

General Approach to the Electromagnetic Origin of Cosmic-Ray Energy*

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If, in an assigned piece of matter, the energy of motion, chemical energy, or even nuclear energy is shared among all the nucleons, there cannot amount to more than a small fraction of 10^9 ev for each nucleon. Hence the acquirement of cosmic-ray energies must invoke processes by which a small amount of matter steals from a much larger amount of matter energy which it stores up for subsequent conversion into cosmic-ray energy. By setting up electric currents with their accompanying magnetic fields within itself, a small quantity of matter can hoard an amount of energy which, estimated per nucleon, can readily yield cosmic-ray energies. The general theorem relating rate of diminution of electromagnetic energy to rate of doing work on the particles and to the Poynting flux tells us that if the energy of such an electromagnetic field can be caused to disappear, the whole of it must go into cosmic-ray energies of the particles, apart from that involved in the Poynting flux to infinity, which part, in certain cases, can be small. The destruction of the electromagnetic energy can be shown to occur in general by the spreading out of the system concerned as a result of its normal dynamical activities. The matter is illustrated by considering the case of a ring of gas shot out from a region of primary magnetic field. The ring carries the magnetic flux with it, and the corresponding electromagnetic energy subsequently disappears by the expansion of the ring to infinite size with an accompanying conversion of the said energy into cosmic-ray energies of the nucleons.

I. INTRODUCTION

THE complete conversion of all the mass energy of a number of protons or neutrons into cosmic-ray energy cannot provide more than about 10^9 ev per particle if the energy is shared equally among the particles; and mass for mass, such a process is a more potent source of energy than any so-called atomic explosion. If a quantity of matter moves with a velocity practically equal to that of light, but not so near to that limit as to cause the mass to become relativistically increased, and if the whole of this motional energy could become shared equally among the atomic nucleons, there could not result more than about 10^9 ev for each nucleon. In order, therefore, to account for cosmic ray energies, we must envisage a process by which a small amount of matter can steal, from a much larger amount of matter, energy which it stores up for the subsequent purpose of converting it into cosmic-ray energies for its nucleons.

Now how can matter steal energy from other matter and store it up? The answer is to be found in processes by which the energy of the larger amount of matter becomes responsible for the production of a magnetic field produced by electric currents associated with the smaller amount of matter. The coherence of the magnetic fields produced by the individual particles in the electric currents enables the currents to store up in magnetic form an amount of energy which, if shared among the particles, can, under suitable conditions, provide an amount of energy per particle which is enormous compared with the energy which that particle could hope to attain by what we may term its unilateral efforts born of the possibilities inherent in the normal processes by which nucleons acquire high energy. It then remains only for the system to dissolve its com-

munal activities and share its stolen goods in the form of energy among its particles in such manner that each possesses its share in the form of particle kinetic energy.

In what follows, we shall use the word sunspot. It is to be distinctly understood, however, that what we have to say is not necessarily bound up with actual sunspots—or star spots. We simply use the word sunspot as a convenient means of referring to a place where a magnetic field is to be found.

It is not the writer's present purpose to enter into the problem involved in the process by which the magnetic energy of the spot has been stolen from the kinetic or compressional energy of the matter around it. While he has some ideas on this matter, it is not his purpose to expound them now. He proposes to rely simply upon the experimental fact that such magnetic fields exist, but he does wish to discuss the details of a second theft, in which a new thief, in the form of a small amount of matter in the spot, runs off, carrying with it, in the form of a magnetic field, an amount of energy which is, as it were, out of all proportion to its fair share, and indeed, of such amount that when shared with its nucleons it can provide for them cosmic-ray energies.

II. DEVELOPMENT OF THE MECHANISM

We are all familiar with the oft-quoted statement to the effect that a mass of gas, shot out from the magnetic field of a spot, carries with it a magnetic field stolen, as it were, from the spot. The writer wishes to discuss the circumstances of this matter in some detail and to that end will, for simplicity, consider a conducting ring of gas, Fig. 1, which is at present at O, Fig. 2, and is in motion upwards with its plane perpendicular to the lines of force of the sunspot.

In the first place, we shall discuss matters as though this ring can be treated as an ordinary ring of high ohmic conductivity—a copper ring, for example—and shall apply the laws of electrodynamics to it in elemen-

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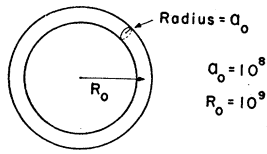


FIG. 1. Numerical data for gas ring.

tary form. The writer is well aware of the vulnerability of this procedure. The kind of ring we are considering is a ring of particle density of the order of 10^4 particles per cc or less. The mean free path of such particles will be of the order 10^{11} cm. Collisions will not occur, and the whole notion of Ohmic conductivity as applied to our particular problem evaporates. A rigorous treatment demands that we attack the problem as one of free-particle motion under the proper electrodynamic laws applicable in such cases. However, the writer will later show that the end results following from the more rigorous treatment are essentially the same as those following from the more elementary treatment; and for this reason, he will, in accordance with common practice, follow the elementary procedure, at first in any case, on account of its greater intuitive appeal.

If the ring is in motion in the upward direction, as seen in Fig. 2, an induced current will be set up of such amount that the change of flux through the ring resulting from the external field is just compensated by the flux set up by the induced current, so that the ring is able to defend itself against change of flux. Presently we shall discuss some limitations of this power of the ring to defend itself, but for the moment we shall regard it as complete, so that by the time the ring has reached a region well outside of the primary magnetic field, it will have, in virtue of its own induced current, the same total flux—say N_0 —which it enjoyed when at O, where, however, the flux was due entirely to the external magnetic field. The induced current I in electrostatic units will be given at this stage by

$$LI/c = N_0, \quad (1)$$

where L is the self-induction of the ring.

At this stage it is of interest to consider the energy in the ring, and to calculate how much an individual nucleon would possess if all the energy could be divided equally among the nucleons and if they could be set free from one another.

The energy W in the ring is given by

$$W = (1/2c^2)LI^2,$$

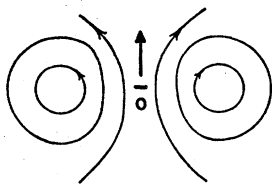


FIG. 2. Gas ring in external magnetic field.

so that, from (1),

$$W = (LI)^2/2c^2L = N_0^2/2L.$$

If the field at O due to the spot is H_0 , and R_0 is the radius of the ring when at O, then

$$N_0 = \pi R_0^2 H_0,$$

and

$$W = \pi^2 R_0^4 H_0^2 / 2L. \quad (2)$$

The energy comes, of course, from the agency responsible for shooting the ring out from O in opposition to the force of attraction of the primary magnetic field on the induced currents set up in the ring.¹

Now, of course, the ring can defend itself against change of total flux in the above manner only to the extent that it can evolve an induced current to that end, and the maximum induced current density i is given by

$$i = nec, \quad (3)$$

where n is the number of ions per cc. However, provided that, with n assigned, the induced current density demanded does not exceed this amount, the energy in the ring is given by (2), with L having the value which it has attained when the ring has reached the point at which we are now considering it. If the subsequent expansion of the ring to infinite size destroys the magnetic energy and transfers it all into energy of the nucleons, the energy per nucleon will be w , where

$$w = W / [2\pi R(\pi a^2 n)]$$

or, using (2),

$$w = R_0^2 H_0^2 / (4LRa^2 n). \quad (4)$$

However, we must not insert a value of n which is too small to yield the value of I necessary to yield H_0 . This value of I is given by (1), with $N_0 = \pi R_0^2 H_0$, so that, expressing I in terms of the saturation current density nec , we have, from (1),

$$\begin{aligned} L\pi a^2 nec/c &= 1R_0^2 H_0, \\ La^2 n &= R_0^2 H_0/e. \end{aligned} \quad (5)$$

Inserting the value of n given by (5) in (4),

$$\begin{aligned} w &= (R_0/R)(R_0/4)H_0e \\ &= 300(R_0/R)(R_0/4)H_0 \text{ ev.} \end{aligned}$$

Putting $R_0 = 10^9$ cm and $H_0 = 1000$ gauss,

$$w = 0.75 \times 10^{14} (R_0/R) \text{ ev.}$$

The ring expands by repulsion of its current elements. It cannot expand with a velocity greater than c , so that if the distance from O to a point where the primary field is negligible is $10R_0$, and if the average velocity of the ring is $0.1c$, the time to travel $10R_0$ is $100R_0/c$, and $R < 100R_0$, so that

$$w > 0.75 \times 10^{12} \text{ ev.}$$

¹ See Appendix to this paper for comments on this matter.

Of course, if n is greater than the value corresponding to saturation, w will be less than the above value.

It is of interest to calculate the value of n_0 necessary to correspond to saturation, n_0 being the value of n at O . The value of n is given by (5). We cannot directly determine n from (5) because we do not know a^2 . However, a_0^2 is assigned, and

$$\pi a^2 n (2\pi R) = \pi a_0^2 n_0 (2\pi R_0),$$

so that

$$na^2 R = n_0 a_0^2 R_0. \quad (6)$$

Thus (6) and (5) yield

$$\begin{aligned} Ln_0 a_0^2 (R_0/R) &= La^2 n = R_0^2 H_0 / e, \\ n_0 &= RR_0 H_0 / La_0^2 e. \end{aligned}$$

Since, for the dimensions concerned $L \sim 10R$,

$$n_0 \sim R_0 H_0 / 10 a_0^2 e.$$

Thus, with $R_0 = 10^9$ cm, $H_0 = 1000$ gauss, and $a_0 = 10^8$ cm,

$$n_0 \sim 20\,000;$$

n is, of course, much smaller.

Using (6), equation (4) may be written

$$w = R_0^3 H_0^2 / 4Ln_0 a_0^2 = 300 R_0^3 H_0^2 / 4Ln_0 a_0^2 \text{ ev},$$

which applies for all values of n_0 greater than the value 20 000 above cited.

Naturally, there is considerable latitude in the choice of the various dimensions assumed, and those we have taken are chosen only for illustration. Moreover, it is to be observed that our result represents a lower limit for the problem treated, since energy is also transferred to the nucleons during their journey out of the primary field.

III. TRANSFER OF THE ENERGY TO THE NUCLEONS

So far we have discussed only the energy stored in the magnetic field of the ring; and while we have calculated the amount per nucleon, we have not actually provided for its being shared among the nucleons, so that each possesses its share independently of its fellows. We now appeal to the well-known theorem which, as applied to a volume V bounded by a surface S , is

$$\begin{aligned} \iiint \mathbf{E} \cdot \rho \mathbf{u} d\tau &= -\frac{1}{8\pi} \frac{d}{dt} \iiint (E^2 + H^2) d\tau \\ &\quad - \frac{c}{4\pi} \iint [\mathbf{E} \times \mathbf{H}]_n dS, \quad (7) \end{aligned}$$

where \mathbf{u} is the velocity of the density ρ .

Neglecting for the moment the Poynting flux represented by the surface integral, and integrating from $t=0$ when the fields have finite values to $t=\infty$, we have, if Ω represents the total work done by the field on the

nucleons over the period $t=0$ to ∞ ,

$$\Omega = -[\text{energy of field at } t]_0^\infty.$$

Since the energy of the field at $t=\infty$ is zero,² we have

$$\Omega = \text{energy initially in the field.}$$

Thus, if ever a magnetic (or magnetic plus electric) field is made to destroy itself, its energy, apart from the Poynting flux, all goes into work done on the charged nucleons associated with it. The importance of the Poynting flux depends upon the nature of the specific problem. If, instead of a ring, we consider a cylinder, with a current flowing around it as in a solenoid, there is no magnetic field outside the cylinder,³ and consequently no Poynting flux. As a matter of fact, a cylinder is probably a more natural model for our problem than is a ring. We shall not, at this time, trace the matter of the Poynting flux in detail. It represents one of those cases where it is more easily possible to show that it is of small importance in a special case than it is to work out formally its exact magnitude.

Concerning the Meaning of Ω

It is to be noted that the Theorem (7) is a direct algebraical consequence of the Maxwell-Lorentz field equations without the force equation of motion of a particle which, indeed, is not contained in or deducible from those equations. Thus, without some further statement, the work done on the particles and represented by Ω is simply a name for the space and time integrals of $\rho \mathbf{u} \cdot \mathbf{E} d\tau$. However, suppose we assume an equation of motion of a proton of the form

$$m_0 \frac{d}{dt} \frac{\boldsymbol{\omega}}{(1-\beta^2)^{\frac{1}{2}}} = \left(\mathbf{E}_0 + \frac{[\boldsymbol{\omega} \times \mathbf{H}_0]}{c} \right) e, \quad (8)$$

where $\boldsymbol{\omega}$ is the average velocity for the proton as a whole, \mathbf{E}_0 and \mathbf{H}_0 are the fields, excluding the field of the proton, the fields being assumed uniform over the proton, and m_0 is the rest mass of the proton. Then we have the following: The integrated value J of $\mathbf{E} \cdot \rho \mathbf{u} d\tau$ over the proton is given by

$$J = \mathbf{E}_0 \cdot \iiint \rho \mathbf{u} d\tau + \iiint \mathbf{E}_i \cdot \rho \mathbf{u} d\tau,$$

where \mathbf{E}_i is the field due to the proton itself.

$$J = \mathbf{E}_0 \cdot \boldsymbol{\omega} e + \iiint \mathbf{E}_i \cdot \rho (\boldsymbol{\omega} + \delta \boldsymbol{\omega}) d\tau$$

² In our problem, the ring expands without limit on account of the mutual repulsion of its current elements: Energy of magnetic field $= \frac{1}{2} LI = \frac{1}{2} (LI)^2 / L$. Since LI is constant as the ring expands, the energy is zero at $t=\infty$ since $L=\infty$ at $t=\infty$.

³ When the current in the cylinder is changing, there is a magnetic field, but one of small order.

where $\delta\omega$ is the departure of \mathbf{u} from the average ω .

$$J = \mathbf{E}_0 \cdot \omega e + \omega \cdot \int \int \int \mathbf{E}_i \rho d\tau + \int \int \int \mathbf{E}_i \cdot \rho \delta\omega d\tau. \quad (9)$$

It is well known that the first of the above integrals leads to the value

$$\int \int \int \mathbf{E}_i \rho d\tau = -m_e \frac{d}{dt} \frac{\omega}{(1-\omega^2/c^2)^{1/2}}, \quad (10)$$

where m_e is the electromagnetic mass $2e^2/3\alpha c^2$, α being the radius of the proton.

Again, since the scalar product of ω and $[\omega \times \mathbf{H}_0]/c$ is zero, (8), when incorporated in the first term of (9), causes that equation, with the further use of (10), to lead to

$$\begin{aligned} J &= m_0 \omega \cdot \frac{d}{dt} \frac{\omega}{(1-\omega^2/c^2)^{1/2}} - m_e \omega \cdot \frac{d}{dt} \frac{\omega}{(1-\omega^2/c^2)^{1/2}} \\ &\quad + \int \int \int \mathbf{E}_i \cdot \rho \delta\omega d\tau \\ &= (m_0 - m_e) \omega \cdot \frac{d}{dt} \frac{\omega}{(1-\omega^2/c^2)^{1/2}} + \int \int \int \mathbf{E}_i \cdot \rho \delta\omega d\tau. \end{aligned} \quad (11)$$

From this it readily follows that

$$\begin{aligned} J &= (m_0 - m_e) c^2 \left\{ \frac{1}{(1-\omega^2/c^2)^{1/2}} - 1 \right\} \\ &\quad + \int \int \int \mathbf{E}_i \cdot \rho \delta\omega d\tau. \end{aligned} \quad (12)$$

The last integral represents a phenomenon which involved serious consideration in the early days of electrodynamics and had to do with what was then regarded as a violation of the conservation of energy by pure electrodynamic theory.⁴ This matter was bound up with Lorentz's pure electromagnetic theory of mass. The said integral, and the quantity m_e occurring in (12) cease to give trouble if we relieve electromagnetic theory of the responsibility of providing a meaning to mass. The term m_e cannot be omitted even though we cease to regard electromagnetic theory as the sole origin of mass, but it can be made to play a subordinate role if $m_0 \gg m_e$. It is regrettable that we have become involved in such a lengthy discussion of a matter which is primarily a concern of electrodynamics rather than cosmic-ray theory, but it has been felt necessary to deal with the matter to the end of providing the basis for regarding the time integral of the left-hand side of (7) as giving for *each proton* the increase of kinetic energy for that proton as ordinarily understood.

⁴ See H. A. Lorentz, *The Theory of Electrons* (G. E. Stechert and Company, New York, 1909), pp. 213-216; also W. F. G. Swann, Bull. Nat. Research Council (U. S.) No. 24, 4, 40-44 (1922).

A Difficulty Concerning the Development of the Induced Current in the Gas Ring

We have started with the gas ring in the position O, Fig. 1, under conditions in which it has through it a certain flux N_0 resulting from the primary field. Strictly speaking, however, it is not fair to start in this manner, for the gas had to be brought into this position from a place where there was no flux through it or the primary magnetic field had to be created while it was at O. In either case, if the gas has the properties of a conductor, induced currents will be generated during the process of arriving at the condition in which the gas finds itself; and by the fundamental principle which we have invoked, the gas may well find itself in a condition with no flux and will maintain that condition throughout, even to the point where it has been ejected from the primary field, so that the ring will arrive in this position with no magnetic energy, and our primary purpose will have been defeated. We can avoid this calamity by the following considerations:

As already stated, the ring cannot defend itself against change of flux to an extent which calls for it to acquire a current greater than the saturation current, which we shall call I_s . Let I represent a general value of the ring current, so that we must have $|I| \leq I_s$.

Suppose that the horizontal axis, Fig. 3, represents the path of the ring from $-\infty$ to the left to $+\infty$ to the right. Let the ordinates in Fig. 3 represent N , the flux through the ring resulting from the external field alone. On passing from left to right, I goes from zero according to the law, $LI = Nc$, by which procedure it keeps the total flux through the ring zero. This continues until we reach a value N_s of N such that $cN_s/L = I_s$. At this stage, the power of the ring to defend itself against change of total flux ceases, because the saturation current has been reached. As the ring continues to move to the right, the total flux starts to increase until, at the point Q , the resulting flux is $N_M - LI_s/c$. A further movement to the right now results in a decrease in N , so that the ring is once more able to defend itself against change of total flux by experiencing a decrease in its current from the value I_s . This decrease continues until a point C is reached such that $BD = AP = LI_s/c$, at which point the actual current on the ring becomes zero. We now have a ring with zero current and a flux equal to $N_M - LI_s/c$ resulting entirely from the external field. A further decrease of the flux due to the external field as the ring continues its journey toward the right calls forth a growth of the current I in a direction opposite to the original direction; and by the time the ring has passed out of the external field, it will have acquired a current I_s , or a current given by $N_M - LI_s$, whichever is the smaller.

In the light of the foregoing considerations, we see how it is possible to realize a ring current when the ring has left the magnetic field, in spite of the arguments presented at the beginning of this section.

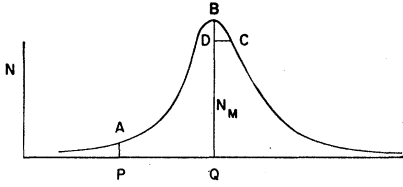


FIG. 3. External flux through ring as a function of distance traveled.

Another procedure consists in starting with the gas ring as part of the system responsible for the primary magnetic field, in which case it carries a current in the opposite direction to the induced current in the discussion already given. It of course becomes necessary to have a mechanism which will hurl the ring to outer space. However, if the ring is hurled out, it will carry with it whatever total flux it possessed when at O.

The Problem from the Standpoint of Rigorous Electrodynamics

The Lagrangian function \mathcal{L} , for a charged particle in a field of vector potential \mathbf{U} and scalar potential φ , is given by

$$\mathcal{L} = -m_0c^2(1-\beta^2)^{1/2} + (e/c)(\mathbf{U} \cdot \mathbf{v}) - e\varphi,$$

where \mathbf{v} is the velocity and $\beta = v/c$. In cylindrical coordinates R, θ, z , and for a case of axial symmetry, and where $\varphi = 0$ and U_r and U_z are zero,

$$\mathcal{L} = -m_0c^2(1-R^2\dot{\theta}^2/c^2 - \dot{R}^2/c^2 - \dot{z}^2/c^2)^{1/2} + (e/c)R\dot{\theta}U. \quad (13)$$

The Lagrangian equations, in terms of generalized coordinates q_r , are of the form

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_r} \right) - \frac{\partial \mathcal{L}}{\partial q_r} = 0, \quad (14)$$

and these equations yield the equations of motion.⁵ As applied to the θ coordinate, they yield

$$m_0R^2\dot{\theta}(1-\beta^2)^{-1/2} + ReU/c = \text{const}, \quad (15)$$

the units here being electrostatic. Now

$$2\pi RU = \text{Total flux through ring} = N + LI/c. \quad (16)$$

Writing $R\dot{\theta} = v_\theta$, where v_θ is the tangential velocity

$$\frac{m_0Rv_\theta}{(1-\beta^2)^{1/2}} + \frac{e}{2\pi c} \left(N + \frac{LI}{c} \right) = \text{const}.$$

Since, at $t=0$, $v_\theta=0$, $I=0$, and $N=N_0$, the constant is $N_0e/2\pi c$. Hence

$$\frac{m_0Rv_\theta}{(1-\beta^2)^{1/2}} + \frac{eLI}{2\pi c^2} = \frac{e}{2\pi c} (N_0 - N). \quad (17)$$

⁵ The equations of motion have been developed for the case of axial symmetry by the writer in a paper, J. Franklin Inst. 258, 383 (1954).

If \bar{n} is the total number of ions in the ring,

$$I = \bar{n}ev_\theta.$$

Hence,

$$\frac{m_0R}{(1-\beta^2)^{1/2}} v_\theta + \frac{eL\bar{n}e}{2\pi c^2} v_\theta = \frac{e}{2\pi c} (N_0 - N).$$

The term involving m_0 is negligible compared with the term involving \bar{n} , provided that

$$L\bar{n} \gg \frac{2\pi c^2}{e^2} \frac{m_0R}{(1-\beta^2)^{1/2}}.$$

Since $L \sim 10R$, this calls for

$$\bar{n} \gg \frac{2\pi c^2 m_0}{10e^2(1-\beta^2)^{1/2}}. \quad (18)$$

Now

$$\bar{n} = 2\pi^2 R_0 a_0^2 n_0.$$

Hence, (18) becomes

$$\pi R_0 a_0^2 n_0 \gg \frac{m_0 c^2}{10e^2(1-\beta^2)^{1/2}}.$$

Putting $R_0 = 10^9$, $a_0 = 10^8$ cm, and $n_0 = 20\,000$, this gives

$$6 \times 10^{29} \gg 1 \times 6 \times 10^{-24} \times 10^{21} / 10 \times 25 \times 10^{-20} (1-\beta^2)^{1/2},$$

i.e.,

$$10^{29} \gg 10^{14} / (1-\beta^2)^{1/2}.$$

In other words, not until $(1-\beta^2)^{-1/2}$ became of the order 10^{15} and the energy became 10^{13} times the rest energy would the first term on the left-hand side of (15) become important. Thus, (15) may, for practical purposes, be written as

$$LI/c = N_0 - N,$$

which is the form we have used in our previous discussion, a form leading to

$$LI/c = N_0,$$

when the ring has passed out of the primary field.

Additional Comment

Strictly speaking, the quantity U is given by

$$cU = \iiint \frac{[i]}{R} d\tau,$$

where $[i]$ is the current density calculated at retarded time in accordance with principles involved in retarded potentials. Thus, the quantity L occurring in (16) and in the equations which follow is really a special kind of self-induction depending upon the retarded feature. However, in the discussion following (16), and in the earlier discussions where L has been used, we have been concerned only in calculating the magnetic energy in the

ring at the time when it had just emerged from the primary field and before it had expanded to an entity which could no longer be regarded as a ring. The use of L up to this point was a convenient approximate procedure for calculating the magnetic energy of the ring before that energy had been largely converted into cosmic ray energy. Once this energy has been calculated, the subsequent story of its complete conversion to cosmic ray energy depends entirely upon (7). It is completely general and independent of all considerations involved in the use of L .

APPENDIX

The concept of gas carrying a magnetic field and induced currents shot out from stellar bodies is frequently employed. There is sense, therefore, to our evading the responsibility of accounting for the mechanism of the process on the plea that "everyone believes it can occur." However, the mechanism of ejection presents difficulties since the direction of the induced

currents is such that the ejection is opposed by the primary magnetic field. Pressure gradients are unrealistic in the light of the long mean free path. Mere mechanical inertia of the ring alone is ineffective in providing the magnetic energy necessary to endow all the nucleons in the ring with cosmic-ray energies. One conceivable method of ejection involves the invocation of a second primary magnetic field B which is superposed on the already invoked primary field A , the field B being associated with a mass of matter moving upwards from below in Fig. 1. The action by B on the induced currents provided by B tends to drive the ring upwards, and this phenomenon can prevail over the force of attraction which would exist if B were absent. The ring will move ahead of the system providing B and become expelled. If the said system becomes destroyed before it itself emerges, the ring will ultimately be found in free space enjoying, however, a total magnetic flux N_0 equal to that produced by the field A when the ring was at O .