Large-Time Exponential Decay and "Hidden Variables"*

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If "hidden variables" exist, they might become observable in radioactive samples that have decayed for many half-lives. No large deviations from exponential decrease occur in the decay of the intensity of the 1.8-Mev gamma rays from Mn^{56} during the first 34 half-lives.

THERE exist no compelling philosophic or experiential reasons for adopting "causal" or "hidden variables" interpretations[1] of quantum results. Such theories cannot be excluded, however, and those phenomena that might make observable "hidden variables" should be examined. The large-time decay of quasistationary states is one such phenomenon.

According to conventional quantum theory, quasistationary states begin decaying exponentially shortly after their creation. The decay rate then continues to be exponential for many mean lives, but eventually behaves differently. The time t of the appearance of nonexponential effects depends on details of the initial state, but is given roughly by[2,3]

$$t=K\tau \ln(E\tau/\hbar),$$

where τ is the mean life, and E is the resonance energy of the spectrum, with the lowest energy in the spectrum taken to be zero. K is a numerical factor, typically in the range from 2 to 10, that depends on the shape of the low-energy end of the spectrum. For the radioactive decay of Mn^{56}, with $\tau=2.576$ hr/ln 2 and $E=2.81$ Mev,[4] $\ln(E\tau/\hbar)=59$; nonexponential effects should not occur before roughly 200 half-lives. In this example, as with all the usual radioactive materials, nothing observable should be left long before the end of the exponential region. This conclusion could be wrong only if there has been a vast underestimate of the low-energy wing of the spectrum.

"Hidden variables" theories have not been developed to the point where they give predictions about the large-time decay of radioactive nuclei. We can, however, make some speculative remarks. It is possible to construct "hidden variables" theories that predict exponential decay. Suppose that the configuration of a nucleus is specified by the coordinates $x_1, \cdots, x_n$, and that the evolution of the system is described by the constant velocity motion of a point in the $x_1, \cdots, x_n$ space. Suppose further that decay of the nucleus is caused whenever this point touches the surface of any of a large number of small domains in the space. If these domains are distributed randomly for the nuclei of the sample, exponential decay results.

If a particular method of preparation of a sample does not lead to an entirely random distribution of domains, deviations from exponential decay can result. One might expect that observations on a sample that has already decayed for many half-lives would be the most likely to show such deviations. Long decay might remove preferentially those nuclei whose "hidden variables" lie in certain regions and leave nuclei with average properties differing from those of the original sample.

The speculations outlined above make it interesting to observe the decay of a radioactive source many half-lives after its creation. The isotope Mn^{56} seemed particularly suitable. The element Mn occurs entirely as Mn^{55}, and has a thermal-neutron capture cross section[5] of 13.3×10^{-24} cm^2.

Several samples, irradiation methods, and counting schemes were tried; the following proved to be the most successful. A 65-g sample[6] of metallic Mn was irradiated for 2.0 hr at the core of a 200-kw "swimming pool" reactor, in a thermal-neutron flux density of 2×10^{12}/sec cm^2. After it was allowed to decay for 22 half-lives, the sample was removed to a low-background area and mounted 7 in. from a 3-in. diameter, 3-in. long NaI(Tl) scintillation crystal with a 2-in. diameter lead collimator. A 128-channel pulse-height analyzer was used to record every hour the counts accumulated during 26.2 min of analyzer live time.

In 29% of its disintegrations, Mn^{56} emits a 1.81-Mev gamma ray.[4] Since the background in that region was low, the counting rate in three channels covering 1.77 to 1.85 Mev was used.

The Mn^{56} activity in the 1.8-Mev neighborhood finally disappeared into a background that consisted of a mixture of 40-hr La^{140} and 15-hr Na^{24}, plus a small amount of apparently constant background. The constant background was subtracted first. Then the amounts of La^{140} and Na^{24} were determined by a least-squares fit in the time interval from 38 to 58 Mn^{56} half-lives, where the Mn^{56} contribution was negligible. The La^{140}

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[1] J. P. Wesley, Phys. Rev. **122**, 1932 (1961), and references therein.
[2] L. A. Khalfin, Soviet Phys.—JETP **6**, 1053 (1958).
[3] R. G. Winter, Phys. Rev. **123**, 1503 (1961) and references therein.
[4] D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).
[5] *Neutron Cross Sections*, compiled by D. J. Hughes and R B. Schwartz, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed.
[6] Electrolytically refined Mn, obtained from Professor A. J. Shaler of The Pennsylvania State University Metallurgy Department.

and Na^{24} activities were then subtracted. The results for the time between 30 and 40 Mn^{56} half-lives after the end of the irradiation are shown in Fig. 1. The size of the error bars is

$$(N+\sigma^2)^{\frac{1}{2}},$$

where N is the number of counts in the interval and σ is the standard deviation in the least-squares fitted background. For all points shown in Fig. 1, N is at least 2.5 times as large as σ^2.

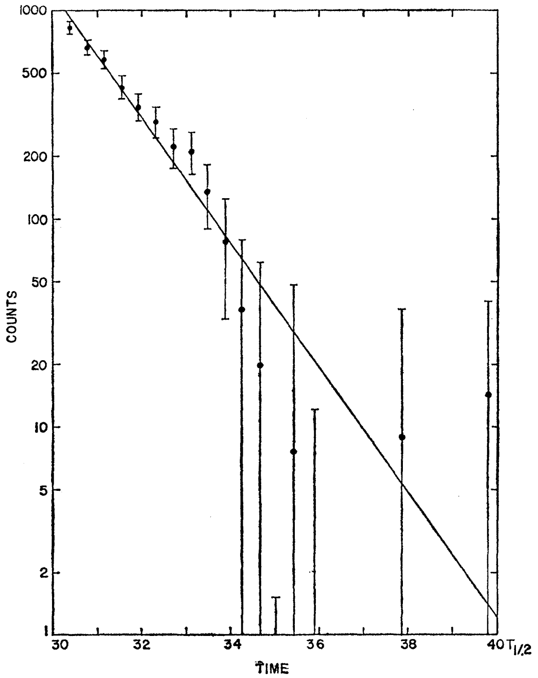

FIG. 1. Mn^{56} activity, in counts per 26.2-min live time, between 30 and 40 half-lives after the end of irradiation, with background subtracted. The time is in units of 2.576 hr, the Mn^{56} half-life. The slope of the straight line corresponds to decay with a 2.576-hr half-life.

The time to which exponential decay can be said to have been verified depends of course on the criteria and confidence limits chosen. It is more useful to let Fig. 1 describe the result, and to observe that there are no large deviations from exponential decay at less than around 34 half-lives, where less than one atom in 10^{10} of the original sample remains.

There may well be unpublished work in which exponential decay has been verified for sources that are very many half-lives old. The best previous large-time study that has been published was made by Lord Rutherford[7] in 1911. He measured the activity of a sample of 3.8-day Rn^{222} at two times separated by 27 half-lives.

It would be of some interest to find examples in which exponential decay can be verified for a larger number of half-lives. At present, we can conclude that "hidden variables" are hidden very well.

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[7] E. Rutherford, Akad. Wiss. Wien. **120.2a**, 303 (1911).