Distortion Effects in the $Ca^{40}(d,p)Ca^{41}$ Reaction*

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A series of calculations has been carried out in which the optical-model parameters have been varied in the distorted-wave Born approximation. In this manner the theoretical (d,p) cross section has been obtained as a function of V, W, R, and a for the $Ca^{40}(d,p)Ca^{41}$ reaction. Some preliminary comments on the stripping due to the interior of the nucleus are also made. Using the results of these first calculations, an attempt is made to obtain best fits to the (d,p) angular distributions from four levels of Ca^{41} at 4.13- and 4.69-Mev incident energy.

INTRODUCTION

THE Butler theory of deuteron stripping reactions[1] has been found to be of great use in the field of nuclear spectroscopy over the past few years. It has been suggested[2] that, by the use of the distorted wave Born approximation (DWBA), the applicability of this method could be extended to lower projectile energies

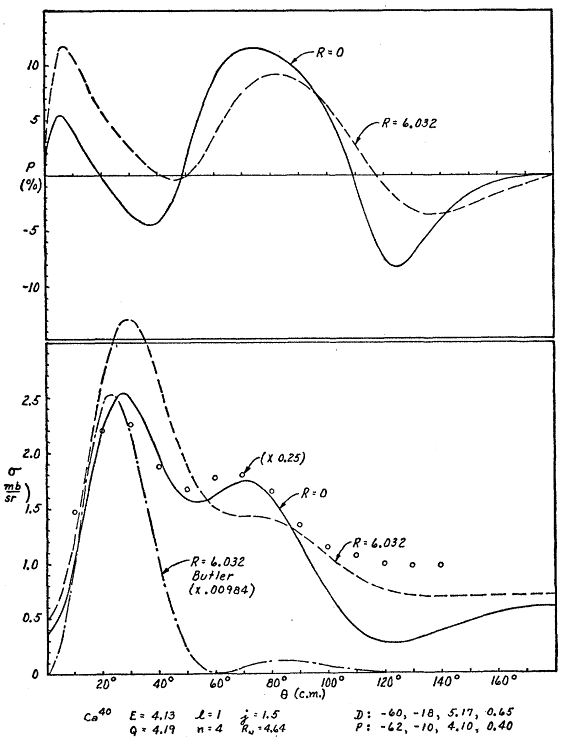

FIG. 1. Differential cross section and polarization for the $Ca^{40}(d,p)Ca^{41}$ reaction. The incident energy is 4.13 Mev and the Q is 4.19 Mev. The curves were calculated by means of the plane wave, cutoff Born approximation or Butler theory (dot-dash curve), the cutoff distorted wave Born approximation (dashed curve), and the non-cutoff distorted wave Born approximation (full curve). The experimental points are due to Rusk and Class.[3] The parameters used in these calculations are listed in Table I.

where the Butler theory is no longer valid. One of the difficulties arising from this approach is that a relatively large number of parameters are necessary to describe the wave functions used in the calculation. Hence, it is necessary to understand the dependence of the stripping cross section on the optical-model parameters in order to understand the implications of the fits obtained. For this reason a systematic variation of the optical parameters was undertaken and will be presented in the next section of this paper.

In attempting to apply the Butler theory to some cases it has been thought necessary to assume that two values of angular momentum transfer contribute since two peaks of comparable magnitude were found in the stripping angular distribution. This seems a reasonable assumption since the nuclei treated in this manner have large quadrupole moments and hence are distorted in shape. However, the calculations of Tobocman[2] (see especially Pb^{207}) tend to show that the second peak may arise naturally when the contribution due to the interior of the nucleus is included and more realistic wave functions are used than in the Butler theory. This effect will be discussed more fully in the present paper.

Since the data of Rusk and Class[3] had just become available it was decided to attempt a fit to the several groups of protons at the two deuteron energies at which the measurements were done. Since Ca^{40} is doubly magic, it is to be expected that the captured neutron is left in a good single-particle shell-model state. Hence, the reduced width should be that of a single particle and the theory should give the magnitude of the cross section exactly and not just an upper limit. This will also be investigated in this paper.

A program was written for an IBM 704 which was essentially identical to the one for the IBM 650 which was used by Tobocman.[2] By using the larger machine the running time per case was cut from 20 hr to 10 min, thus making feasible the series of variations which have been carried out here.

The calculations were carried out in the same manner as in reference 2, where a detailed explanation of the methods may be found.

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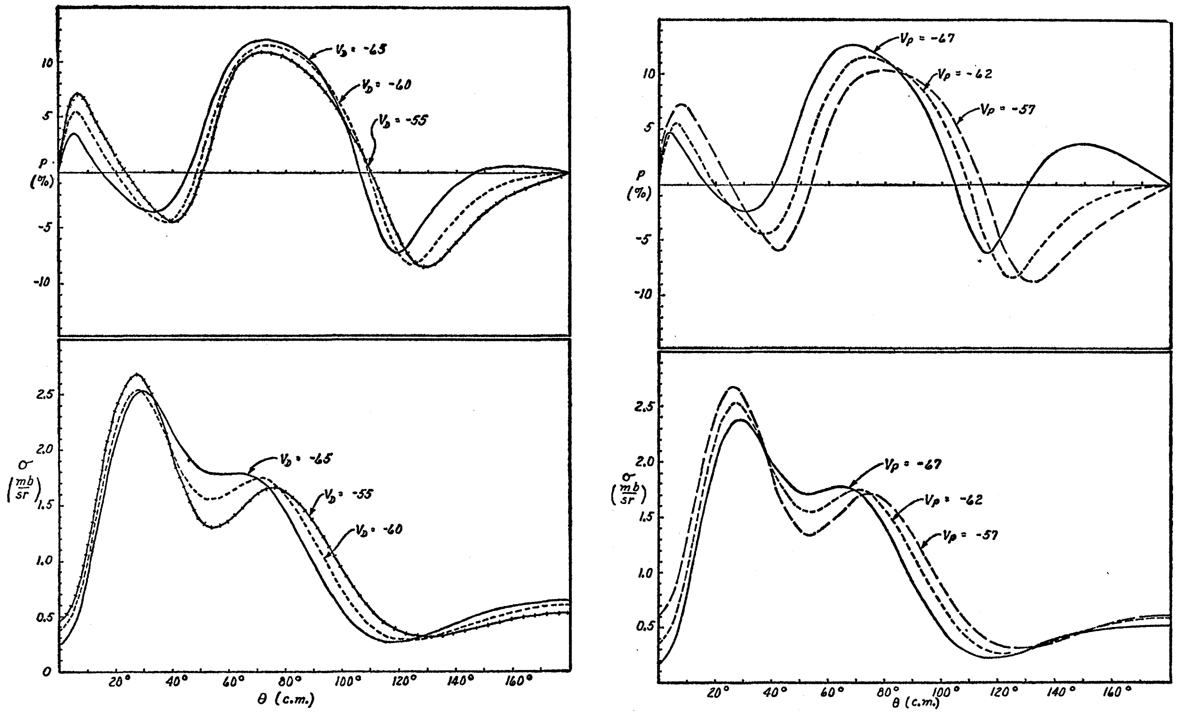
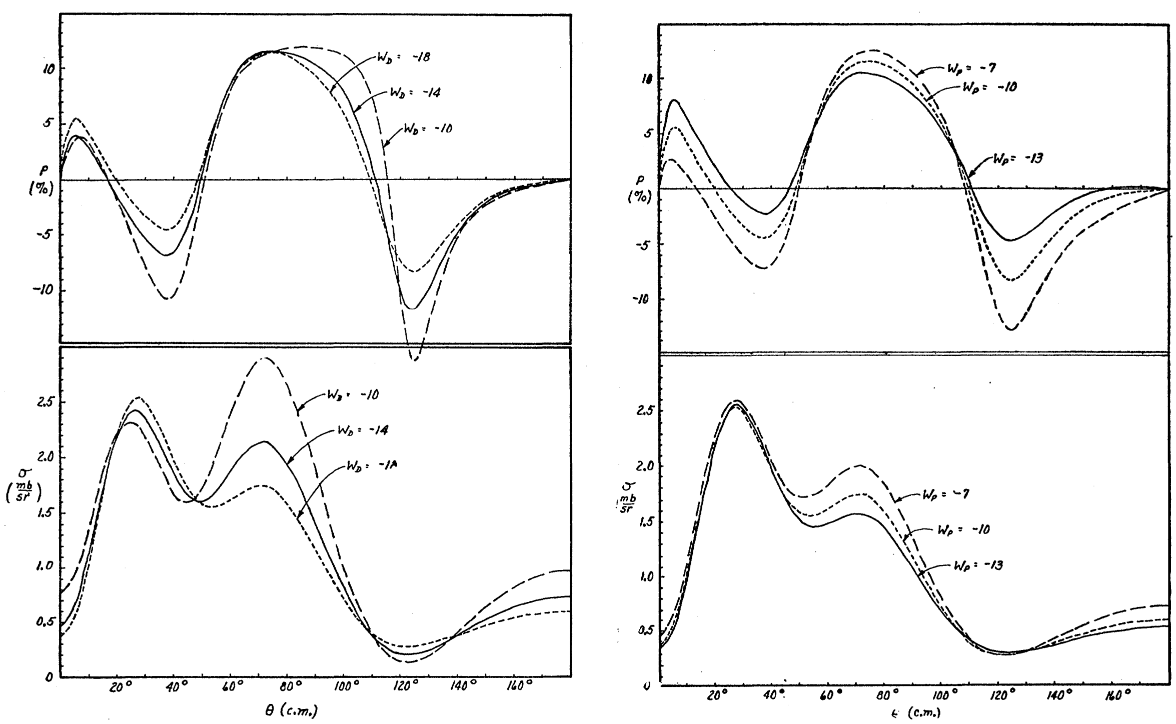
[1] S. T. Butler, Phys. Rev. **80**, 1095 (1950).
[2] W. Tobocman, Phys. Rev. **115**, 98 (1959).
[3] S. Rusk and C. M. Class (unpublished).



(a) (b)

FIG. 2. Variation with the depth of the real part of the optical potential of the (d,p) cross section and polarization given by the distorted wave Born approximation.



(a) (b)

FIG. 3. Variation with the imaginary part of the optical potential of the (d,p) cross section and polarization given by the distorted wave Born approximation.

RESULTS

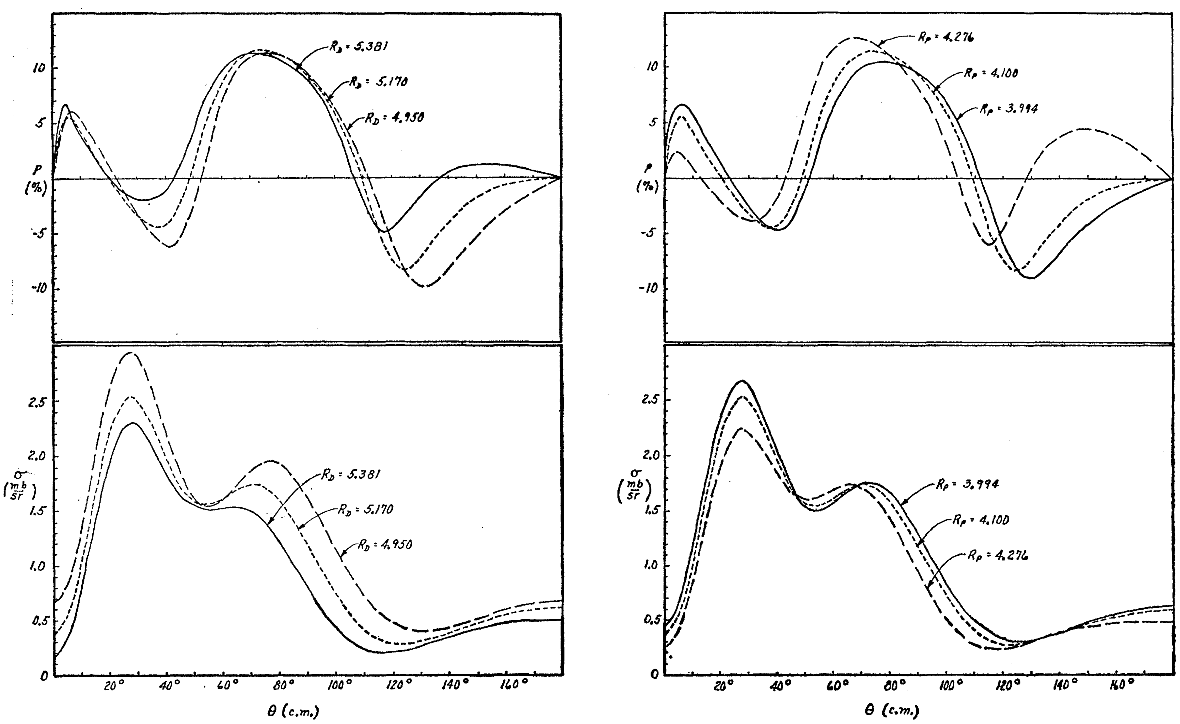
Figure 1 shows a comparison of a cutoff plane wave Born approximation calculation (Butler theory), a cutoff distorted wave Born approximation calculation, and a non-cutoff distorted wave Born approximation (DWBA) calculation for the $Ca^{40}(d,p)Ca^{41}$ reaction cross section and polarization. The experimental points are plotted as circles. It may be seen that Butler theory does produce a second relative maximum but this second peak is much too low and at the wrong angle. The cutoff case brings up the relative height of the second maximum but does not change its position. It is seen to be still much too low. The non-cutoff case brings in the second peak with the correct position and relative height. It would appear that the inclusion of the contribution due to the interior of the nucleus, together with distortion effects, is sufficient to explain the appearance of the second stripping peak. Very similar effects have been seen in calculations involving Zr^{90} and Ce^{140}. This effect seems to be typical of medium Q reactions ($Q\sim 5$ Mev) and, in fact, the second peak may dominate the first (see e.g., Pb^{207} and Ti^{48} in reference 2).

Figure 2 shows the effect of varying the depth of the real part of the optical potential. It may be seen that increasing the depth of either the deuteron or proton potential has the effect of moving the two stripping peaks closer together with the second peak being more affected. It is to be expected that the second peak will be more sensitive to distortion changes since it is entirely due to distortion effects while the first peak is present with no distortion.

Figure 3 shows the variation of the cross section and polarization with changes in the depth of the imaginary part of the optical potential. It is seen that the primary effect of this parameter is to change the relative size of the second stripping peak. This again indicates that the second peak comes from stripping occurring inside of the nucleus since there exists a close correlation between the size of the imaginary part of the optical potential and the probability of finding the particle inside of the nucleus. The variation goes in the direction expected, i.e., the greater the probability of finding the particle inside, the greater the second peak. As in the case of the real part of the potential, it is difficult to draw any general conclusions about the dependence of the polarization on the well depth.

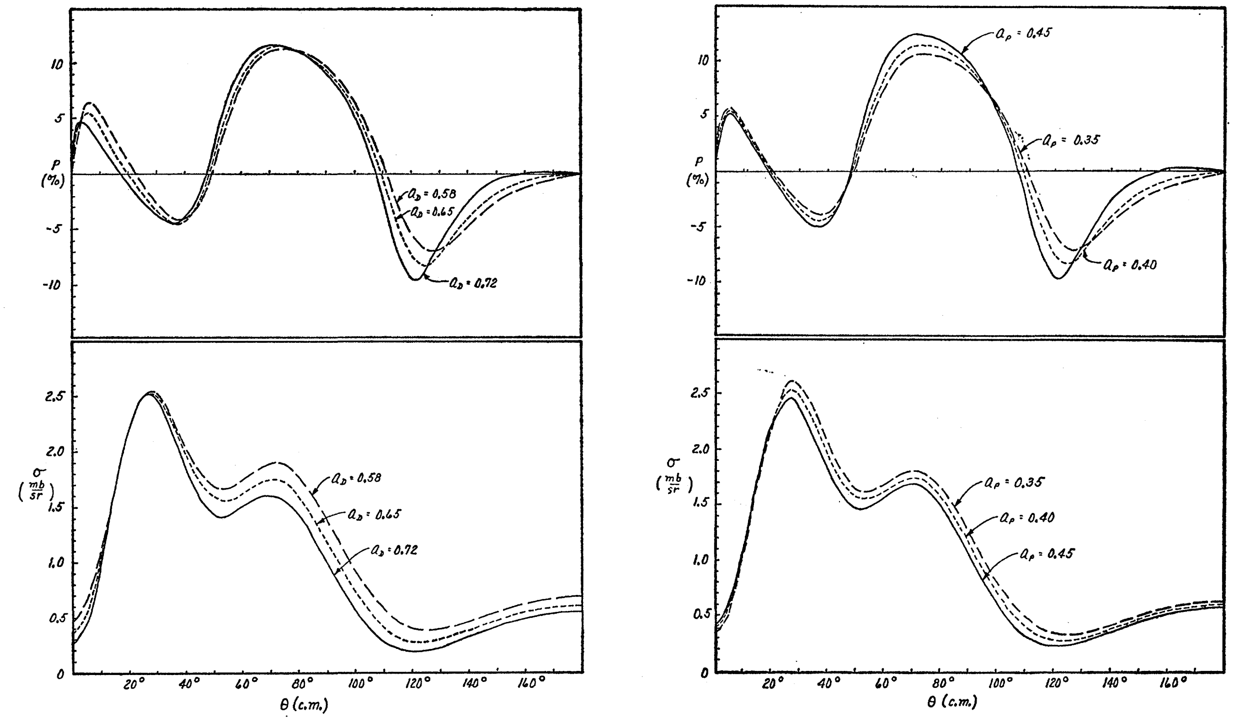
Figure 4 shows the dependence on the radius of the Saxon well. The general behavior is similar to that found in Fig. 2 in accordance with the usual VR^2 ambiguity. The curves have been calculated such that Figs. 2 and 4 consist of three pairs of curves, each member of a pair having the same VR^2. It may be noted that the position of the main stripping peak is essentially independent of R.

Figure 5 gives the dependence of the cross section and polarization on the diffuseness of the Saxon well.



(a) (b)

FIG. 4. Variation with the radius of the optical potential of the (d,p) cross section and polarization given by the distorted wave Born approximation.



(a) (b)

FIG. 5. Variation with the diffuseness of the optical potential of the (d,p) cross section and polarization given by the distorted wave Born approximation.

This is seen to have very little effect on the cross section, particularly in the case of the proton. What effect is observed is of the same character as that of W. This is analogous to the result obtained by Glassgold *et al.*[4] for elastic scattering. (See Table I.)

Figure 6 shows the variation of the cross section and polarization with changes in the bound state wave function. Curve a is computed using a neutron wave function which is strongly peaked inside of the nucleus while curve b is computed using a neutron wave function which is not so strongly peaked. In case a there is a larger relative probability of finding the neutron inside of the nucleus than in case b. Taking the normalization condition into account, this means that there is a smaller probability of finding the neutron

TABLE I. Standard set of parameters from which the variations in Figs. 2–5 were carried out. Energies are in Mev and lengths in fermis.

V_d	−60
W_d	−18
R_d	5.17
a_d	0.65
V_p	−62
W_p	−10
R_p	4.10
a_p	0.40

[4] A. E. Glassgold, W. B. Cheston, M. L. Stein, S. B. Schuldt, and G. W. Erickson, Phys. Rev. **106**, 1207 (1957).

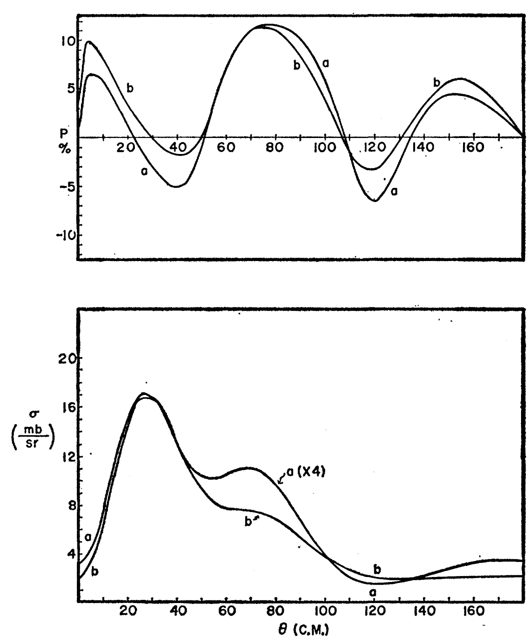


FIG. 6. Variation with the extension of the wave function of the captured neutron of the (d,p) cross section and polarization given by the distorted wave Born approximation. The bound-state wave function used in the calculation of curve a was such as to confine the captured neutron to a smaller volume than that used to calculate curve b.

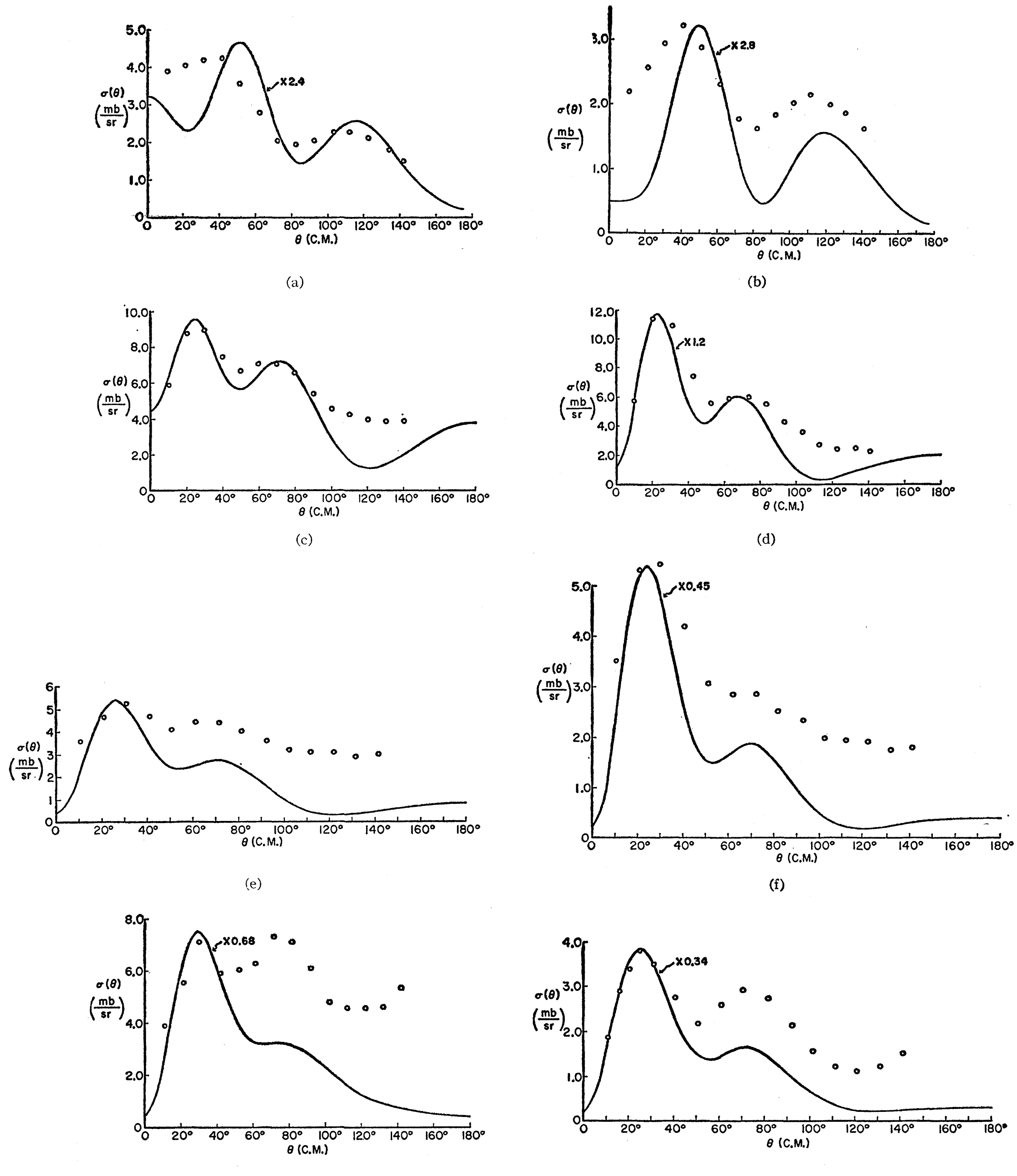


FIG. 7. Best fits found to the $Ca^{40}(d,p)Ca^{41}$ cross sections measured by Rusk and Class.[3] The curves were calculated with the distorted wave Born approximation. The parameters used in each case are given in Table II.

outside of the nucleus. Thus, case *a* should have a smaller main peak than case *b*, as is observed. The height of the second peak above the background of the first peak stays about constant. Since this second peak depends critically on the shape of the internal wave function, it may be possible to learn something about the radial part of this wave function by simply observing the position and relative height of the second peak.

In Fig. 7 are shown the best fits obtained to the data of Rusk and Class by variation of the optical potential and bound state parameters in our program (see Table II). The magnitudes are in reasonable

TABLE II. Important parameters characterizing the plots in Fig. 7. σ_e/σ_t is the ratio of the experimental to theoretical cross section at the peak. Energies are in Mev and lengths in fermis.

	$Q=6.14$		$Q=4.19$		$Q=3.67$		$Q=2.19$	
	$E_d=4.13$	$E_d=4.69$	$E_d=4.13$	$E_d=4.69$	$E_d=4.13$	$E_d=4.69$	$E_d=4.13$	$E_d=4.69$
V_p	−37	−40	−57	−57	−52	−52	−42	−47
W_p	−18	− 4	− 4	− 9.1	−15	−15	0	− 9
R_p	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
a_p	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
V_p	−60	−60	−60	−60	−60	−60	−60	−60
W_d	−10	−10	−10	−10	−10	−10	−10	−10
R_d	5.55	5.55	5.55	5.55	5.55	5.55	5.55	5.55
a_d	0.72	0.72	0.60	0.72	0.72	0.72	0.72	0.72
l	3	3	1	1	1	1	1	1
j	$\frac{7}{2}$	$\frac{7}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
σ_e/σ_t	2.5	2.8	1.2	1.0	1.0	0.45	0.68	0.34
Fig.	7(a)	7(b)	7(c)	7(d)	7(e)	7(f)	7(g)	7(h)

agreement in all cases except the ground state [Figs. 7(a) and 7(b)]. The ground-state case also has its first peak at too large an angle. This effect is also seen in the Butler fits at higher energies.[5] This would tend to indicate that this may not be $l=3$ stripping although the shell model is very definite on this assignment. A possible resolution of the difficulty may come from the strong compound-nucleus effects which are present here, although this still does not explain the discrepancies at higher energies.

Figures 7(c) and 7(d) show the reaction leading to the 1.95-Mev state. The magnitudes are in good agreement and the angular distributions follow the experimental points well except in the region of 120° where the theoretical curves fall consistently too low. This discrepancy is possibly due to compound nucleus interference.

As the Q value decreases the theoretical curve looks more like a Butler angular distribution since the second peak decreases in magnitude. It is interesting to note that in this case the experiment shows an increase of the second peak with decreasing Q value.

DISCUSSION

We have found the dependence of the stripping cross section on the optical parameters involved in the DWBA treatment. From this it seems that certain features, in particular the position of the main peak, are not much affected by a variation of these parameters. This should be true as long as the energy of the particles is above the Coulomb barrier. If the energies are too low the angular distribution loses the characteristic Butler shape.

It has also been seen that the second stripping peak arises naturally in the DWBA treatment. Since this peak depends critically on parameters describing the interior of the nucleus it may be possible to obtain information on this region directly by studying the second peak. The formation of this peak depends on the Q value and the amount of imaginary potential required so that a second peak might not exist at all. This limits the usefulness of this method to those cases in which conditions are right for the formation of a second peak.

Our fits to the data of Class and Rusk are the result of a limited sequence of trial and error runs. Certainly, more systematic parameter searches are to be hoped for in the future.

ACKNOWLEDGMENTS

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[5] C. K. Bockelman and W. W. Buechner, Phys. Rev. **107**, 1366 (1957).