Gamma\rangle$ is larger than the one listed by a factor of less than 3.
[f] In this case, Γ certainly includes more than electromagnetic transitions because the photofission threshold is well below 7 Mev.

states reached if the ground state $I_g=0$ is excited by dipole radiation. These values are obtained from the data with the aid of Eq. (24) which is based on the Porter-Thomas distribution; if all levels of spin 1 had the same width, or if $\langle\Gamma_0\rangle$ is close to Γ, the values listed in column 5 should be multiplied by 3.

Column 6 is analogous to column 5 except that a specific assumption is included about the relative probability of exciting states of different spin when $I_g\neq 0$. The listed values correspond to $\langle\Gamma_0/D\rangle_I=\langle\Gamma_0/D\rangle_{I'}$, which is Case A discussed in Sec. II(F). The formulas used, and the changes which would be implied if Case B were correct can be found in Table I.

Column 7 contains the calculated value of $\langle\Gamma_0/D\rangle$ from Eq. (34); it applies to all spin values if $\langle\Gamma_0/D\rangle$ is the same in a given nucleus for all spins which can be reached by dipole absorption (i.e., if Case A of Sec. II(F) applies). Comparing the experimental values (columns 5 and 6) with the predictions (column 7) gives the value of $\langle\Gamma_0\rangle/\Gamma$ in column 8. For $I_g=0$, the value in column 8 is $\langle\Gamma_{01}\rangle/\Gamma$; for $I_g\neq 0$, the value in column 8 is $\langle\Gamma_{0I_g}\rangle/\Gamma$. In accordance with the assumptions used to obtain these values, $\langle\Gamma_{0I_e}\rangle=\langle\Gamma_{0I_g}\rangle(2I_g+1)/(2I_e+1)$.

The inferred values of $\langle\Gamma_0\rangle/\Gamma$ might seem somewhat larger than are usually found in neutron capture studies. However, the levels reached by 7-Mev photons often have a larger spacing, D, than is customary in neutron capture; $\langle\Gamma_0\rangle$ might then be expected to be proportionately higher. Furthermore, there is often a high-energy capture gamma ray which accounts for 5–10% of the observed radiation width; these might well be analogous to the ground-state gamma rays in nuclei where selection rules prohibit $E1$ transitions to the ground state following neutron capture.

A more revealing test of Eqs. (32) and (34) could be obtained if better experimental values of D were available with which to compare D values inferred from the data in Table II with the aid of measured values of Γ. Without additional data, one can be gratified by qualitative features such as the implied differences in D (7

TABLE III. Ground-state radiation widths from resonant absorption.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Thickness 10^{23} atom/cm^2		Percentage absorbed resonantly			Widths		
	Total	Average[a]		Γ_0[b,c] (10^{-3} ev)	Γ_0[c] (10^{-3} ev)	Γ_{01} (10^{-3} ev)	Γ_{0I_g} (10^{-3} ev)	D (kev)
Cu	3.0	1.7	36±13	280	490		125±45	$D_{\frac{3}{2}}=19$
Ag	2.1	1.05	30±24	250	500		120±95	$D_{\frac{1}{2}}=4.4$
Sn	1.1	0.29	28±19	150	610	125±85		$D_1=3.6$
Hg	1.3	0.29	34±21	130	580	125±80		$D_1=0.89$
Pb^{206}	0.67	0.57	53±23	470	560	130±55		$D_1=0.84$
Bi	0.73	0.73	29±13	550	550		120±55	$D_{9/2}=0.75$

[a] Effective number of atoms/cm^2 capable of absorbing resonantly. Average assumes that both isotopes of Cu and Ag contribute equally and that only even-even isotopes of Sn, Hg, and Pb^{206} contribute equally.
[b] Uncorrected for average isotopic abundance.
[c] Uncorrected for Porter-Thomas distribution.

Mev) for odd A and even-even nuclei, or the general decrease in D as A increases.

B. Resonant Absorption and Partial Radiation Width, Γ_0

The partial width of a level for radiative decay to the ground state Γ_0 can be obtained directly from a resonant absorption measurement if $\Gamma<\Delta$ by using Eqs. (16), (17), and for small t, Eqs. (26–28). (If the total level width is much bigger than the Doppler broadening, this measurement gives instead Γ_0/Γ as shown by Eqs. (18) and (19). In the intermediate case of $\Gamma\approx\Delta$, more than a single absorption measurement must be made to determine both Γ_0, and Γ.) In order to illustrate the effect of a Porter-Thomas distribution, some absorption data will be re-interpreted using the extreme Doppler approximation. The inferred values of Γ_0 are small enough to make $\Delta>\Gamma$ a reasonable approximation even though it is by no means proven.

Table III contains the resonant absorption data of Reibel and Mann[9,10] together with the derived parameters. Column 2 contains the total number[9] of atoms/cm^2 while column 3 contains the average number of atoms per scattering isotope. [Inasmuch as different isotopes will not usually resonantly absorb the radiations other isotopes scatter, a number smaller than $\mathfrak{N}_{A\ \mathrm{total}}$ should be used in Eq. (15), unless the element consists of a single isotope. For Cu and Ag, it was assumed that each isotope had the same scattering cross section and the same $\langle\Gamma_0\rangle/D$; the correction factors were 1.75 and 2.0. For Sn, Hg, and Pb^{206}, it was assumed that only the even-even isotopes contributed to the elastic scattering, and that each even-even isotope gave the same contribution; this introduced correction factors of 4.1, 4.5, and 1.2. These assumptions would be doubtful if the D values of the contributing isotopes varied significantly.]

The observed resonant absorption is given in column 4. The errors listed are those Reibel assigned to the values of Γ_0 he deduced from the absorption when he calculated Γ_0 directly from the Doppler approximation[9]; his values Γ_0 [obtained using the equivalent of Eq. (27) but including a more exact treatment of electronic absorption[9]] are given in column 5. When these values are corrected for the isotopic abundances discussed above, one obtains the widths listed in column 6. (If any one isotope dominated the scattering, it would have a larger value of Γ_0 than that deduced by assuming equal contributions from the different isotopes.) The widths listed in column 6 would therefore be correct if all excited levels had the same value of Γ_0 as Reibel assumed.[9]

Columns 7 and 8 give the average values of Γ_0 if a Porter-Thomas distribution is assumed. These values were calculated directly from the data of columns 3 and 4 by using Eqs. (26) and (39). The main differences between these corrected values and those of column 6 are the reductions in $\langle\Gamma_0\rangle$ obtained by using Eq. (26); the reduction factors are 4.55, 4.69, 4.71, 4.65, 4.41, and 4.70, respectively. The other minor changes are due to the different assumptions made in averaging over spin; the approximate treatment of electronic absorption in the scatterer does not seem to contribute significantly.

The values of $\langle\Gamma_0\rangle$, at first glance, seem rather large compared with values of both[14,26a] Γ_0 and[13] Γ encountered in neutron studies. However, these large values can be understood, at least in part. For the elements studied, the 7-Mev photons excite states well below the neutron threshold, implying D values considerably above those usually encountered in neutron capture. If D varies more rapidly with energy than does $\langle\Gamma_0/D\rangle$, larger values of $\langle\Gamma_0\rangle$ might be expected at these lower energies. Furthermore, the large experimental errors in columns 7 and 8 would admit values of $\langle\Gamma_0\rangle$ well below those listed as nominal. Finally, it should be noted that the elements appearing in Table III (except for Ag and Hg) are not typical because they have particularly large values of $\langle\Gamma_0\rangle/\Gamma$ in Table II.

The values of D listed in column 9 were obtained from the values of $\langle\Gamma_0\rangle$ in columns 7 and 8 (Table III) and the predictions appearing in column 7 of Table II. Thus, these D values have experimental uncertainties directly proportional to those given for $\langle\Gamma_0\rangle$. Within these possible errors, these D values are consistent with reasonable energy extrapolations of the D values obtained in other experiments.[13]

C. Inelastic Scattering Measurements and $\langle\Gamma_0\rangle/\Gamma$

A direct measurement of the inelastic-scattering cross section would not be affected by a distribution of widths inasmuch as only the first power of Γ_0 or Γ_i appear (see Table I). On the other hand, the interpretation of some indirect measurements of inelastic scattering can be affected strongly by a distribution of level widths.

The changes that are introduced by the Porter-Thomas distribution in some cases can be illustrated well by the measurements Reibel and Mann[9,10] used to infer Γ_0/Γ. They based their value on the ratio of the absorption observed when detecting inelastic scattering to the absorption observed when detecting elastic scattering. This measurement gives the ratio $\langle\Gamma_0^3\rangle/\Gamma_i\langle\Gamma_0^2\rangle$ which according to the Porter-Thomas distribution is $5\langle\Gamma_0\rangle/\Gamma_i$. Thus, the quantity Reibel and Mann called[10] Γ_0/Γ is, according to the Porter-Thomas distribution, $5\langle\Gamma_0\rangle/(\Gamma_i+5\langle\Gamma_0\rangle)$. $\langle\Gamma_0\rangle$ is understood as $\langle\Gamma_{01}\rangle$ for even-even targets and $\langle\Gamma_{0I_g}\rangle$ for odd-A targets with $I_g\geq\frac{3}{2}$. For $I_g=\frac{1}{2}$, the corresponding expression is $10\langle\Gamma_0\rangle/(3\Gamma_i+10\langle\Gamma_0\rangle)$.

In order to illustrate the effect of this change, Table IV lists both the old values of Γ_0/Γ and the revised values.

It should be emphasized that the values listed in Table IV are cited to illustrate the effect of the Porter-Thomas distribution rather than to give corrected values for $\langle\Gamma_0\rangle/\Gamma$. The experimental values are quite uncertain, mainly because of extremely large backgrounds of unknown origin. Even the large errors assigned have probably been reduced somewhat[9] by using the consistency of the elastic scattering, absorption, and inelastic scattering data, and the implications of this consistency change when a Porter-Thomas distribution is assumed. (The actual percentage changes in the counts in the inelastic region caused by the absorbers were, except for Cu, only about 10%; since the assigned cross sections with and without absorbers also have quoted errors of 10% or greater[9] it is impossible to estimate the error in the ratio of differences on which Γ_0/Γ depends.)

Furthermore, there are systematic errors which could increase or decrease the values significantly. The values would decrease if the energy region chosen as representative of the inelastic scattering included less than one gamma ray per inelastic cascade. On the other hand, the values would increase due to two effects which tended to overestimate the inelastic scattering. First, elastic scattering events would be counted as inelastic when only a portion of the photon energy produced ionization in the crystal. In addition, the detector efficiency would be greater for lower energy photons. These last two corrections, which were probably neglected because of the large uncertainties already caused by the large unexplained background, could increase Γ_0/Γ considerably.

TABLE IV. Effect of Porter-Thomas distribution on indirectly inferred $\langle\Gamma_0\rangle/\Gamma$.

	$(\Gamma_0/\Gamma)^{a}$	$(\langle\Gamma_0\rangle/\Gamma)^{b}$ Max	Min
Cu	0.15 ± 0.1^{c}	0.07^{c}	0.01^{c}
Ag	0.06 ± 0.04	0.03	0.006
Sn	0.2 ± 0.1	0.08	0.02
Hg	0.07 ± 0.03	0.02	0.008
Pb^{206}	0.6 ± 0.3^{c}	$0.65^{c,d}$	0.08^{c}
Bi	0.3 ± 0.2^{c}	0.17^{c}	0.02^{c}

[a] Values taken from reference 10 which assumed all levels in each target had a single value of Γ_0 and a single value of Γ_i.
[b] Values calculated using Porter-Thomas distribution for Γ_0 and a constant value of Γ_i.
[c] The level spacing may be too large for the measurement to involve enough levels to define a meaningful average. If only one level were involved, the value in column 2 would be correct.
[d] For values of $\langle\Gamma_0\rangle/\Gamma$ which are large, some levels will have Γ_0 values large enough to influence Γ. The resonant absorption ratio might then depend on $\langle\Gamma_0^2\rangle/\Gamma_i\langle\Gamma_0\rangle$ rather than $\langle\Gamma_0^3\rangle/\Gamma_i\langle\Gamma_0^2\rangle$; this would change the inferred values of $\langle\Gamma_0\rangle/\Gamma$, but such changes are not included in the listed value.

D. Other Photon Absorption Measurements

Most of the other photon interaction data below threshold come from inelastic scattering which leads to an isomeric state, but there is also a recent measurement of the total absorption cross section. A detailed study of the interaction of 7-Mev photons with Th^{232}, 5 isotopes of U, and Np^{237} gives a total absorption cross section[43] of about 50 mb, in excellent agreement with Eq. (32) (see values given in Table II).

The data on isomer production give a lower limit to the absorption cross section; the total inelastic scattering is expected to be considerably larger because the ground state would be highly favored over the isomer in the inelastic cascade.[40] Unfortunately, the standard calculations are least reliable when they try to predict weak branches such as those encountered in the photoexcitation of isomers. On the other hand, estimates can be made, and one has some confidence that the correction factor will increase as the spin change between ground state and isomer increases. Within the uncertainties involved in this correction the observed partial cross sections at 7 Mev for $Cd^{111}(\gamma,\gamma')Cd^{111m}$ (0.4 mb),[44] $In^{115}(\gamma,\gamma')In^{115m}$ (1.6 mb),[44] $Au^{197}(\gamma,\gamma')Au^{197m}$ (about 3 mb),[45] and $Rh^{103}(\gamma,\gamma')Rh^{103m}$ (1.9 mb),[46] are consistent with the estimates of the total cross section listed in Table II [calculated from Eq. (32)].

Before closing this discussion of photoabsorption data near 7 Mev, it should be noted that we have purposely excluded the wealth of data above particle emission threshold. Matching equations analogous to Eq. (31) and Eq. (32) to existing information on photoinduced reactions may be a fruitful avenue toward later refinements, but it would not contribute very much right now

[43] J. R. Huizenga, K. M. Clarke, J. E. Gindler, and R. Vandenbosch (to be published).
[44] J. R. Huizenga and R. Vandenbosch (to be published).
[45] L. Meyer-Schützmeister and V. L. Telegdi, Phys. Rev. **104**, 185 (1956).
[46] O. V. Bogdankevich, L. E. Lazareva, A. A. Moiseev, J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 1224 (1960) [translation, Soviet Phys.—JETP **12**, 853 (1961)].

toward deciding whether the elastic scattering data at 7 Mev are intimately connected with the giant resonance. Equations (31) and (32) are close enough to most data available just above thresholds to qualify as first approximations in attempting to understand the limited data available at 7 Mev. Furthermore, comparing precise data just above and just below the threshold would not particularly show that the lower energy region is connected to the main features of the giant resonance except insofar as other studies show that the region just above threshold is intimately connected with the giant resonance. For example, the relatively radical suggestion that special phenomena exist in the threshold region[47,48] does not necessarily imply a discontinuity at threshold.

The discussion above should not be misconstrued as a criticism of the interesting and important detailed comparisons that have been made near thresholds. (Examples can be found in references 35 and 46.) In principle, these comparisons could produce information about discontinuities at threshold. The present uncertainties and experimental errors are so great, however, that the comparisons could be used instead to give additional insight into unknown branching ratios below the threshold energy by assuming continuity. In order to emphasize this point consider the particular case of Bi^{209} which has been studied by Fuller and Hayward.[35] They obtain an absorption cross section with a dip at 8 Mev which rises somewhat at lower energies. At 7.75 Mev their total cross section is above 40 mb. [Incidentally, Eq. (32) predicts a total cross section of 49 mb at 7.75 Mev.] In order to match this to their measured scattering cross section of 21 mb at 7 Mev they assume $\Gamma_0/\Gamma=\frac{1}{2}$ (using Reibel and Mann's values of Γ_0/Γ for Pb and Bi as justification). Simultaneously they assume that $\Gamma_0/\Gamma_i=1$ for all energies in order to infer the total absorption cross section at other energies from the measured elastic scattering. The same data can be interpreted in terms of a Porter-Thomas distribution and Eq. (31) as in Table II. The value of $\sigma_s(E)=21$ mb (instead of 15 mb given in Table II) implies $\langle\Gamma_0\rangle/\Gamma=0.19$; for the reasons listed in note e of Table II, the actual value of $\langle\Gamma_0\rangle/\Gamma$ is expected to be somewhat larger. The value of Γ_0/Γ would be expected to vary with energy unless the energy dependences of (Γ_0/D), D, and Γ combined to produce a constant, energy-independent Γ_0/Γ.

IV. CONCLUSIONS AND SUMMARY

Although the existing data are too fragmentary to support the claim that either the Porter-Thomas distribution or the extrapolation of the giant resonance have been proven, both of these are certainly more reasonable working hypotheses than the alternatives that have been used most commonly. The analysis presented in this paper removes the main discrepancies which had existed, and thereby renders premature the conclusion that a new phenomenon is implied[47] (such as the existence of threshold states[48]). Additional data from both neutron capture and the nuclear interaction of intermediate-energy photons will be needed to test the suggested formulas more critically; such tests will probably lead to refinements such as might be expected from shell effects. In order to emphasize the justification for the change to the suggested analysis, its successes will be summarized briefly.

A. Porter-Thomas Distribution of Γ_0

Data on individual neutron resonances in a single nucleus,[14,15] and on the variation of Γ_0/D values obtained from thermal neutron capture gamma-ray studies[26a] provide direct evidence for a distribution of ground-state transition widths. The nuclear interaction of 7-Mev photons provides strong, albeit indirect, support for the idea of a distribution. (This confirmatory information is indirect because it depends in part on nuclear parameters which are plausible rather than proven.)

The application of a Porter-Thomas distribution to the analysis of elastic scattering contributes a factor of 3 to the expected relation between the observed average values of the elastic scattering and total absorption cross sections: $\langle\sigma_s\rangle=\langle\sigma_a\rangle(3\langle\Gamma_0\rangle/\Gamma)$. This factor weakens objections which have been raised[47] against the statistical interpretation of the observed σ_s/σ_a ratios. If the extrapolated value of σ_a (7 Mev) is credible, the Γ_0/Γ values (of Table II column 8) support this factor of 3 because they are already large and would be three times as large without it. Even if the extrapolation is questioned, isolated measurements (e.g., of Hg^{200}) give values of $\langle\Gamma_0\rangle$, D, and Γ which agree better with the values inferred from photon experiments if the factor of 3 is included.

Another important result of using a Porter-Thomas distribution is the reduction by almost 5 it implies in $\langle\Gamma_0\rangle$ values obtained from resonant absorption data (Table III). Consider for example the data on Hg in Table III. If $\langle\Gamma_0\rangle$ were about 0.5 ev, it would stretch one's credulity to expect $\langle\Gamma_0\rangle/\Gamma$ to be much below $\frac{1}{2}$; the observed value of $\langle\sigma_s\rangle$ would then imply D_1 (7 Mev) = 17 kev whereas D_1 (8 Mev) has been measured[13] in Hg^{200} as only 0.09 kev. Appealing to the tendency of photon absorption to select states with large values of Γ_0 (and therefore larger D)[10] by no means reduces the need for a distribution function; on the contrary, such explanations are merely qualitative expressions of the existence of a distribution.

Although one might reasonably demand additional confirmation before accepting the universal validity of a distribution of widths, there is no justification for pre-

[47] A. M. Badalyan and A. I. Baz', J. Exptl. Theoret, Phys. (U.S.S.R.) **40**, 549 (1961) [translation, Soviet Phys.—JETP **13**, 383 (1961)].

[48] A. I. Baz', *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1959), Vol. 8, p. 349.

ferring the more special assumption that all neighboring levels have identical values of Γ_0. On the other hand, it should be stressed that the Porter-Thomas distribution has been assumed rather than established by the data. Other distributions could undoubtedly fit the existing data equally well.

B. Extrapolation to Low Energy of the Giant Dipole Resonance

Neutron capture data call for a more rapid energy dependence of Γ_0/D than is predicted by the single particle model [as discussed in Secs. II(D) and II(E)]. Additional data will be needed before the E and A dependences and shell effects can be sorted out, but an energy dependence of the form of Eq. (31) and even the crude estimates given by Eqs. (32–35) are consistent with neutron data now available.

The available photon interaction data are fit surprisingly well by the crude extrapolation obtained from giant dipole systematics. As shown in Table II the observed scattering cross sections are reasonably consistent with predicted total absorption cross sections. The odd A, even A variation of the elastic scattering is explained naturally if Γ and Γ_0/D are essentially uniform, and if the larger D values in even-even nuclei produces a larger $\langle\Gamma_0\rangle/\Gamma$. (A larger Γ_0/Γ was also cited by Reibel and Mann as the probable cause of the larger elastic scattering cross sections in even-even nuclei.[10])

Equation (34) seems supported because the D values predicted from observed $\langle\Gamma_0\rangle$ values (Table III), are reasonable. Finally, the few total absorption cross sections and partial inelastic cross sections [Sec. III(D)] fit very well.

Much more data are needed before one can conclude that the photon interaction data well below the giant dipole resonance are intimately related to this resonance. (There may be $M1$ or $E2$ effects, or there may be $E1$ effects not specifically related to the giant resonance.) Further, it remains to be seen whether a detailed curve of the giant resonance is needed or whether a crude fit [perhaps somewhat more refined than Eqs. (31)–(35)] will be sufficient. In spite of the open questions, there seems little doubt that the suggested extrapolation of the giant dipole resonance gives a better account of the data than does any minor modification of the single-particle model.

ACKNOWLEDGMENTS

The author wishes to thank the Physics Staff at the University of Washington in Seattle, and particularly Professor R. Geballe and Professor I. Halpern for the hospitality he received as a summer visitor. He is also grateful to Professor J. Blair, Professor I. Halpern, and Professor E. Henley for stimulating discussions related to this paper. It is also a pleasure to acknowledge informative talks with J. Huizenga, G. Ravenhall, and K. Reibel.