$I=2$ S-Wave π-π Scattering Length*

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Through investigation of the reaction $\pi^+ p \rightarrow \pi^+\pi^+ n$ at an incident pion kinetic energy of 357 Mev, we have found the $I=2$ $\pi-\pi$ scattering length $|a_2| \lesssim 0.15\lambda_\pi$. This work is compared with other experiments bearing on this scattering length.

A NUMBER of recent experimental and theoretical papers relate directly or indirectly to a_2, the isotopic spin $=2$ S-wave $\pi-\pi$ scattering length. These are discussed in the latter part of this article. Because these papers are frequently widely divergent in their conclusions, we have undertaken a more definitive measurement of this scattering length by investigating the production of pions in the reaction $\pi^+ + p \rightarrow \pi^+ + \pi^+ + n$ in the 72-in. Lawrence Radiation Laboratory hydrogen bubble chamber.

We see no clear evidence for a low-energy $\pi-\pi$ interaction in the isotopic spin-2 state. An upper limit $|a_2| \lesssim 0.15\lambda_\pi$ is indicated by this experiment.

The pion beam kinetic energy was 357 ± 5 Mev at the chamber center with a total energy loss in the chamber fiducial region of 40 Mev. The energy chosen was as high as possible, in order to assure a reasonable production cross section while still permitting visual separation (on the basis of track ionization) of this reaction from much more common elastic scattering. The film was scanned twice, giving a scanning efficiency $>95\%$. The events were measured on digital measuring projectors, and kinematic fits were obtained by using IBM 709 programs PANG and KICK. Among a total of 67 000 interactions 213 events of the type $\pi^+ + \pi^+ + n$ were observed. Normalizing to a total $\pi^+ - p$ cross section of 40.5 mb,[1] we obtain $\sigma(\pi^+ + p \rightarrow \pi^+ + \pi^+ + n) = 0.12\pm0.01$ mb. Figure 1 shows this point and that of Willis[2] at 500 Mev, the only other measure of this cross section. The solid curve shows the cross section as calculated by Schnitzer[3] using a 1-pion peripheral-collision model and including corrections due to rescattering of one of the pions [illustrated in Figs. 2(a) and 2(b)]. The total cross section in this model is proportional to a_2^2, where a_2 is the S-wave $\pi-\pi$ scattering length in the $I=2$ state.[4] The value used for a_2 was $0.16\lambda_\pi$. To fit our measurement this model would require $a_2 = 0.08\lambda_\pi$. The dashed curve in Fig. 1 shows the dependence of three-body phase space on energy, normalized to our cross section.

Figure 3 shows the number of dipions vs relative pion momentum in the center-of-mass system of the dipion. The curve, normalized to the total number of events, represents phase space. A strong final-state interaction between the pions might be expected to distort this in the manner suggested by Watson.[5] We see no statistically significant deviation from phase space. The interaction between the π^+ and n is quite small in this experiment, since their relative momentum is always below the $\frac{3}{2}$, $\frac{3}{2}$ resonance and furthermore, the charge state is predominantly $I=\frac{1}{2}$; averaged over all momenta it is about 20 mb and virtually inde-

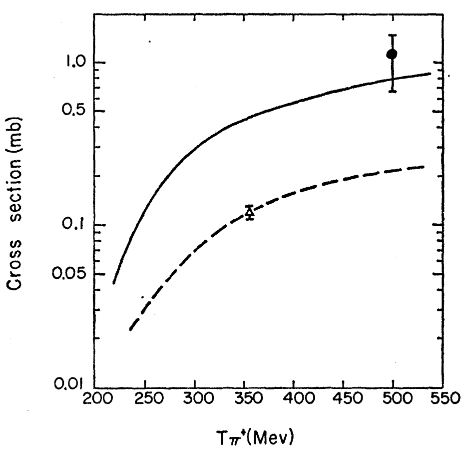

FIG. 1. Cross sections for $\pi^+ + p \rightarrow \pi^+ + \pi^+ + n$. The point at 500 Mev is that of Willis.[2] Our point is shown at 357 Mev. The solid curve is the expected energy dependence calculated by Schnitzer[3]; the dashed curve is three-body phase space scaled to our point.

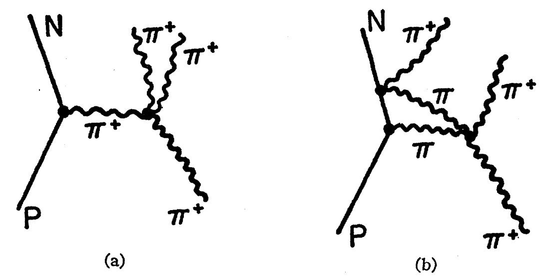

FIG. 2. Diagrams for the peripheral collision model: (a) one-pion exchange and (b) rescattering of one of the pions.

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[1] N. P. Klepikov, V. A. Meshcheryakov, and S. N. Sokolov, Joint Institute for Nuclear Research Report D-584, 1960 (unpublished).
[2] W. J. Willis, Phys. Rev. **116**, 753 (1959).
[3] Howard J. Schnitzer, Phys. Rev. **125**, 1059 (1962); Charles J. Gobel and Howard J. Schnitzer, Phys. Rev. **123**, 1021 (1961).
[4] Figure 2(b) includes a contribution which is proportional to a_1. This should, however, be small in our energy region.
[5] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

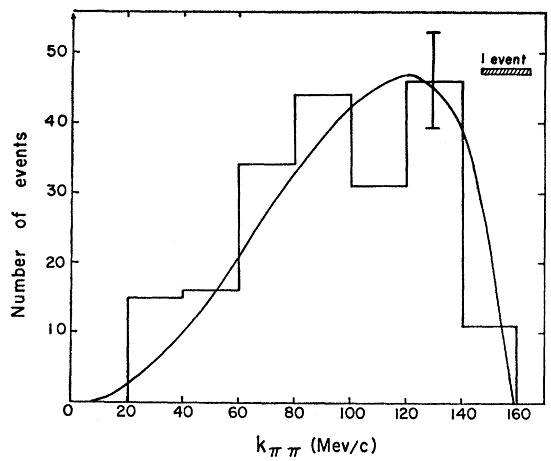

FIG. 3. Histogram of the number of dipions vs relative pion momentum in the dipion center-of-mass system. The curve represents phase space expected on the statistical model.

pendent of the relative dipion momentum. A plot similar to Fig. 3 but with the π^+-n relative momentum as abscissa is more sensitive to the $\pi-n$ interaction. This plot has been found to fit phase space very well.

Figure 4 is a plot of the events as a function of p^2, the square of nonrelativistic momentum transfer to the nucleon in units of m_π^2. The solid curve represents phase space, whereas the dashed curve is the distribution to be expected from a peripheral collision model.[6,7] The curves are normalized to give the same maximum as the data. The data clearly disagree with phase space and, for $p^2>4$, with the peripheral collision model. To study the latter in more detail we have made a Chew-Low extrapolation,[7] shown in Fig. 5. The data are divided into two ω^2 intervals, $\omega^2=4$ to 6 and 6 to 8.5 (where ω = total dipion energy in its center-of-mass system in units of m_π). The quantity $(p^2+1)^2\partial^2\sigma/\partial p^2\partial\omega^2 = 2\{(f^2/2\pi)p^2\omega[(\omega^2/4)-1]^{\frac{1}{2}}/q^2\}\sigma_{\pi\pi}$ is constructed for various p^2 intervals. f^2 is the π-n coupling constant and q is the incident pion momentum (laboratory system). A good linear fit is made for $p^2<4$, yielding extrapolated cross sections at $p^2=-1$ of $\sigma_{\pi\pi}=1\pm6$ mb and -2 ± 7.5 mb for the lower and higher ω^2 intervals, respectively.

Additional uncertainty arises from the assumption of linearity in the extrapolation. This uncertainty is difficult to evaluate, but from Fig. 5 it seems unlikely that $\sigma_{\pi\pi}$ could exceed 10 mb. With $\sigma_{\pi\pi}=8\pi a_2^2$ this leads to $|a_2|\lesssim 0.15\lambda_\pi$.

Although one expects the extrapolation curve to show deviations from linearity for large p^2, it is rather surprising to see the deviations occur as abruptly as indicated in Fig. 5. This again suggests that the $I=2$ $\pi-\pi$ interaction is extremely small, and perhaps the dominant mechanism for this reaction is not via peripheral collisions. It should be remarked that calculations by Rodberg based on a static model (containing no $\pi-\pi$ interaction),[8] although yielding too small a cross section for $\pi^-+p\rightarrow\pi^-+\pi^++n$, are in agreement with this experiment so far as total cross section is concerned.[9]

Other evidence bearing on a_2 generally involves the difference a_2-a_0, where a_0 is the $I=0$ scattering length. They are:

(1) The analysis by Khuri and Treiman[10] of τ-meson decay, considering all deviations from phase space in τ decay to be due to final-state π-π interactions, gives for the most recent data[11] $a_2-a_0=(+0.9\pm0.1)\lambda_\pi$.

(2) Experiments by Batusov *et al.*[12] on the reaction $\pi^-+p\rightarrow\pi^-+\pi^++n$ near threshold, and using the theory of Anselm and Gribov,[13] yield

$$a_2-a_0=(-0.35\pm0.30)\lambda_\pi.$$

(3) Either of the above two results combined with the interpretation of the experiment by Abashian,

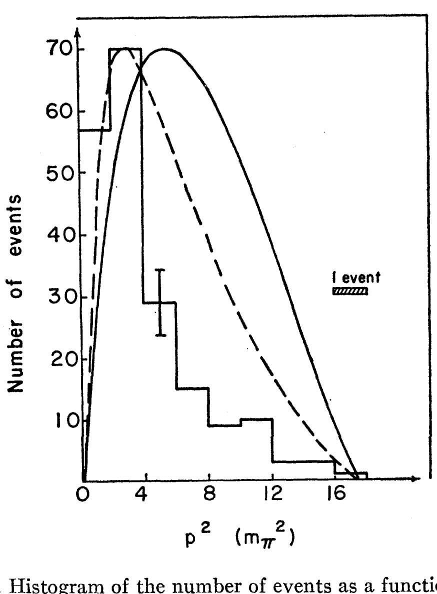

FIG. 4. Histogram of the number of events as a function of the square of the nonrelativistic momentum transfer to the nucleon (units of pion mass squared). The solid curve represents phase space; the dashed curve is the dependence expected from the peripheral collision model. Both curves are normalized to give the same maximum as the data.

[6] C. Goebel, Phys. Rev. Letters 1, 337 (1958).
[7] G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[8] L. S. Rodberg, Phys. Rev. 106, 1090 (1957).
[9] Other calculations using the static model but with different approximations result in a larger cross section than we observe. For a comparison of the various approximations and other references see E. Kazes, Phys. Rev. 107, 1131 (1957).
[10] N. N. Khuri and S. B. Treiman, Phys. Rev. 119, 1115 (1960).
[11] M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and R. D. Tripp, Nuovo cimento 22, 1087 (1961).
[12] Yu. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, in *Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 79.
[13] A. A. Anselm and V. N. Gribov, Soviet Physics—JETP 37, 501 (1959).

Booth, and Crowe[14] on the anomaly in He^3 production from $p+d$ collisions as a strong final-state π-π interaction in the $I=0$ state ($a_0\sim 2.5\lambda_\pi$) yields a very large a_2, inconsistent with our result.

(4) As remarked previously, calculations of pion production in pion nucleon collisions indicated a_2 to be small and $a_0\approx 0.5\lambda_\pi$ to $0.65\lambda_\pi$,[3] considerably less than that of reference 14.

Theoretical calculations by Desai[15] based on crossing symmetry relate the π-π scattering lengths in the three isotopic spin states. His calculations lead to our result for a negative π-π coupling constant λ, whose magnitude is less than 0.15. This value of the coupling constant would yield an $a_0 \lesssim 1.9\lambda_\pi$.

Note added in proof. Treiman and Yang [Phys. Rev. Letters **8**, 140 (1962)] have recently suggested some tests that peripheral collisions must satisfy. They point out that in the center of mass of the beam particle, the plane of the outgoing pions and that of the two nucleons (incoming proton and outgoing neutron) should be uncorrelated. The spinless pion, which "connects" these two planes in the peripheral picture, cannot transmit any orientation information.

We have calculated this correlation for $p^2\leq 4$ and have found a 2-standard-deviation effect, tending to make the planes perpendicular. Taking all p^2, the correlation is weaker.

[14] N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters **7**, 35 (1961).

[15] B. R. Desai, Phys. Rev. Letters **6**, 497 (1961).

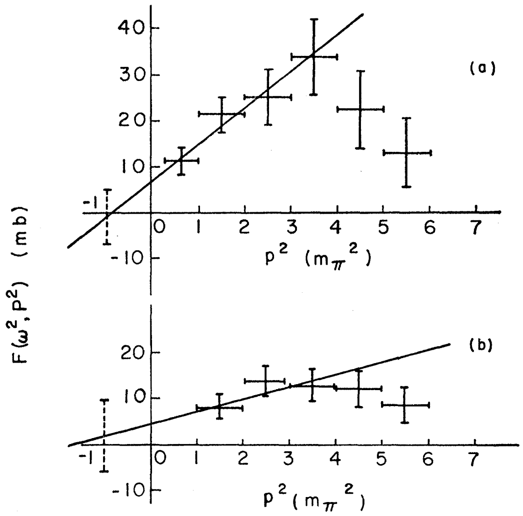

FIG. 5. Chew-Low extrapolation for (a) $4<\omega^2<6$ and (b) $6<\omega^2<8.5$. The plot shows values of the function $F(\omega^2,p^2)$ obtained in the physical region by use of the expression

$$F(\omega^2,p^2)=\frac{q^2}{2(f^2/2\pi)\omega(\omega^2/4-1)^{\frac{1}{2}}}(p^2+1)^2\frac{\partial^2\sigma}{\partial p^2\partial\omega^2}\underset{p^2\to -1}{=}p^2\sigma_{\pi\pi}.$$

The linear fits are made for points with $p^2<4$. The extrapolated values at $p^2=-1$ are (a) $\sigma_{\pi\pi}=1\pm 6$ mb and (b) $\sigma_{\pi\pi}=-2\pm 7.5$ mb.

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