Dependence of Neutron Production in Fission on Rate of Change of Nuclear Potential*

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Most prompt neutrons from fission are thought to come from separated fission fragments excited during scission. Can some come out in a prior process: excitation of individual nucleons in the dividing nucleus by nonadiabatic changes in the nuclear potential? The question can be raised: Can enough energy be transferred to some neutrons to free them by the nonadiabatic distortion of the potential as the nucleus fissions? To study this possibility we consider a square well containing a Fermi sea of nucleons in the middle of which is "erupting" an inverted square well "volcano." It rises and divides the original square well into the two fission fragments. The energy transferred to the nucleons can be calculated analytically for the two limiting cases, adiabatic and sudden changes of potential. The transition between these limiting situations has been studied on a computer. One finds that there exist reasonable rates of rise of the volcano (i.e., scission times$\cong 5\times 10^{-22}$ sec) for which the number of neutrons ejected is of the same order of magnitude as the total number of neutrons ejected on the average per act of fission due to all mechanisms. By varying the ratio of the width of the volcano to the width of the square well, or by smoothing the volcano out in space (rounded volcano instead of an inverted square well volcano), the number of particles ejected can be changed also. For a particular set of parameters, consistent with the experimentally known facts, we find that about 1% of the neutrons in the nucleus are ejected. For a heavy nucleus such as Cf^{252} this corresponds to about 1.5 neutrons per fission.

It is not proposed that all prompt fission neutrons are ejected by this mechanism, nor even a fraction necessarily anywhere as large as this example might suggest; but it is emphasized that certainly *some* fraction of the neutrons is produced by this mechanism.

I. INTRODUCTION

MANY physical effects depend primarily on the time rate of change of force rather than the magnitude of force. As a prototype of all such effects consider the energy transferred to a swing by the wind. For wind of a given peak strength, this energy transfer will depend on the time rate of change of wind strength. It is clear that if the wind increases very slowly ("adiabatically") to its peak strength and then decreases in the same fashion the swing will not be excited at all. In contrast, a departure from an adiabatic change in the force exerted by the wind (such as a gust) will result in energy being transferred to the swing.

At first sight it would seem that the problem of neutron emission in nuclear fission has nothing to do with adiabaticity, or the departures from adiabaticity, of a time-dependent potential. The usual picture is that the neutrons emitted in fission evaporate from the neutron-rich, separated fission fragments. However, the potential that binds the nucleons has been changing with time during the process of fission. Thus the question arises: Can some neutrons be emitted prior to scission in a distinct process? Can enough energy be transferred to some neutrons in the dividing nucleus by the nonadiabatic distortion of nuclear potential during fission to permit their escape? Thus, this problem is seen to possess features in common with the swing blown by the wind.

We consider a square-well nucleus containing a Fermi sea of nucleons in the middle of which is "erupting" an inverted square-well "volcano." It rises and divides the original square well into the two idealized fission fragments. This potential imparts excitation to some of the nucleons as it rises. More precisely, it has a certain probability to give this, that, or the other amount of energy to the nucleons in the Fermi sea. These probabilities of imparting specified amounts of excitation depend upon the rate of rise of the potential. We study the fraction of particles ejected and the mean energy transfer as a function of time of rise of the volcano, making allowance for the effect of the Pauli exclusion principle on nucleon transitions.

We do not study a specific problem with particular values of the parameters associated with a fissioning nucleus because the exact duration of the division of the fragments is not known, nor is the exact shape of the fissioning nucleus known once the fission neck has snapped. Therefore, we are forced to examine the dependence of neutron emission on the degree of nonadiabaticity of the scission, to see if there exist reasonable lengths of time for the division that do indeed imply the liberation of a reasonable fraction of the neutrons in the original nucleus.

These neutrons ejected by the nonadiabatic change of nuclear potential could be the "scission neutrons" indicated in the work of Bowman *et al.*[1] They have measured the angular distribution of the prompt neutrons in the spontaneous fission of Cf^{252}. Their results are consistent with up to 10–20% more neutrons in the region perpendicular to the line of flight of the fragments above the neck than would be expected on the basis of a purely isotropic evaporation of neutrons. They call this surplus (10–20%) of neutrons in the region perpendicular to the neck the "scission neutrons." The present study indicates that the presence of these scission neutrons can be explained as being due to the nonadiabatic

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[1] H. Bowman, S. Thompson, J. Milton, W. Swiatecki, Lawrence Radiation Laboratory, University of California, Berkeley, California. UCRL-9713-Rev. (to be published).

distortion of nuclear potential and the resulting transfer of energy to nucleons in the nucleus. For reasonable values of the available parameters (size of neck, rate of scission) we find that the number of neutrons ejected is consistent with the data of Bowman *et al.*[1]

II. SPECIFICATION OF MODEL AND ANALYTICAL TREATMENT

The division of the original square well nucleus into the two fission fragments is shown in Fig. 1. It is seen that in the center of the original square well the potential depth is constantly decreasing. Eventually, the potential in this region is not low enough to bind nucleons. Finally, the fragments separate. The volume of nuclear matter is known to remain nearly constant during fission. Therefore, for proper nuclear matter, the well width should increase as the volcano rises.

The model on which our calculations have been based differs from a real nucleus in two respects. First, we have taken the square well to be of infinite depth. The volcano rises until it is as far from the bottom of the well (the zero of potential) as the true nuclear well is deep (about 41 Mev). Second, we have kept the square well at constant width during the volcano rise. This, of course, results in the particles finding themselves in a reduced volume of phase space at the end of the volcano rise; hence they automatically gain in energy, this gain having nothing to do with the excitation energy transferred to the particles in the well by the rising volcano. We must subtract off this energy rise, in the limiting case of no excitation to higher quantum levels, when we compute the mean energy transfer to particles in the well. We can exclude it by measuring the energy of the particles after the volcano rise *relative to the energy transfer in the adiabatic case.* In the case of adiabatic volcano rise, we know that no energy will be transferred to the particles in the well by the volcano. The increase in energy will be due solely to the reduced volume in phase space allowed the particles because of the presence of the volcano. Thus the energy transferred to the particles in the well by the rising volcano will be the difference between the actual mean energy transfer and the mean energy transfer in the adiabatic case.

An analogy may throw further light on this situation. Imagine someone leaping into a tub of water. What we want to measure is the splash, the displacement of water over and above the rise in water level that would accompany his slow descent beneath the original water level. If he enters the tub adiabatically, there will be no splash, but no matter how he enters there will be a rise in water level due to his displacing water. The particles we are looking for in the subsequent analysis are "those in the splash."

In the model we shall study (Fig. 1) the well width is $2a$; the volcano width is $2l$, and its height at time t is $V(t)$, or simply V. Its final height, V_0, occurs for

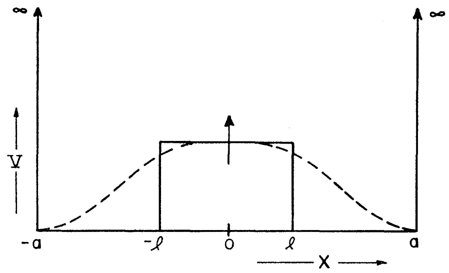

FIG. 1. Square-well (solid) and rounded (dashed) volcanos rising in time and dividing the nucleus.

$t=2\tau$, so $V_0=V(2\tau)$. In numerical work V_0 will have the value of the nuclear well depth, i.e., about 41 Mev. By varying τ, different rates of change of potential result. For the numerical work we must pick some particular functional dependence of $V(t)$ on t. So long as the potential starts out at $t=0$ with a smooth slope, the exact functional dependence of $V(t)$ on t is not of great importance; of course, it is not known experimentally. We want something that starts off smoothly at $t=0$. We have picked the function

$$V(t)=(V_0/2)\{1+\tfrac{1}{2}[3(t/\tau-1)-(t/\tau-1)^3]\}. \quad (1)$$

It is seen that $V'(0)=V'(2\tau)=0$. Since $V(\frac{3}{2}\tau)-V(\frac{1}{2}\tau)=(22/32)V_0$, most ($\approx 2/3$) of the volcano rise occurs during the time interval τ. This function is the simplest one that has these desirable properties in a finite time interval. The exact form of the potential will enter into the calculation only when we examine the region intermediate between the adiabatic and sudden limits.

Since the momenta in the y and z directions (perpendicular to the length of the square well, i.e., parallel to the sides of the volcano) remain constant during the rise of the volcano, we can treat the x degree of freedom in and by itself. This is the point of the one-dimensional idealization.

III. ADIABATIC AND SUDDEN LIMITS

If we apply the criterion for the validity of the adiabatic approximation to the time-dependent potential, $V(t)$, we find as the criterion for the validity of the adiabatic approximation: $\tau \gg 5\times 10^{-22}$ sec. Our numerical calculations will substantiate this. It will be seen that for $\tau=10^{-21}$ sec the transition probability to any state other than the originally occupied state is negligible. For all $\tau \geqq 10^{-21}$ sec we know that the quantum numbers are adiabatic invariants. Therefore, if the particle starts out before the volcano erupts in the jth state it will—after the volcano has risen—still be in the jth state.

To determine the energy of this state we have only to solve the time-independent Schrödinger equation after the volcano has risen. There is an easier—approximate—way to determine the eigenvalues than by solving the transcendental equation that results if we solve Schrödinger's equation. We can determine the

quantum number of a state of energy E approximately by dividing the phase space available to such a particle by Planck's constant. Before the volcano rises the quantum number, n_i, of a state of energy E_i is given by

$$n_i = (2a)[2(2mE_i)^{\frac{1}{2}}]/h. \tag{2}$$

Here the length $(2a)$ is the extension of the square well (see Fig. 1). The quantity $2(2mE_i)^{\frac{1}{2}}$ is the difference between the two values of the momentum that correspond to E_i, $\pm(2mE_i)^{\frac{1}{2}}$. Once the volcano has risen to a height, V_0, the quantum number, n_f, of an eigenvalue of energy $E_f < V_0$ is again given by the same method. We have

$$n_f = [2(a-l)][2(2mE_f)^{\frac{1}{2}}]/h. \tag{3}$$

To an energy $E_f \geqq V_0$ corresponds the quantum number

$$n_f = \{2(a-l)2(2mE_f)^{\frac{1}{2}} + 4l[2m(E_f - V_0)]^{\frac{1}{2}}\}/h. \tag{4}$$

From these equations we can determine for any quantum number, n_f, the eigenenergy, E_f, that corresponds to it in the final potential configuration. Actually, n_f will be equal to the number of quantum states of both even and odd parity. For example, for $l/a = 1/3$, the energy $E_f = 0.862 \times 10^{-4}$ ergs (≈ 54 Mev) corresponds to 3 decimal places to the quantum number $n_f = 17$. This means the 9th even eigenstate because there are 8 odd eigenstates lying between the 9 even ones. If we are interested in knowing the energy of a particle which was originally in the 9th even eigenstate after the volcano rises we have only to use the appropriate value of l/a in (4), set $n_f = 17$, and solve for E_{17}. In the numerical calculations we treat the case $l/a = 15/48 = 0.3125$ for the adiabatic limit ($\tau = 10^{-21}$ sec). Initially, the particle is in the 9th even eigenstate. Using the quantum mechanical adiabatic theorem, we know that the particle must find itself in the 9th even eigenstate of the final potential. Equation (4) tells us that the energy of this state is 0.849×10^{-4} erg (≈ 53 Mev). The machine calculation gives the probability of the particle being in even state number 9 as 1.002 (Table I) (machine errors will be mentioned later). The transition probabilities to all states other than state 9 are negligible. The numerical result for the energy of this state is 0.836×10^{-4} erg.

The mean energy transfer, $\bar{E}$, follows immediately for the adiabatic limit. The original energy of the particle was the energy of the 9th even eigenstate of the pure square well. This is 0.593×10^{-4} erg. Therefore, $\bar{E} = (0.849 - 0.593) \times 10^{-4}$ erg $= 0.256 \times 10^{-4}$ erg (≈ 16 Mev). The numerical result is 0.261×10^{-4} erg as seen in Table I. In this same adiabatic limit the mean excitation—as distinct from the energy transfer—is, of course, zero.

We turn now to the other extreme—that of very short time of rise, the sudden limit. The criterion of validity for use of the sudden approximation tells us that unless the volcano rises in a time shorter than 1.5×10^{-23} sec the sudden approximation will not be valid.

Rough estimates can be made for the time of snap in fission (i.e., the time of rise of the volcano in our model), which indicate that the process of division takes at least 10 times as long as would be necessary for the sudden approximation to be useful. The fragment velocities after separation have been measured.[2] The light fragment of U^{235} is found by Stein to travel 269 cm in 180×10^{-9} sec implying a velocity of $\approx 1.5 \times 10^{-9}$ cm/sec. The length of the neck (in Fig. 1 the neck has length $2l$) in a fissioning nucleus has an average value of $\approx 1/3$ the length of the distorted nucleus, which itself is 2 or 3 times the diameter of the original nucleus.[3] An estimate of the time of snap is then given by dividing the length of half this neck by the velocity (relative to the laboratory) of one of the fission fragments. We find

$$\begin{aligned}(\Delta t)_{\text{snap}} &\approx (1/3)(\text{length of distorted nucleus})/ \\ &\qquad\qquad (\text{separation velocity}) \\ &\approx (1/3)(2.0 \times 10^{-12}\text{ cm})/(1.5 \times 10^{9}\text{ cm/sec}) \\ &= 4.5 \times 10^{-22}\text{ sec.}\end{aligned}$$

It might be expected that the time interval during which most of the scission occurs is considerably shorter than this because the fragments probably move apart much more rapidly when closer together, being slowed up as they approach the point of division by the increased surface tension of the elongated shape. This effect would be expected to reduce the velocity of separation below its average value during the process of fission, making this estimate of the time of snap too high. Even with this qualification we would expect to be well outside the range of validity of the sudden approximation. Thus we cannot expect to get useful numbers out of an analysis based on the sudden approximation.

IV. THE SINGLE-PARTICLE PROBLEM

In this section we consider a single particle in a given eigenstate of the original well. A volcano rises in the well and energy is transferred to the particle. The dependence of the transition probabilities (to the different eigenstates in the final potential) on the *rate* of volcano rise (τ) is examined. This is done for two volcanos of different shapes (Fig. 1). First the inverted square-well volcano, and second, a volcano, smoothed out in space, of the form

$$V(x,t) = V(t)[1 + \cos(\pi x/a)]/2. \tag{5}$$

We shall refer to this volcano as the "rounded" volcano.

[2] W. Stein, Phys. Rev. **108**, 94 (1957). J. Milton and J. Fraser, Phys. Rev. **111**, 877 (1958).

[3] W. J. Swiatecki and S. Cohen, *The Deformation Energy of a Charged Drop* (Aarhus University, Det Fysiske Institut, Aarhus, Denmark, 1961), Vol. 4.

Both volcanos have the same smooth time dependence, $V(t)$, described by Eq. (1).

Of course, what is really of interest for fission is the many-particle problem, i.e., given a well filled with a Fermi sea of nucleons, what is the probability of them picking up a certain amount of energy from the erupting volcano? We shall show how this information can be deduced from the knowledge of the single-particle transition probabilities. The Pauli exclusion principle will be built into the many-particle problem. It will be shown that it is a consequence of the many-particle formalism that no more than one particle can occupy a given state. The single-particle results, aside from being necessary (and sufficient) for the solution of the many-particle problem, have little to do with fission. The results are intrinsically interesting, however, viewed simply as the solution to a problem in quantum mechanics.

The computations for this problem divide very naturally into two parts. First one calculates the time development of a given initial wave function until the volcano has stopped rising, getting the wave function for $t=2\tau$, $\psi(2\tau)$. Now to determine the probability that the particle is in any one of the allowed eigenstates one must determine these eigenstates, ϕ_m, and expand $\psi(2\tau)$ in terms of them.

$$\psi(2\tau)=\sum_{m=1}^{N} c_m\phi_m. \tag{6}$$

The ϕ_m are eigenstates in the final potential configuration, $V(2\tau)$. At each instant of time the eigenstates and the corresponding eigenvalues are completely different. Initially the eigenstates are those of a pure square well, simply sines and cosines. The second part of the numerical procedure is then the determination of the eigenstates at $t=2\tau$. The transition probabilities, $|c_m|^2$, follow very simply by taking scalar products once these two basic parts of the problem have been completed. The mean energy transfer is also immediately available. $\psi(2\tau)$ depends on the initial value of ψ, $\psi(0)$, which was one of the eigenstates of the pure square well before the volcano appeared. To indicate which state was originally filled, we shall henceforth index $\psi(2\tau)$ with a subscript n indicating the initial eigenstate. $\psi_n(0)$ is, of course, real. In this notation, we have for the probability of transition for a particle originally in the nth state in the pure square well to the mth state in the final potential configuration,

$$P_{nm}=|c_m|^2=|\langle\psi_n(2\tau)|\phi_m\rangle|^2. \tag{7}$$

As a check on all the parts of the program the special case of $V=0$ has been run, i.e., no volcano rising. The analytical solution to the time-dependent equation is then simply $\psi_n(0)e^{-iE_nt/\hbar}$, and consequently $P_{nm}=\delta_{nm}$. In the numerical work, however, the E_m corresponding to the final eigenstates, ϕ_m, average about 2% in error. Also, the numerically computed $\psi(2\tau)$ is within 2% of the theoretical $\psi(2\tau)$. These numerical errors, however, do not create a correspondingly large error in P_{nm}. One finds $P_{nm}=1.003\delta_{nm}+$a term of order $<10^{-13}$. Thus the numerical calculations tell us that the probability of a transition to any state other than state n is less than 10^{-13}; the probability of staying put in state n is 1.003. Comparing with the exact result of $P_{nm}=\delta_{nm}$ we see that the agreement is excellent despite the 2% error made in the eigenvalue-eigenvector subroutine. The model being treated is a very rough one, and the accomplishment of the aims of this investigation are not dependent upon great numerical accuracy. These errors in the numerical procedures are quite tolerable in view of the aims of this study.

All of the following single-particle results are for a particle that originally had the Fermi energy. In the present problem this is the 9th eigenstate in the original square well, the eigenenergies of which are given by

$$E_m^{(\text{old})}=\hbar^2k_m^2/2(\text{mass})_n, \tag{8}$$

where $(\text{mass})_n$ is the neutron mass and $k_m=(\pi/2a)(2m-1)$; $m=1, 2, \cdots$. The value of m which makes E_m closest to 35 Mev is $m=9$ for $a=2.0\times10^{-12}$ cm. Actually, $E_9=37$ Mev. This value of a is estimated from the liquid drop calculations of Swiatecki,[3] which shows the distorted nucleus undergoing fission as having 2 to 3 times the radius of a heavy nucleus (e.g., uranium).

A. Square Volcano: Dependence of Transition Probabilities and Mean Energy Transfer on Time of Rise

In Table I is shown the effect of volcano rise time, τ, on the probability of a single particle transition, P_{9m}, from state 9 (in the original square well) to state m (in the final potential after volcano has erupted). Also the sum of all the probabilities is shown which in the absence of numerical error would be exactly one. It will be seen that for all volcano rise times, τ, which were considered, the transition probabilities P_{nm} were less than 10^{-4} when both $n\leqq9$ and $m>16$. As m increases beyond 16 the P_{nm} fall rapidly to numbers of the order of magnitude of 10^{-14}. Also included for comparison with theory is the mean energy transfer defined by

$$\bar{E}=\sum_{m=1}^{\infty} P_{9m}(E_m^{(\text{new})}-E_9^{(\text{old})}). \tag{9}$$

From Table I it is clear that the adiabatic limit has been reached to 2% accuracy for $\tau=10^{-21}$ sec. The sudden limit has almost been reached for $\tau=10^{-23}$ sec, consistent with the estimate based on the criterion of validity for the sudden approximation. For the two limiting cases the mean energy transfer, $\bar{E}$, can easily be calculated analytically. In the adiabatic case we have only to determine the 9th eigenstate in the final potential. This has been done previously where it was found that the 9th energy eigenstate in the final

TABLE I. Dependence of transition probabilities and mean energy transfer on time of rise for square volcano. A blank space indicates a probability of less than 10^{-3}. All P_{9m} for $m \leq 4$ or $m \geq 15$ are less than 10^{-3}. The ratio of "volcano" width to over-all well width is $l/a = 0.3125$. All eigenstates listed here are even. Beneath the quantum number of a state is the energy of that state in Mev.

		P_{9m}										
τ (sec)	$\bar{E}$ (10^{-4} erg)	$m=5$ 24.4	6 34.5	7 44.0	8 45.8	9 52.2	10 60.2	11 69.7	12 78.7	13 90.7	14 100.1	$\sum_{m=1}^{48} P_{9m}$
10^{-23}	0.191	0.002	0.559	0.032	0.065	0.020	0.035	0.169	0.091	0.026	0.003	1.004
5×10^{-23}	0.189		0.490	0.038	0.095	0.024	0.060	0.200	0.071	0.012	0.002	0.992
7.5×10^{-23}	0.197		0.428	0.043	0.137	0.022	0.098	0.210	0.050	0.006	0.001	0.994
9.0×10^{-23}	0.207		0.392	0.045	0.168	0.016	0.135	0.207	0.034	0.003		1.003
10^{-22}	0.210		0.366	0.045	0.191	0.012	0.155	0.203	0.025	0.002		1.002
1.75×10^{-22}	0.212	0.002	0.203	0.033	0.323	0.067	0.301	0.071	0.001			1.002
2.5×10^{-22}	0.215		0.087	0.009	0.273	0.447	0.153	0.019				0.990
3.75×10^{-22}	0.222		0.016	0.005	0.104	0.856	0.008	0.001				0.991
5×10^{-22}	0.235		0.002	0.004	0.211	0.792	0.004					1.014
7.5×10^{-22}	0.224				0.069	0.907	0.005					0.982
10^{-21}	0.261				0.017	1.002						1.020

potential had the value 0.849×10^{-4} erg. Therefore, $\bar{E}=(0.849-0.593)\times10^{-4}$ erg $=0.256\times10^{-4}$ erg which is seen to be in close agreement with Table I, $\tau=10^{-21}$, where $\bar{E}$ is given as 0.261×10^{-4} erg. For the sudden limit a quick estimate of $\bar{E}$ may be made by multiplying l/a times the height that the volcano rises. Here we have taken l/a as a measure of the probability for the particle to be over the volcano. A more accurate account takes, instead of l/a, the quantity $\int_{-l}^{l}|\psi_{\text{initial}}|^2dx$, where ψ_{initial} is the original eigenfunction of the particle in the pure square well before the volcano rises. Evaluating the integral one finds that it gives 0.296 whereas $l/a=0.3125$. Since the volcano rises instantaneously it carries everything above it up with it to its maximum height, 0.6568×10^{-4} erg. For $\bar{E}$ then we find 0.194×10^{-4} erg which compares favorably with Table I, $\tau=10^{-23}$, where $\bar{E}$ is listed as 0.191×10^{-4} erg.

The effect on $\bar{E}$ of changing the ratio of volcano width to well width for a fixed volcano rise time of 10^{-22} sec has also been studied. We find that $\bar{E}$ depends almost linearly on l/a. This is not surprising since this would certainly be the case for the sudden limit, and there is no reason to expect the behavior of $\bar{E}$ as a function of l/a to vary with τ.

B. Rounded Volcano: Dependence of Transition Probabilities and Mean Energy Transfer on Rise Time

In Table II is shown the effect of volcano rise time on the probability of a single particle transition from state 9 (in the original square well) to state m (in the final potential) after the rounded volcano has risen. This table is to be compared to Table I which contains the same information for the case of the square volcano. It will be noted that for a given $\tau \geqq 10^{-22}$ sec, the probability, P_{99}, of no transition is always greater for the rounded volcano than for the square volcano. Furthermore, for $\tau > 10^{-22}$ sec, the sum of the transition probabilities to states of energy greater than the 9th (i.e., $\sum_{m=10}^{\infty} P_{9m}$) is always less for the rounded volcano than for the square volcano. Therefore we draw the conclusion that for a potential $V(x,t)=V_1(x)V_2(t)$ which varies slowly in space, *or* slowly in time, the probability for a particle to change its quantum number (as distinct from its energy) is found to decrease as the rate of space or time variation decreases. That is, one can substitute adiabaticity in space for adiabaticity in time; under either of these conditions the particle becomes less apt to change its quantum number.

V. THE MANY-PARTICLE PROBLEM

A. General Formalism Connecting Single- and Many-Particle Problems

It has been stated that if the single-particle transition probabilities, P_{nm}, are known, then the expectation values of the occupation numbers, $\langle N_m \rangle$, of the final states for the many-particle problem follow from this knowledge alone. Furthermore, if the single particles are fermions (as they are in this case, being nucleons) the Pauli exclusion principle must be incorporated into the formalism that extracts the expectation values of the occupation numbers from the single-particle transition probabilities. In other words, one must be able to prove from this formalism that $\langle N_m \rangle \leqq 1$ for all m. These results will be established first; then we shall discuss the numerical results of the many-particle calculations.

We prove first that the time development of the annihilation and creation *operators*, the q-numbers A_k and $A_k^\dagger$, is governed by exactly the same equation as governs the time development of the c-number *probability amplitudes*, a_k and a_k^* of the quantum theory for a single particle. Specifically this means that the A_k obey the time-dependent Schrödinger equation that we have already solved numerically that governs the time development of the probability amplitudes a_k, whereas the $A_k^\dagger$ obey the complex conjugate equation.

In second quantization the wave function becomes an operator which can be expanded in terms of the annihilation operators. Likewise, the complex conjugate

TABLE II. Dependence of transition probabilities and mean energy transfer on time of rise for "rounded volcano." All eigenstates considered here are even. Beneath the quantum number of a state is the energy of that state in Mev.

τ (sec)	$\bar{E}$ (10^{-4} erg)	P_{9m} $m=3$ 22.9	4 30.1	5 36.2	6 40.6	7 44.6	8 50.6	9 58.0	10 66.5	11 75.7	12 86.0	$\sum_{m=1}^{48} P_{9m}$
10^{-23}	0.316	0.003	0.024	0.074	0.085	0.171	0.163	0.003	0.297	0.162	0.020	1.004
5×10^{-23}	0.308	0.003	0.020	0.065	0.080	0.170	0.188		0.313	0.142	0.016	0.996
10^{-22}	0.311	0.001	0.011	0.045	0.064	0.161	0.254	0.020	0.337	0.101	0.007	1.002
1.75×10^{-22}	0.305		0.002	0.012	0.026	0.096	0.317	0.247	0.270	0.025		0.997
2.5×10^{-22}	0.332				0.004	0.022	0.184	0.707	0.092	0.001		1.013
5×10^{-22}	0.317						0.014	0.961	0.006			0.981
10^{-21}	0.355							1.021				1.021

of the wave function becomes an operator which can be expanded in terms of the creation operators. We shall only deal here with the annihilation operators A_k, showing that the operators A_k obey the same equation as do the c-number amplitudes a_k. The proof that the $A_k^\dagger$ obey the complex conjugate equation is structurally identical. We have, therefore, the *operator equation*

$$\psi(x,t)=\sum_k A_k(t)u_k(x,t)\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right)$$

where $\psi(x,t)$ and $A_k(t)$ are *operators* and $H(t)u_k(x,t)=E_k(t)u_k(x,t)$; the u_k are thus the instantaneous eigenfunctions of $H(t)$: a complete orthonormal set of functions in terms of which $\psi(x,t)$ can be expanded. The time dependence of the expectation value of the operator $\psi(x,t)$ is given by the expectation value of the commutator of ψ with the Hamiltonian. Using this along with the well-known result $H=\sum_i A_i^\dagger A_i E_i$ we have

$$\begin{aligned} i\hbar\dot{\psi}&=i\hbar\sum_k[\dot{A}_k u_k+A_k\dot{u}_k-(i/\hbar)A_k u_k E_k] \\ &\quad\times\left[\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right)\right]=[\psi,H] \\ &=\left[\sum_k A_k u_k \exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right)\right](\sum_i A_i^\dagger A_i E_i) \\ &\quad-(\sum_i A_i^\dagger A_i E_i)\left[\sum_k A_k u_k\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right)\right] \\ &=\sum_k[A_k(\sum_i A_i^\dagger A_i E_i)-(\sum_i A_i^\dagger A_i E_i)A_k]u_k \\ &\quad\times\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right). \end{aligned} \tag{10}$$

This last expression may be broken up into two kinds of terms: $i=k$ and $i\neq k$. In Fermi statistics the A_i and A_k (or $A_k^\dagger$) anticommute for $i\neq k$ and in Bose statistics the A_i and A_k (or $A_k^\dagger$) commute for $i\neq k$. Therefore it follows that in both cases the final term in (10) *for* $i\neq k$ becomes

$$\sum_k\left[\sum_{i\neq k}(A_i^\dagger A_i A_k E_i-A_i^\dagger A_i A_k E_i)\right]u_k \times\exp\left(-\frac{i}{\hbar}\int_0^t E(t')dt'\right)\equiv 0.$$

Therefore the final term in (10) reduces to

$$\begin{aligned}&\sum_k(A_kA_k^\dagger A_k-A_k^\dagger A_kA_k)E_ku_k\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right) \\ &=\sum_k[A_k,A_k^\dagger]A_kE_ku_k\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right).\end{aligned} \tag{11}$$

In Bose statistics this commutator is 1. In Fermi statistics $[A_k,A_k^\dagger]A_k=A_k$ so we have the result that for either statistics (11) becomes

$$\sum_k A_kE_ku_k\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right).$$

Using this simplification in (10) we find

$$\sum_k[\dot{A}_ku_k+A_k\dot{u}_k]\exp\left(-\frac{i}{\hbar}\int_0^t E_k(t')dt'\right)=0.$$

That this result, which is independent of statistics, is equivalent to Schrödinger's equation for the probability amplitudes, a_k, follows immediately by taking the scalar product of this equation with u_k.[4]

However, in the numerical work already outlined we have found the coefficients F_{mn} ("scattering" or "transition" amplitudes) in the equation

$$a_m(2\tau)=\sum_n F_{mn}a_n(0). \tag{12}$$

Since at $t=0$ the particle is in a single eigenstate it follows that $a_n(0)=\delta_{nj}$. Therefore, $F_{mj}=a_m(2\tau)$ if originally the particle was in the jth eigenstate.

Now since the annihilation (and creation) operator satisfies the same equation as does a_k (and a_k^*), and since (12) represents the solution for a_k (and a_k^*), it

[4] Compare with L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), 2nd Ed., Eq. (31.4).

TABLE III. Single-particle probability, P_{nm}, for transition from state n to state m. Square volcano. $l/a=1/3$. $\tau=10^{-22}$ sec. *Even eigenstates*, numbered serially. The expectation values of occupation numbers for many particle problem are in the last row: $\langle N_m\rangle=\sum_n P_{nm}$. Blank indicates $P_{nm}<10^{-4}$. As accuracy check $\sum_m P_{nm}$ is included in last column. Entry in last column, last row is $\sum_{n,m} P_{nm}$ which in absence of error would be exactly 9. See Fig. 2 for graphical representation of these numbers. Beneath the quantum number of the final states (m) is the energy (E_m) of that state in Mev. Alongside the quantum numbers of the initial states (n) are the energies (E_n) of those states in Mev.

n	E_n \ $m=$ / $E_m=$	1 1.1	2 4.2	3 9.5	4 16.7	5 25.8	6 36.4	7 42.1	8 47.4	9 52.4	10 61.8	11 69.7	12 80.4	13 91.3	14 100.2	$\sum_m P_{nm}$
1	0.1	0.3106	0.0832	0.0505	0.0390	0.0321	0.0437	0.4368	0.0011	0.0044	0.0002					1.0016
2	1.1	0.6505	0.0040	0.0030	0.0037	0.0054	0.0152	0.3157	0.0040							1.0015
3	3.2	0.0328	0.6728	0.0534	0.0232	0.0113	0.0038	0.1498	0.0388	0.0153	0.0002					1.0014
4	6.2	0.0060	0.2378	0.3446	0.0980	0.0611	0.0395	0.0324	0.1150	0.0639	0.0009					0.9992
5	10.3		0.0010	0.5259	0.0599	0.0497	0.0617	0.0003	0.1703	0.1254	0.0041	0.0006				0.9989
6	15.3	0.0001	0.0014	0.0185	0.6457	0.0065	0.0115	0.0129	0.1085	0.1653	0.0229	0.0069	0.0003			1.0005
7	21.6		0.0002	0.0032	0.1292	0.5100	0.0710	0.0131	0.0039	0.1517	0.0877	0.0299	0.0012			1.0011
8	28.5			0.0002	0.0004	0.3166	0.2593	0.0004	0.0850	0.0565	0.1937	0.0836	0.0040	0.0006		1.0003
9	36.1				0.0005	0.0077	0.4487	0.0318	0.1195	0.0018	0.1693	0.1832	0.0211	0.0051	0.0004	0.9991
$\langle N_m\rangle$		1.0000	1.0004	0.9993	0.9996	1.0004	0.9544	0.9932	0.6461	0.5943	0.4790	0.3042	0.0266	0.0057	0.0004	9.0036

follows that

$$A_m(2\tau)=\sum_n F_{mn}A_n(0),$$

and also

$$A_m^\dagger(2\tau)=\sum_n F_{mn}^*A_n^\dagger(0),$$

where all the F_{mn} are known from single-particle calculations. The occupation number operator follows as the product of the annihilation and creation operators.

$$N_m(2\tau)=A_m^\dagger(2\tau)A_m(2\tau)=\sum_{p,q} F_{mp}^*A_p^\dagger(0)F_{mq}A_q(0)$$
$$=\sum_p F_{mp}^*F_{mp}A_p^\dagger(0)A_p(0)$$
$$+\sum_p\sum_{q\neq p} F_{mp}^*F_{mq}A_p^\dagger(0)A_q(0).$$

It can be shown that the expectation value of the second term after the final equality is zero. Therefore, we have for the expectation value of the occupation number operator after the volcano has risen ($t=2\tau$):

$$\langle N_m(2\tau)\rangle=\sum_p|F_{mp}|^2\langle A_p^\dagger(0)A_p(0)\rangle$$
$$=\sum_p P_{pm}\langle N_p(0)\rangle, \quad (13)$$

where $P_{pm}=|F_{mp}|^2$ is just the transition probability from state p to state m. If the first k states are originally filled with one particle to a state, we have $\langle N_p(0)\rangle=1$ for $1\leqq p\leqq k$ and $\langle N_p(0)\rangle=0$ for $p>k$. Therefore, (13) reduces to

$$\langle N_m(2\tau)\rangle=\sum_{p=1}^{k} P_{pm} \quad (14)$$

for the case of only the lowest k states filled.

We now show that (14) satisfies the Pauli principle in that $\langle N_m(2\tau)\rangle\leqq 1$ for all m. Also, it must be true that the sum of the expectation values of all the occupation numbers must equal the number of particles originally present in the well. This demand is met:

$$\sum_{m=1}^{\infty}\langle N_m(2\tau)\rangle=\sum_{m=1}^{\infty}\sum_{p=1}^{k}P_{pm}=\sum_{p=1}^{k}(\sum_{m=1}^{\infty}P_{pm})=k.$$

To see if (14) satisfies the Pauli principle we ask: What is the maximum value of $\langle N_m\rangle$ for a given value of m? It is clear that this maximum occurs for $k=\infty$, i.e., all the states in the original well filled.

$$\max\langle N_m\rangle=\sum_{p=1}^{\infty}P_{pm}. \quad (15)$$

Now $P_{pm}=|\langle\psi_p(2\tau)|\phi_m\rangle|^2$ where $\psi_p(2\tau)$ is the wave function of the system which at $t=0$ was the pth eigenstate of the pure square well; ϕ_m is the mth eigenstate in the final potential [Eq. (5)]. The $\psi_p(2\tau)$ form a complete set because the $\psi_p(0)$ form a complete set and the Schrödinger equation preserves orthonormality in time. Therefore, ϕ_m can be expanded in terms of this complete set:

$$\phi_m=\sum_p \alpha_{mp}\psi_p(2\tau). \quad (16)$$

Since

$$\langle\phi_m|\phi_n\rangle=\delta_{mn} \quad\text{and}\quad \langle\psi_p(2\tau)|\psi_q(2\tau)\rangle=\delta_{pq},$$

it follows that

$$\sum_p|\alpha_{mp}|^2=1.$$

Substituting (16) into (15), we have

$$\max\langle N_m\rangle=\sum_{p=1}^{\infty}|\langle\psi_p(2\tau)|\sum_q \alpha_{mq}\psi_q(2\tau)\rangle|^2$$
$$=\sum_{p=1}^{\infty}|\alpha_{mp}|^2=1. \quad (17)$$

If the upper limit in the summation is any finite number—meaning a finite number of particles in the original well [which is our case ($k=9$)]—then the summation of positive-definite terms of (17) will give less than one. Therefore, it follows that we have $\langle N_m\rangle\leqq 1$ for all m. The Pauli principle has been incorporated into the formalism in going from step (13) to step (14) where we explicitly use the principle in assuming that the initial occupation numbers are all

TABLE IV. P_{nm} and $\langle N_m \rangle$ for rounded volcano with $\tau = 10^{-22}$ sec, in the same format as Table III. Again we have *even* eigenstates, numbered serially. See Fig. 3 for graph. Beneath the quantum number (m) of a state is the energy (E_m) of that state in Mev. The energies of the initial states (n) are given in Table III.

$n \backslash m$, E_m	1 6.7	2 15.1	3 23.0	4 30.1	5 36.2	6 40.6	7 44.6	8 50.6	9 58.0	10 66.5	11 75.7	12 86.0	13 96.8	14 100.8	$\Sigma_m P_{nm}$
1	0.0689	0.1043	0.1374	0.1749	0.2400	0.2482	0.0264	0.0004							1.0005
2	0.3702	0.2204	0.0644		0.0748	0.2270	0.0450	0.0012							1.0030
3	0.3676	0.0043	0.1874	0.1815	0.0085	0.1542	0.0959	0.0059	0.0002						1.0055
4	0.1560	0.2839	0.0632	0.0908	0.1634	0.0346	0.1816	0.0275	0.0013						1.0023
5	0.0343	0.2843	0.1356	0.0980	0.1043	0.0180	0.2119	0.1030	0.0096	0.0003					0.9993
6	0.0043	0.0902	0.2865	0.1019	0.0458	0.1128	0.0585	0.2452	0.0529	0.0034					1.0015
7	0.0003	0.0130	0.1106	0.2529	0.1296	0.0034	0.0605	0.2084	0.1899	0.0278	0.0013				0.9977
8		0.0010	0.0162	0.0876	0.1838	0.1322	0.1217	0.0023	0.3078	0.1394	0.0141	0.0005			1.0066
9			0.0011	0.0113	0.0452	0.0639	0.1609	0.2543	0.0195	0.3374	0.1010	0.0073	0.0002		1.0021
$\langle N_m \rangle$	1.0016	1.0014	1.0024	0.9989	0.9954	0.9943	0.9624	0.8482	0.5812	0.5083	0.1164	0.0078	0.0002		9.0185

either 1 or 0. If we were dealing with Bose statistics this restriction would not be made and the proof above that $N_m(2\tau) \leqq 1$ for all m would be impossible.

Equation (14) is, therefore, established and provides the rule whereby single-particle transition probabilities can be combined to give expectation values of the occupation numbers of the allowed states for the many particle problem after the volcano has risen. To determine the occupation number of the jth state from the single-particle transition probabilities we simply sum up the transition probabilities from all originally occupied states to the jth state. This sum will always be less than or equal to one—it is the expectation value of the occupation number of the jth state. We shall now apply this rule to specific numerical cases.

B. Numerical Results for the Square and Rounded Volcanos

In the single-particle problem there was only one particle in the well. It had the Fermi energy which put it in the 9th even state. Now we fill up all the even states beneath the Fermi level.

For both the square and rounded volcanos we have studied the following: (1) the expectation values of the occupation numbers of the final states; (2) the fraction of particles ejected $= \sum^{\infty}_{m=10} \langle N_m \rangle / 9$. The fraction of particles ejected is concerned only with states of energy greater than E_9. If E_9 was not greater than the nuclear well depth (41 Mev) the fraction of particles ejected would be measured from the nuclear well depth. These quantities have been determined for a single rise time for the square volcano and for two rise times for the rounded volcano. In Table III this information is presented for the square volcano with $\tau = 10^{-22}$ sec. In Tables IV and V the same information is presented for the rounded volcano with $\tau = 10^{-22}$ sec and $\tau = 2.5 \times 10^{-22}$ sec, respectively. It is clear that the rounded volcano with $\tau = 10^{-22}$ sec is more nearly adiabatic in its effects than the square volcano of the same τ. Also the rounded volcano with $\tau = 2.5 \times 10^{-22}$ sec is far more gentle than the rounded volcano with $\tau = 10^{-22}$ sec. It causes about 1% of the nucleons originally in the well to be ejected (Table VI). For a nucleus of 250 nucleons of which about 150 are neutrons, this is about 1.5 neutrons which is of the same order of magnitude as the total number of neutrons ejected on the average per act of fission due to *all* mechanisms. Obviously we can get any number of neutrons out we like by varying the shape of the rising volcano and the time of rise. Earlier an estimate of the time of scission was made. From fragment velocity data a rise time of $2\tau = 4.5 \times 10^{-22}$ sec was found reasonable, although scission could occur more quickly. The $\tau = 2.25 \times 10^{-22}$ sec resulting from this is reasonably near our value of 2.5×10^{-22} sec for which the ejection probability for the rounded volcano was about 1%.

For a given n we list only those P_{nm} for which $P_{nm} > 10^{-4}$. This is accomplished in all three cases by listing P_{nm} for $1 \leqq m \leqq 14$. In Tables III, IV, and V the final row shows the expectation values of the occupation numbers of the final states $\langle N_m \rangle = \sum_n P_{nm}$. We see that this is just the sum of the P_{nm} in the mth column. The sum of each row, $\sum_m P_{nm}$, would equal exactly 1 in the absence of numerical error. The sum of the numbers in the last row, and also of the numbers in

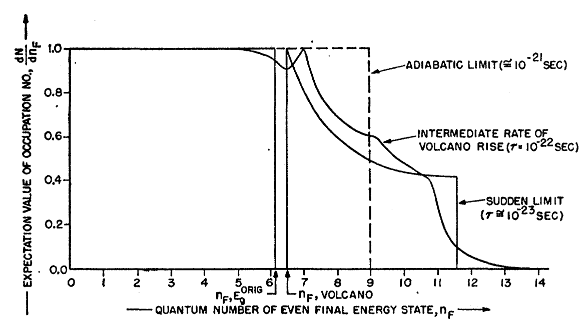

FIG. 2. Many-particle problem: square volcano; even eigenstates only; $l/a = 1/3$. The figure shows dN/dn_F vs n_F for two limiting cases: adiabatic and sudden; plus the computed distribution for intermediate time of rise of the volcano. Note that this latter curve lies "between" two limiting cases. n_F, E_9^{orig} is the quantum number in the final potential corresponding to the 9th energy level in the original pure square well. n_F, volcano is the final quantum number corresponding to the final height of the volcano.

TABLE V. P_{nm} and $\langle N_m \rangle$ for rounded volcano with $\tau = 2.5 \times 10^{-22}$ sec, in the same format as Table III (*even* eigenstates). See Fig. 3 for graph. The energies of the initial states (n) are given in Table III. The energies of the final states (m) are given in Table IV.

$n \diagdown m$	1	2	3	4	5	6	7	8	9	10	11	12	$\Sigma_m P_{nm}$
1	0.1090	0.1443	0.1605	0.1731	0.2084	0.1856	0.0175	0.0002					0.9986
2	0.4851	0.1593	0.0101	0.0201	0.1165	0.1864	0.0242	0.0004					1.0021
3	0.3261	0.1167	0.2603	0.0735	0.0128	0.1731	0.0430	0.0012					1.0067
4	0.0740	0.4137	0.0136	0.2411	0.0434	0.1176	0.0910	0.0046					0.9990
5	0.0067	0.1509	0.3873	0.0128	0.2140	0.0276	0.1774	0.0199	0.0005				0.9971
6	0.0003	0.0148	0.1571	0.3642	0.0812	0.0325	0.2684	0.0815	0.0033				1.0033
7		0.0005	0.0132	0.1096	0.2822	0.1933	0.0927	0.2777	0.0252	0.0004			0.9948
8			0.0003	0.0057	0.0392	0.0829	0.2612	0.4178	0.1831	0.0062			0.9964
9					0.0010	0.0037	0.0225	0.1845	0.7072	0.0925	0.0012		1.0126
$N\langle_m\rangle$	1.0012	0.0002	1.0024	1.0001	0.9987	1.0027	0.9979	0.9878	0.9193	0.0991	0.0012		9.0106

the last column are equal. In each case this sum would be exactly the number of particles in the well originally (9) in the absence of error. It is seen that the deviation from 9 is, in each case, less than 0.2%. In Table VI is shown the dependence of the fraction of particles ejected on type of volcano and time of rise of volcano.

In Fig. 2 we consider different *final* states for the particle after the square volcano has risen, and for each such state plot the expectation value of the occupation number for that state. If we are to connect these isolated points with a continuous line, we must shift our point of view to that of a continuum of states. Then instead of expectation values of occupation numbers we consider d(number of particles)$/d$(quantum number of final states) which we shall write dN/dn_f. The dependence of dN/dn_f on n_f can be computed analytically for the square volcano in the two limiting cases of adiabatic and of sudden rates of change of potential. These cases are included in Fig. 2 for comparison with the intermediate rate of potential change ($\tau = 10^{-22}$ sec). It is seen that the probability distribution belonging to the intermediate rate of change of potential lies intermediate between the distributions for the adiabatic and sudden limits. This situation suggests that we can view the distribution for intermediate time of rise of the volume as a "mixture" of these two limiting cases.

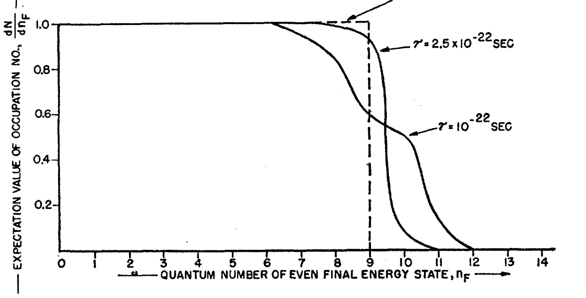

FIG. 3. Many-particle problem: rounded volcano; even eigenstates only. The figure shows dN/dn_F vs n_F for two intermediate times of rise of the rounded volcano. Note that the distribution belonging to the greater τ approaches the limiting adiabatic case.

The expectation values of the occupation numbers are plotted vs the quantum numbers of the final states in Fig. 3 for the two rounded volcanos. The quantum numbers in Fig. 3 refer to different eigenenergies than do the quantum numbers of Fig. 2 for the square volcano.

Although the occupation numbers themselves do total up to 9 (the original number of particles) to within 0.2%, the distributions of Figs. 2 and 3—based on drawing a *continuous* curve through the finite number of points computed on a machine—do not preserve number of particles so well. If conservation obtained, the deficit of area beneath the limiting adiabatic would be exactly compensated for by the surplus in area outside it. But a glance shows us that this is not the case. There is more area outside the adiabatic curve than deficit inside it, although the *sum of the ordinates for integral* quantum numbers of the deficits inside equals the surpluses outside to within 0.2%. The failure of conservation of the areas beneath the distribution curves has resulted because the continuous curve has been based on a finite set of points; namely, only the integral quantum numbers of the even states.

VI. APPLICATION TO FISSION

We have mentioned the experiments of Bowman and Thompson on the angular distribution of prompt neutrons from the spontaneous fission of Cf^{252}. The present study indicates that the existence and the quantity of the *scission* neutrons may be explained in terms of the nonadiabatic change of nuclear potential which transfers enough energy to some neutrons in the nucleus to eject them.

In particular we have found one case, the rounded

TABLE VI. Fraction of particles ejected for the three cases, when attention is limited to even states.

Volcano	τ (sec)	Fraction of particles ejected
Square ($l/a = 1/3$)	10^{-22}	9.1%
Rounded	10^{-22}	7.0%
Rounded	2.5×10^{-22}	1.1%

volcano rising in a time corresponding to $\tau = 2.5\times10^{-25}$ sec, in which about 1% of the neutrons originally present in the square well nucleus are ejected (Table VI). For Cf^{252} with 154 neutrons, this gives about 1.5 neutrons ejected. The number of scission neutrons found for Cf^{252} would be roughly 10–20% of the average number of neutrons per fission which is about 4 for Cf^{252}; that is, the number of scission neutrons per fission would be about 0.4 to 0.8. Our specific case does not exactly fit these data, but by a slight increase in the adiabaticity of the rate of rise of the volcano, the number of neutrons ejected by the mechanism of nonadiabatic potential change can be reduced to within the experimentally observed range for still quite reasonable rates of rise of the volcano (i.e., times of scission).

A rough estimate of the energy of the neutrons emitted is also available from the figures computed here. We know that if the volcano rises adiabatically the particles in the nucleus will gain in energy simply because of the reduced phase space available to them. To determine the energy of an emitted particle we must subtract off this artificial energy gain. Thus in the case of Table V ($\tau = 2.5\times10^{-22}$ sec) for which 1% of the particles in the well are ejected, we see that almost all of this 1% is raised in energy to state $m=10$ which has an energy of 66.5 Mev (Table IV). But from this energy must be subtracted the energy of the state to which the adiabatic volcano raises the particles, that is, state $m=9$, with energy 58.0 Mev. Thus about 8.5 Mev is given to 1% of the particles which for Cf^{252} is about 1.5 neutrons. Each scission neutron would have an energy of about 5–6 Mev. For a slightly less gentle volcano one could find four neutrons ejected of approximately 2 Mev each.

The mechanism discussed here in relation to neutron production could perhaps also be applied to the problem of heavier particles (tritons, alphas) in fission.

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Mike Results—Implications for Spontaneous Fission*

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The November 1, 1952, thermonuclear explosion "Mike" produced isotopes through mass number 255. Since neutron irradiation time is short compared with possible beta-decay lifetimes, we conclude that, through $A=255$, nuclei far from the line of beta stability on the neutron-rich side are at least as likely to decay by negatron emission as by spontaneous fission.

I. INTRODUCTION

THE heavy isotope yields of the Mike thermonuclear device[1] afford a unique opportunity for the examination of nuclear properties far from the stability curve. Both the occurrence of high-mass-number isotopes and the shape of the yield curve are of significance. In this paper, we examine implications of these factors for spontaneous fission.

II. SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES

Early interpretations of spontaneous fission lifetimes were based on the liquid-drop model.[2] Marked deviations from such a simple model were evident and many attempts were made to improve the model as well as to formulate new approaches. One of the most successful was that of Swiatecki who was able to show a regular dependence of the fission half-life on ground state masses.[3] His work has recently been slightly modified and extended by the author.[4] This modification predicts spontaneous fission half-lives which, for a given z, decrease very rapidly with increasing A (Fig. 1).

Foreman and Seaborg[5] plotted the logarithm (base 10) of the spontaneous fission half-life vs mass number. They observed that all the half-life curves approached the same linear dependence on mass number at the line characterizing 152 neutrons (Fig. 2).

Recently, Johansson[6] has analyzed spontaneous fis-

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