General Formalism Obeying Intrinsic Exclusion Principle

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It is shown here that the formalism for fermions introduced previously by the author is the most general one, if p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, and Ξ^- are the only elementary baryons, and ν, ν', e^-, and μ^- the only elementary leptons (ν and ν' are two kinds of neutrinos). The most general form of the mass operator for all possible baryons is derived. On this level, π and K mesons are treated phenomenologically.

Next, the formalism is extended to mesons and the most general form of the mass operator for all possible mesons is obtained. The formalism allows for two new elementary mesons, a singly-strange isoquartet R and doubly-strange isotriplet D. Also there is a possibility of two new excited mesons. Masses of new mesons are estimated. A symmetry between strong interactions of baryons and mesons of different kinds is considered.

INTRODUCTION

In previous papers of the author[1] a formalism for fermions has been introduced, in which the following "intrinsic exclusion principle" (IEP) holds: (B) isodoublet p, n, isosinglet Λ, isotriplet Σ^+, Σ^0, Σ^-, and isodoublet Ξ^0, Ξ^- are the only possible elementary baryons; (L) isosinglet ν, isodoublet ν', e^-, and isosinglet μ^- are the only possible elementary leptons (ν and ν' are two kinds of neutrino).

This formulation enables us to write down strong, electromagnetic, and weak interactions in concise forms by means of one many-component baryon field, one many-component lepton field, and some boson fields.[1] In other words, all baryons and all leptons can be treated in this formalism as states of one baryon-particle and one lepton-particle, respectively. In this sense, the new formalism is a natural extension of the $\boldsymbol{\tau}$-isospin formalism for nucleons enabling us to consider p and n as states of one nucleon-particle. Also, the advantage of the present formalism is similar to that of the $\boldsymbol{\tau}$ formalism. One gets here a natural tool to write down and discuss symmetries of interactions, e.g., the charge independence and possible higher symmetries of strong interactions.[2] In a similar way the $\boldsymbol{\tau}$ formalism provides the simplest instrument to express the charge independence of nucleon-pion interactions.

In Sec. I of the present paper it is demonstrated that this formalism is the most general one having properties (B) and (L). Therefore, it does not impose by itself any symmetry on interactions such as the charge independence and some higher symmetries in the case of strong interactions. On the other hand, the charge independence and some possible higher symmetries, like the global symmetry (GS), doublet symmetry (DS) [called also restricted global symmetry (RGS)], and a "generalized doublet symmetry" (GDS), can be expressed by this formalism in a quite natural way.

To demonstrate the above statement we show that the most general form of Yukawa strong interactions between baryons listed in (B) and pions and K mesons is expressible in terms of this formalism. In this case the charge independence is additionally assumed from the very beginning and also higher symmetries may be imposed. In the case of weak interactions the argument is similar. For electromagnetic interaction, it is obvious. The universal Fermi weak interaction was discussed by means of this formalism in reference 1, the weak interaction for nonleptonic hyperon decays in another paper.[3]

In Sec. II we determine the most general form of the mass operator for all possible baryons, elementary as well as excited or composite, if only they follow from a charge-independent theory obeying (B). The existence of other baryons than the elementary ones listed in (B) is, from the point of view of the present formalism, a matter of dynamics and depends in some cases on the possibility of higher symmetries of strong interactions like the GS. The point of interest within this problem was previously the well-known $\frac{3}{2}$–$\frac{3}{2}$ πN resonance and now is the 1385-Mev $\pi\Lambda$ resonance.[4–7]

In Sec. III an extension of the formalism to mesons, pointed out in reference 2, is assumed to be true. By this extension a list of elementary mesons is determined, but no symmetries are imposed on interactions. The formalism allows for two new elementary mesons, a singly-strange isoquartet and doubly-strange isotriplet. The most general form of the mass operator for all possible mesons is discussed. A possibility, but not necessity, of two new excited mesons is pointed out. The masses of the new mesons are estimated under some additional assumptions.

In Sec. IV a possible symmetry is discussed between Yukawa strong interactions of elementary baryons and elementary mesons of different kinds. This symmetry may be called "strangeness independence."

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[1] W. Królikowski, Nuclear Phys. **23**, 53 (1961) and previous papers of the author referred to therein.

[2] W. Królikowski, Nuclear Phys. **26**, 91 (1961).

[3] W. Królikowski, Nuovo cimento **20**, 797 (1961).

[4] D. Amati, A. Stanghellini, and B. Vitale, Nuovo cimento **13**, 1143 (1959); Phys. Rev. Letters **5**, 524 (1960).

[5] Ph. Mayer, J. Prentki and Y. Yamaguchi, Phys. Rev. Letters **5**, 442 (1960).

[6] T. D. Lee and C. N. Yang, Phys. Rev. **122**, 1954 (1961).

[7] R. H. Dalitz, Phys. Rev. Letters **6**, 239 (1961).

I. BARYONS, LEPTONS, AND YUKAWA STRONG INTERACTIONS

It is well known that the charge properties of nucleons can be described by matrices $\boldsymbol{\tau}=(\boldsymbol{\tau}_{AB})$, where $A=1$, 2 (and also $B=1$, 2) is an isospinor index corresponding to charge $+e$, 0;

$$\tfrac{1}{2}(\tau_3+1)e=\begin{pmatrix} e & 0 \\ 0 & 0 \end{pmatrix}.$$

The isospin and charge of nucleons are given by

$$\mathbf{T}^N=\int d_3x\, N^*(x)\tfrac{1}{2}\boldsymbol{\tau}N(x)$$

and

$$Q^N/e=\int d_3x\, N^*(x)\tfrac{1}{2}(\tau_3+1)N(x)=T_3{}^N+\tfrac{1}{2}N^N, \quad (1)$$

where N^N is the number of nucleons and

$$N(x)=N_A(x), \quad (2)$$

describes the nucleon field. We have here $(p,n)=N_A$.

It was shown in reference 1 that the charge properties of baryons require introducing, besides the matrices $\boldsymbol{\tau}$, some new matrices ξ_α and $\xi_\alpha{}^*$ defined by commutation relations,

$$\{\xi_\alpha,\xi_\beta{}^*\}=\delta_{\alpha\beta}, \quad \{\xi_\alpha,\xi_\beta\}=0, \quad (3)$$

where $\alpha=1$, 2 (and also $\beta=1$, 2) is an isospinor index corresponding to charge 0, $-e$;

$$\tfrac{1}{2}(\tau_3-1)e=\begin{pmatrix} 0 & 0 \\ 0 & -e \end{pmatrix}.$$

It follows from (3) that ξ_α and $\xi_\alpha{}^*$, when acting on the baryon field, are annihilation and creation operators of α degree of freedom of baryons. The choice of anticommutation relations for ξ_α and $\xi_\alpha{}^*$ incorporates to the formalism the property (B). The isospin and charge of baryons are described by

$$\mathbf{T}^B=\int d_3x\, B^*(x)(\tfrac{1}{2}\boldsymbol{\tau}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)B(x) \quad (4)$$

and

$$Q^B/e=\int d_3x\, B^*(x)[\tfrac{1}{2}(\tau_3+1)+\tfrac{1}{2}\xi_\alpha{}^*(\tau_3-1)_{\alpha\beta}\xi_\beta]B(x) = T_3{}^B+\tfrac{1}{2}(N^B+S^B), \quad (5)$$

where

$$N^B=\int d_3x\, B^*(x)B(x)$$

and

$$S^B=-\int d_3x\, B^*(x)\xi_\alpha{}^*\xi_\alpha B(x) \quad (6)$$

are the number of baryons and the strangeness, respectively, and

$$B(x)=\begin{bmatrix} B_A(x) \\ B_{A\alpha}(x) \\ B_{A\alpha_1\alpha_2}(x) \end{bmatrix} \quad (7)$$

is the baryon field. Formula (7) describes the baryon field $B(x)$ in a representation, which could be called "intrinsic configurational representation" of the field $B(x)$. We have here

$$\begin{aligned} &(p,n)=N_A=B_A, && \Lambda=(B_{12}-B_{21})/\sqrt{2}, \\ &\Sigma^+=B_{11}, && \Sigma^0=(B_{12}+B_{21})/\sqrt{2}, \\ &\Sigma^-=B_{22}, && (\Xi^0,\Xi^-)=\Xi_A=(B_{A12}-B_{A21})/\sqrt{2}. \end{aligned} \quad (8)$$

It follows from (3) that the formalism has the property (B). This is evident from (7) and (8).

We can see from (4) that $\mathbf{T}^B=\mathbf{T}_F{}^B+\mathbf{T}_S{}^B$, where

$$\mathbf{T}_F{}^B=\int d_3x\, B^*(x)\tfrac{1}{2}\boldsymbol{\tau}B(x)$$

and (9)

$$\mathbf{T}_S{}^B=\int d_3x\, B^*(x)\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta B(x)$$

have been called the fundamental and strange isospins of baryons,[2] respectively. So far as baryons are considered, they correspond to the **I** and **K** isospins discussed by Pais.[8]

It was shown in reference 1 that the charge properties of leptons can be described by matrices ξ_α and $\xi_\alpha{}^*$ alone, provided there are two kinds of neutrino. Then, leptons have the following isospin and charge:

$$\mathbf{T}^L=\int d_3x\, L^*(x)\tfrac{1}{2}\xi^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta L(x) \quad (10)$$

and

$$Q^L/e=\int d_3x\, L^*(x)\tfrac{1}{2}\xi_\alpha{}^*(\tau_3-1)_{\alpha\beta}\xi_\beta L(x) = T_3{}^L+\tfrac{1}{2}S^L, \quad (11)$$

where

$$S^L=-\int d_3x\, L^*(x)\xi_\alpha{}^*\xi_\alpha L(x) \quad (12)$$

can be called the strangeness of leptons, and

$$L(x)=\begin{bmatrix} L_0(x) \\ L_\alpha(x) \\ L_{\alpha_1\alpha_2}(x) \end{bmatrix} \quad (13)$$

is the lepton field. We have here

$$\nu=L_0, \quad (\nu',e^-)=L_\alpha, \quad \mu^-=(L_{12}-L_{21})/\sqrt{2}. \quad (14)$$

Formulas (10) and (11) are consistent with the experimental list of leptons, provided there are two neutrinos ν and ν'. The formalism has, therefore, also the property

[8] A. Pais, Nuovo cimento **18**, 1003 (1960); Phys. Rev. **122**, 317 (1961).

TABLE I. Quantum numbers of elementary baryons, leptons, and mesons.[a]

S	T_S	Baryons	J	T_F	T	Leptons	J	T_F	T	Mesons	J	T_F	T
0	0	pn	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	ν	$\frac{1}{2}$	0	0	$\pi^+\pi^0\pi^-$	0	1	1
-1	$\frac{1}{2}$	Λ	$\frac{1}{2}$	$\frac{1}{2}$	0	$\nu'e^-$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\bar{K}^0K^-$	0	1	$\frac{1}{2}$
		$\Sigma^+\Sigma^0\Sigma^-$	$\frac{1}{2}$	$\frac{1}{2}$	1					$R^+R^0R^-R^{--}$(?)	0	1	$\frac{3}{2}$
-2	0	$\Xi^0\Xi^-$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	μ^-	$\frac{1}{2}$	0	0	$D^0D^-D^{--}$(?)	0	1	1

[a] The charge is given by $Q=T_3+\frac{1}{2}(N^B+S)$. R and D denote predicted new elementary mesons with masses $M_R\sim 600$ Mev and $MD\sim 910$ Mev $\sim M_N$.

(L) as a consequence of (3). This is evident from (13) and (14).

We can see from (10) that $\mathbf{T}^L=\mathbf{T}_S{}^L$. Leptons have no fundamental isospin.

Values of S, T_F, T_S, and T for elementary baryons and leptons are listed in Table I. We shall assume that the parities and spins of baryons are equal.

Now we are going to demonstrate that the above formalism is the most general one obeying the IEP. To this end let us write down in the present formalism the following Yukawa strong interaction (without derivatives) between baryons and pions:

$$H^{B\pi}=i\bar{B}\gamma_5[g_F{}^\pi(\xi_\alpha{}^*\xi_\alpha)\boldsymbol{\tau}+g_S{}^\pi(\xi_\alpha{}^*\xi_\alpha)\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta]B\cdot\boldsymbol{\pi}, \quad (15)$$

where $g_{F,S}{}^\pi(\lambda)$ are arbitrary real functions. Making use of representation (7) and formula (8) and the algebraic properties of $\boldsymbol{\tau}$ and ξ_α, $\xi_\alpha{}^*$, we can rewrite (15) in the form

$$H^{B\pi}=i[g^{N\pi}\bar{N}\gamma_5\boldsymbol{\tau}N+g^{\Lambda\Sigma\pi}(\bar{\Lambda}\gamma_5\boldsymbol{\Sigma}+\bar{\boldsymbol{\Sigma}}\gamma_5\Lambda)$$
$$-ig^{\Sigma\Sigma\pi}\bar{\boldsymbol{\Sigma}}\gamma_5\times\boldsymbol{\Sigma}+g^{\Xi\pi}\bar{\Xi}\gamma_5\boldsymbol{\tau}\Xi]\cdot\boldsymbol{\pi}, \quad (16)$$

where

$$\Sigma_1=(\Sigma^--\Sigma^+)/\sqrt{2},\quad \Sigma_2=-i(\Sigma^-+\Sigma^+)/\sqrt{2},\quad \Sigma_3=\Sigma^0, \quad (17)$$

and

$$\begin{aligned} g^{N\pi}&=g_F{}^\pi(0), & g^{\Lambda\Sigma\pi}&=g_F{}^\pi(1)-g_S{}^\pi(1),\\ g^{\Sigma\Sigma\pi}&=g_F{}^\pi(1)+g_S{}^\pi(1), & g^{\Xi\pi}&=g_F{}^\pi(2). \end{aligned} \quad (18)$$

We can see that interaction (16) is the most general Yukawa strong interaction (without derivatives) between baryons and pions.[9]

The following higher symmetries of interaction (15) are of interest.

(a) The DS[8]: $g_S{}^\pi(\lambda)=0$. Then $g^{\Lambda\Sigma\pi}=g^{\Sigma\Sigma\pi}$ $(\equiv g^{Y\pi})$. Interaction (15) can be expressed by baryon doublets,

$$H^{B\pi}=i\bar{B}\gamma_5g_F{}^\pi(\xi_\alpha{}^*\xi_\alpha)\boldsymbol{\tau}B\cdot\boldsymbol{\pi}=i[g^{N\pi}\bar{N}\gamma_5\boldsymbol{\tau}N$$
$$+g^{Y\pi}(\bar{Y}\gamma_5\boldsymbol{\tau}Y+\bar{Z}\gamma_5\boldsymbol{\tau}Z)+g^{\Xi\pi}\bar{\Xi}\gamma_5\boldsymbol{\tau}\Xi]\cdot\boldsymbol{\pi}, \quad (19)$$

where

$$\begin{aligned} &N_A=B_A=(p,n),\quad Y_A=B_{A1}=(\Sigma^+,(\Sigma^0-\Lambda)/\sqrt{2}),\\ &Z_A=B_{A2}=((\Sigma^0+\Lambda)/\sqrt{2},\Sigma^-),\\ &\Xi_A=(B_{A12}-B_{A21})/\sqrt{2}=(\Xi^0,\Xi^-). \end{aligned} \quad (20)$$

In this case $H^{B\pi}$ is invariant under transformations generated by $\mathbf{T}_F{}^B+\mathbf{T}^\pi$, where $\mathbf{T}^\pi$ is the isospin of the pions.

(a′) The GS[10]: $g_S{}^\pi(\lambda)=0$ and $g_F{}^\pi(\lambda)=\text{const}$. Then $g^{N\pi}=g^{\Lambda\Sigma\pi}=g^{\Sigma\Sigma\pi}=g^{\Xi\pi}$.

(b) $g_F{}^\pi(\lambda)=g_S{}^\pi(\lambda)$. Then $g^{\Lambda\Sigma\pi}=0$.

(c) $g_F{}^\pi(\lambda)=-g_S{}^\pi(\lambda)$. Then $g^{\Sigma\Sigma\pi}=0$.

In a similar way we consider in the present formalism the following Yukawa strong interaction between baryons and K mesons:

$$H^{BK}=\frac{i}{2\sqrt{3}}\bar{B}\gamma_5\{g_F{}^K(\xi_\alpha{}^*\xi_\alpha)\boldsymbol{\tau}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}+g_S{}^K(\xi_\alpha{}^*\xi_\alpha)$$
$$\times[\xi_\gamma{}^*\boldsymbol{\tau}_{\gamma\delta}\xi_\delta,\xi_\alpha{}^*]\cdot\boldsymbol{\tau}_{\alpha\beta}\}BK_\beta{}^G+\text{H.c.}, \quad (21)$$

where $K^G=i\tau_2K^*=(\bar{K}^0,-K^-)$, $K=(K^+,K^0)$. Let us note that

$$\bar{B}\gamma_5g_S{}^K[\xi_\gamma{}^*\boldsymbol{\tau}_{\gamma\delta}\xi_\delta,\xi_\alpha{}^*]\cdot\boldsymbol{\tau}_{\alpha\beta}BK_\beta{}^G=3\bar{B}\gamma_5g_S{}^K\xi_\alpha{}^*BK_\alpha{}^G. \quad (22)$$

After a calculation we get from (21)

$$H^{BK}=iK^*(g^{N\Lambda K}\bar{\Lambda}\gamma_5N+g^{N\Sigma K}\bar{\boldsymbol{\Sigma}}\gamma_5\cdot\boldsymbol{\tau}N)$$
$$+i(g^{\Xi\Lambda K}\bar{\Xi}\gamma_5\Lambda+g^{\Xi\Sigma K}\bar{\Xi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\Sigma})K^G+\text{H.c.}, \quad (23)$$

where

$$\begin{aligned} g^{N\Lambda K}&=\frac{3g_F{}^K(1)-3g_S{}^K(1)}{2\sqrt{6}},\\ g^{N\Sigma K}&=-\frac{g_F{}^K(1)+3g_S{}^K(1)}{2\sqrt{6}},\\ g^{\Xi\Lambda K}&=-\frac{3g_F{}^K(2)+3g_S{}^K(2)}{2\sqrt{6}},\\ g^{\Xi\Sigma K}&=\frac{g_F{}^K(2)-3g_S{}^K(2)}{2\sqrt{6}}. \end{aligned} \quad (24)$$

Interaction (23) is the most general Yukawa strong interaction (without derivatives) between baryons and K mesons.[9]

The following higher symmetries of interaction (21) may be of interest.

(a) The DS[8]: $g_F{}^K(\lambda)=0$. Then $g^{N\Lambda K}=g^{N\Sigma K}$ $(\equiv g^{NYK})$ and $g^{\Xi\Lambda K}=g^{\Xi\Sigma K}$ $(\equiv g^{\Xi YK})$. Interaction (21) is expres-

[9] B. d'Espagnat and J. Prentki, Nuclear Phys. 1, 3 (1955).

[10] M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

sible by baryon doublets,

$$H^{BK}=\frac{i}{2\sqrt{3}}\bar{B}\gamma_5 g_S{}^K(\xi_\alpha{}^*\xi_\alpha)$$
$$\times[\xi_\gamma{}^*\boldsymbol{\tau}_{\gamma\delta}\xi_\delta,\xi_\alpha{}^*]\cdot\boldsymbol{\tau}_{\alpha\beta}B\cdot K_\beta{}^G+\text{H.c.}$$
$$=i\sqrt{2}[g^{NYK}(-\bar{Y}\gamma_5 N\bar{K}^0+\bar{Z}\gamma_5 NK^-)$$
$$+g^{\Xi YK}(\bar{\Xi}\gamma_5 Z\bar{K}^0+\bar{\Xi}\gamma_5 YK^-)]+\text{H.c.} \quad (25)$$

In this case H^{BK} is invariant under transformations generated by $\mathbf{T}_F{}^B$ and hence also by $\mathbf{T}_F{}^B+\mathbf{T}^\pi$.

(a′) The GS: $g_F{}^K(\lambda)=0$ and $g_S{}^K(\lambda)=\text{const}$. Then $g^{N\Lambda K}=g^{N\Sigma K}=g^{\Xi\Lambda K}=g^{\Xi\Sigma K}$.

(b) $g_F{}^K(\lambda)=g_S{}^K(\lambda)$. Then $g^{N\Lambda K}=0$, $g^{\Xi\Lambda K}=3g^{\Xi\Sigma K}$.

(c) $g_F{}^K(\lambda)=-g_S{}^K(\lambda)$. Then $3g^{N\Lambda K}=g^{N\Sigma K}$, $g^{\Xi\Lambda K}=0$.

(d) The "generalized doublet symmetry" (GDS)[2]: $g_S{}^K(\lambda)=0$. Then $g^{N\Lambda K}=-3g^{N\Sigma K}$ and $g^{\Xi\Lambda K}=-3g^{\Xi\Sigma K}$. If one introduces the meson field $M_{i\alpha}$ gathering the mesons K and R discussed in Sec. III, then the interaction $H^{BK}+H^{BR}$ can be expressed by baryon doublets, provided the GDS is satisfied. In this case, $H^{BK}+H^{BR}$ is invariant under transformations generated by $\mathbf{T}_F{}^B+\mathbf{T}^\pi+\mathbf{T}_F{}^K+\mathbf{T}_F{}^R$ where $\mathbf{T}_F{}^K$ and $\mathbf{T}_F{}^R$ are the fundamental isospins of K and R mesons (see Secs. III and IV).

(d′) The "generalized global symmetry" (GGS): $g_S{}^K(\lambda)=0$ and $g_F{}^K(\lambda)=\text{const}$. Then $g^{N\Lambda K}=-3g^{N\Sigma K}=-g^{\Xi\Lambda K}=3g^{\Xi\Sigma K}$.

(e) $g_F{}^K(\lambda)=3g_S{}^K(\lambda)$. Then $g^{N\Lambda K}=-g^{N\Sigma K}$, $g^{\Xi\Sigma K}=0$.

(f) $g_F{}^K(\lambda)=-3g_S{}^K(\lambda)$. Then $g^{N\Sigma K}=0$, $g^{\Xi\Lambda K}=-g^{\Xi\Sigma K}$.

Experimental data about the photoproduction of K mesons seem to suggest that $g^{N\Lambda K}\approx -g^{N\Sigma K}$.[11] This relation is satisfied, if the case (e) is nearly realized. Then $|g^{\Xi\Lambda K}|\gg|g^{\Xi\Sigma K}|$.

II. MASS OPERATOR FOR BARYONS

We turn now to a discussion of the mass operator for baryons. First, let us observe that for a regular arbitrary function $f(\lambda_1,\lambda_2)$ we have the following formula, as a consequence of the algebraic properties of $\boldsymbol{\tau}$ and ξ_α, $\xi_\alpha{}^*$:

$$f(s+\xi_\alpha{}^*\xi_\alpha,\,(\mathbf{t}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2)$$
$$=\alpha(s+\xi_\alpha{}^*\xi_\alpha,\,\mathbf{t}^2)+\beta(s,\mathbf{t}^2)(\mathbf{t}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2$$
$$+\gamma(s,\mathbf{t}^2)(\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2, \quad (26)$$

where s is a matrix commuting with $\xi_\alpha{}^*\xi_\alpha$ as well as $\mathbf{t}$ and $\frac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta$, and $\mathbf{t}$ is an isospin matrix commuting with $\frac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta$ and $\xi_\alpha{}^*\xi_\alpha$, e.g., $\mathbf{t}=\frac{1}{2}\boldsymbol{\tau}$. Note that $\mathbf{t}^2=t(t+1)$ is a c number. Formula (26) follows from the relations:

$$(\mathbf{t}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2=\tfrac{1}{3}\mathbf{t}^2(\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2-\mathbf{t}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta,$$
$$(\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^4=3(\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2,$$
$$(\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2(\mathbf{t}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)=3\mathbf{t}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta,$$
$$(\xi_\alpha{}^*\xi_\alpha)(\mathbf{t}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)=\mathbf{t}\cdot\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta,$$

which reduce a power series representing f to the right-hand side of formula (26).

Neglecting electromagnetic and weak interactions, the mass operator for a single baryon (elementary as well as excited or composite) is a function of strangeness $S=-(s^c+\xi_\alpha{}^*\xi_\alpha)$ and isospin squared

$$\mathbf{T}^2=(\mathbf{t}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2$$

of the baryon, and also some other quantum numbers A, e.g., $\mathbf{J}^2=J(J+1)$, determining the space structure of the baryon. Here s^c is a matrix describing the strangeness (with opposite sign) of mesons and baryon pairs bounded to the baryon, and $\mathbf{t}=\frac{1}{2}\boldsymbol{\tau}+\mathbf{t}^c$, where $\mathbf{t}^c$ is a matrix representing isospin of the bounded mesons and baryon pairs. Then, using (26) we can write the mass operator of a single baryon in the form

$$M=M(A,\,s^c+\xi_\alpha{}^*\xi_\alpha,\,(\mathbf{t}+\tfrac{1}{2}\xi_\alpha{}^*\mathbf{t}_{\alpha\beta}\xi_\beta)^2)$$
$$=\alpha(A,\,s^c+\xi_\alpha{}^*\xi_\alpha,\,\mathbf{t}^2)+\beta(A,s^c,\mathbf{t}^2)(\mathbf{t}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2$$
$$+\gamma(A,s^c,\mathbf{t}^2)(\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2. \quad (27)$$

If elementary baryons and excited baryons without excitation of strangeness are considered, then $s^c=0$ and $A=\mathbf{J}^2=J(J+1)$. In this case the mass operator is given by

$$M^B=\alpha^B(\mathbf{J}^2,\xi_\alpha{}^*\xi_\alpha,\mathbf{t}^2)+\beta^B(\mathbf{J}^2,\mathbf{t}^2)(\mathbf{t}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2$$
$$+\gamma^B(\mathbf{J}^2,\mathbf{t}^2)(\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2. \quad (28)$$

For such baryons $\mathbf{T}_F=\mathbf{t}$ and $\mathbf{T}_S=\frac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\mathbf{T}_S{}^B$. In particular, for elementary baryons we have

$$J=\tfrac{1}{2},\qquad \xi_\alpha{}^*\xi_\alpha=\begin{cases}0 \text{ for } N\\ 1 \text{ for } \Lambda \text{ and } \Sigma,\\ 2 \text{ for } \Xi\end{cases}$$

$$\mathbf{t}=\tfrac{1}{2}\boldsymbol{\tau},\ \tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\begin{cases}0 \text{ for } N\\ \text{isospin}-\tfrac{1}{2} \text{ vector for } \Lambda \text{ and } \Sigma;\\ 0 \text{ for } \Xi\end{cases}$$

for $P_{\frac{3}{2}}$ $T_F=\frac{3}{2}$ excited baryons we have

$$J=\tfrac{3}{2},\ \xi_\alpha{}^*\xi_\alpha=\begin{cases}0 \text{ for } N^*\\ 1 \text{ for } \Lambda^* \text{ and } \Sigma^*,\\ 2 \text{ for } \Xi^*\end{cases}$$

$$\mathbf{t}=\text{isospin}-\tfrac{3}{2} \text{ vector},$$

$$\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\begin{cases}0 \text{ for } N\\ \text{isospin}-\tfrac{1}{2} \text{ vector for } \Lambda \text{ and } \Sigma.\\ 0 \text{ for } \Xi\end{cases}$$

If a conjecture is made that

$$\alpha^B(\mathbf{J}^2,\xi_\alpha{}^*\xi_\alpha,\mathbf{t}^2)=\alpha_0{}^B(\mathbf{J}^2,\mathbf{t}^2)+\alpha_1{}^B(\mathbf{J}^2,\mathbf{t}^2)\xi_\alpha{}^*\xi_\alpha, \quad (29)$$

and that β^B and γ^B are approximately constants independent of $\mathbf{J}^2$ and $\mathbf{t}^2$, then formula (28) turns out to be equivalent to that used by Lee and Yang[6] to estimate the masses of Σ^* and Ξ^* ($\Lambda^*=Y^*$, $\Sigma^*=Z^*$ in their

[11] For references see Y. Shimamoto, Phys. Rev. **122**, 289 (1961).

TABLE II. Quantum numbers of excited baryons and mesons.[a]

S	T_S	Excited baryons	J	T_F	T	M(Mev)	Excited mesons	J	T_F	T	M(Mev)
0	0	N^*	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1237	π^*	1	1	1	$\sim$ 660
-1	$\frac{1}{2}$	Λ^*	$\frac{3}{2}$	$\frac{3}{2}$	1	1385	$\bar{K}^*$	1	1	$\frac{1}{2}$	880
		Σ^* (?)	$\frac{3}{2}$	$\frac{3}{2}$	2	$\sim$1539	R^* (?)	1	1	$\frac{3}{2}$	$\sim$ 990
-2	0	Ξ^* (?)	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\sim$1637	D^* (?)	1	1	1	$\sim$1160

[a] The charge is given by $Q=T_3+\frac{1}{2}(N^B+S)$. R^* and D^* denote possible new excited mesons.

notation). This equivalence follows from the relations

$$\mathbf{t}=\tfrac{1}{2}\boldsymbol{\tau}+\mathbf{t}^c=\mathbf{L},\quad \tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\mathbf{M},\quad \tfrac{1}{2}\xi_\alpha{}^*\xi_\alpha=\tfrac{1}{2}-N_3$$

and

$$(\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2=\tfrac{3}{4}-\mathbf{N}^2.$$

Formula (28) does not prove, of course, that some excited baryons follow from the formalism. It only gives their masses, if such baryons exist. It is well known that the existence of excited hyperons Λ^*, Σ^*, and Ξ^* is a dynamical problem depending on the applicability of the GS to strong interactions.[4–6] Interpretation of the observed 1385-Mev $\pi\Lambda$ resonance as the excited hyperon Λ^*, or alternatively as a composite particle being a bound state of N and $\bar{K}$, is yet an open question.[7] A third possibility would be *a priori* provided by an excited nucleon with excitation of strangeness, which would not be identical with a composite state of one N and one $\bar{K}$.

Possible $P_{\frac{3}{2}}$ $T_F=\frac{3}{2}$ excited baryons without excitation of strangeness are listed for completeness in Table II. The masses are given as estimated by Lee and Yang under the tentative assumption that Λ^* is the experimental 1385-Mev $\pi\Lambda$ resonance.

III. MESONS AND MASS OPERATOR FOR MESONS

In Secs. I and II, mesons were treated phenomenologically in the sense that no assumptions were made about their classification. Especially, the IEP was not extended to mesons.

Now we assume that the extension of the formalism to mesons pointed out in reference 2 is correct. We assume, namely, that the charge properties of mesons can be described by isospin-1 matrices $\boldsymbol{\theta}=(\boldsymbol{\theta}_{ij})$ and matrices ξ_α and $\xi_\alpha{}^*$. Here $i=1, 2, 3$ (and also $j=1, 2, 3$) is an isovector index corresponding in a well-known way to charge $+e$, 0, $-e$. Then, the isospin and charge of mesons are given by

$$\mathbf{T}^M=\int d_3x\ (1/i)\partial_t M^*(x)\times(\boldsymbol{\theta}+\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)M(x)+\text{H.c.},\quad (30)$$

and

$$Q^M/e=\int d_3x\ (1/i)\partial_t M^*(x)[\theta_3+\tfrac{1}{2}\xi_\alpha{}^*(\tau_3-1)_{\alpha\beta}\xi_\beta]\times M(x)+\text{H.c.}=T_3{}^M+\tfrac{1}{2}S^M,\quad (31)$$

where

$$S^M=-\int d_3x\ (1/i)\partial_t M^*(x)\xi_\alpha{}^*\xi_\alpha M(x)+\text{H.c.}\quad (32)$$

is the strangeness of the mesons and

$$M(x)=\begin{bmatrix} M_i(x) \\ M_{i\alpha}(x) \\ M_{i\alpha_1\alpha_2}(x)\end{bmatrix}\quad (33)$$

represents the meson field. We make here the following identification:

$$\begin{aligned}&\pi_i=M_i,\\ &K^G \text{ and } R\\ &\quad=\text{doublet and quartet coming out from } M_{i\alpha},\\ &D_i=(M_{i12}-M_{i21})/\sqrt{2},\end{aligned}\quad (34)$$

where a singly-strange quartet R and doubly-strange triplet D are new mesons. $K^G=i\tau_2\bar{K}=(\bar{K}^0, -K^-)$. This formalism extends, therefore, to mesons the IEP in the following way:

(M) isotriplet π^+, π^0, π^-, isodoublet $\bar{K}^0$, $-K^-$, isoquartet R^+, R^0, R^-, R^{--}, and isotriplet D^0, D^-, D^{--}

are the only possible elementary mesons.

We can see from (3) that $\mathbf{T}^M=\mathbf{T}_F{}^M+\mathbf{T}_S{}^M$, where

$$\mathbf{T}_F{}^M=\int d_3x\ (1/i)\partial_t M^*(x)\boldsymbol{\theta}M(x)$$

and

$$\mathbf{T}_S{}^M=\int d_3x\ (1/i)\partial_t M^*(x)\tfrac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta M(x).\quad (35)$$

Values of S, T_F, T_S, and T for elementary mesons are listed in Table I.

We shall assume that parities and spins of mesons are equal.

The mass operator for a single meson has now also the form (27), where s^c is a matrix describing mesons and baryon pairs bounded to the meson, and $\mathbf{t}=\boldsymbol{\theta}+\mathbf{t}^c$, where $\mathbf{t}^c$ are isospin matrices of the bounded mesons and baryon pairs.

If one wants to discuss elementary mesons and excited mesons without excitation of strangeness, then $s^c=0$ and $A=\mathbf{J}^2=J(J+1)$. In this case the mass operator has the form analogous to (28). For such mesons $\mathbf{T}_F=\mathbf{t}$ and $\mathbf{T}_S=\frac{1}{2}\xi_\alpha{}^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\mathbf{T}_S{}^M$. In particular,

for elementary mesons we have

$$J=0, \qquad \xi_\alpha^*\xi_\alpha=\begin{cases}0 \text{ for } \pi\\ 1 \text{ for } \bar{K} \text{ and } R,\\ 2 \text{ for } D\end{cases}$$

$$\mathbf{t}=\mathbf{\theta},\ \tfrac{1}{2}\xi_\alpha^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\begin{cases}0 \text{ for } \pi\\ \text{isospin}-\tfrac{1}{2} \text{ vector for } \bar{K} \text{ and } R;\\ 0 \text{ for } D\end{cases}$$

for $P_1\ T_F=1$ excited mesons (if they exist), we have

$$J=1,\ \xi_\alpha^*\xi_\alpha=\begin{cases}0 \text{ for } \pi^*\\ 1 \text{ for } \bar{K}^* \text{ and } R^*,\\ 2 \text{ for } D^*\end{cases}$$

$$\mathbf{t}=\text{isospin}-1 \text{ vector},$$

$$\tfrac{1}{2}\xi_\alpha^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta=\begin{cases}0 \text{ for } \pi^*\\ \text{isospin}-\tfrac{1}{2} \text{ vector for } \bar{K}^* \text{ and } R^*.\\ 0 \text{ for } D^*\end{cases}$$

In both cases $t=1$.

If a conjecture is made that a formula analogous to (29) holds here also and that β^M and γ^M are approximately constants independent of $\mathbf{J}^2$ and $\mathbf{t}^2$ like β^B and γ^B, then one gets for mesons a mass formula corresponding to the Lee-Yang mass formula for baryons.

Possible $P_1\ T_F=1$ excited mesons without excitation of strangeness are listed in Table II. Masses are obtained as follows.

The excited meson π^* having $J=1$ and $T=1$ is tentatively identified with the $\pi\pi$ resonance discovered theoretically by Frazer and Fulco.[12] The mass of this resonance seems recently to be 47 M_π.[13,14] Further, it is natural to interpret tentatively the excited meson $\bar{K}^*$ with $J=1$ and $T=\frac{1}{2}$ as the 880 Mev $\pi\bar{K}$ resonance. In this way we have in the formula for M^M four known masses M_π, M_K, M_{π^*}, and M_{K^*} and six unknown constants, $\alpha_0^M(0,2)$, $\alpha_0^M(2,2)$, $\alpha_1^M(0,2)$, $\alpha_1^M(2,2)$, β^M, γ^M. It is impossible, therefore, to determine all those constants without any additional assumption. The following argument suggests an approximation procedure.

Under the conjecture that $\alpha^{B,M}=\alpha_0^{B,M}+\alpha_1^{B,M}|S|$ and that $\beta^{B,M}$ and $\gamma^{B,M}$ are nearly constants, the mass operator for elementary and excited baryons and mesons has the form

$$\begin{aligned}M^{B,M}=\alpha_0^{B,M}(J(J+1),\ T_F(T_F+1))\\ +\alpha_1^{B,M}(J(J+1),\ T_F(T_F+1))|S|\\ +\beta^{B,M}(T(T+1))+\gamma^{B,M}(\tfrac{1}{2}\xi_\alpha^*\boldsymbol{\tau}_{\alpha\beta}\xi_\beta)^2,\end{aligned} \quad (36)$$

where in the case of baryons the constants are determined as follows:

$$\begin{aligned}\alpha_0^B(\tfrac{3}{4},\tfrac{3}{4})&=M_N-\tfrac{3}{4}(M_\Sigma-M_\Lambda)\approx 882 \text{ Mev},\\ \alpha_0^B(15/4,15/4)&=M_{N^*}-\tfrac{3}{4}(M_\Sigma-M_\Lambda)\approx 1328 \text{ Mev},\\ \alpha_1^B(\tfrac{3}{4},\tfrac{3}{4})&=\tfrac{1}{2}(M_\Xi-M_N)\approx 190 \text{ Mev},\\ \alpha_1^B(15/4,15/4)&=\alpha_1^B(\tfrac{3}{4},\tfrac{3}{4})+(M_{\Lambda^*}-M_\Lambda)\\ &\quad-(M_{N^*}-M_N)+\beta^B\approx 200 \text{ Mev},\\ \beta^B&=\tfrac{1}{2}(M_\Sigma-M_\Lambda)\approx 38 \text{ Mev},\\ \gamma^B&=\frac{4}{3}\left(\frac{5M_\Lambda+3M_\Sigma}{8}-\frac{M_N+M_\Xi}{2}\right)\approx 20 \text{ Mev}.\end{aligned} \quad (37)$$

We can see from (37) that the terms in M^B containing β^B and γ^B can be treated as small corrections to the remaining terms in (36). Assuming that the same is true for mesons, we can neglect in the first approximation the corresponding small terms in M^M or equate them to appropriate terms in M^B. The latter approximation seems to be more adequate, since it does not change some qualitative features of the mass spectrum. Formula (36) for M^M with $\beta^M=\beta^B$ and $\gamma^M=\gamma^B$ was just used for a rough estimate of M_R, M_D, M_{R^*}, and M_{D^*} given in Tables I and II. Four constants in M^M, besides β^B and γ^B, are determined by four masses M_π, M_K, M_{π^*}, and M_{K^*} as follows:

$$\begin{aligned}\alpha_0^M(0,2)&=M_\pi-2\beta^B\approx 65 \text{ Mev},\\ \alpha_0^M(2,2)&=M_{\pi^*}-2\beta^B\approx 585 \text{ Mev},\\ \alpha_1^M(0,2)&=M_K-M_\pi+(5/4)\beta^B-\tfrac{3}{4}\gamma^B\approx 386 \text{ Mev},\\ \alpha_1^M(2,2)&=M_{K^*}-M_{\pi^*}+(5/4)\beta^B-\tfrac{3}{4}\gamma^B\approx 250 \text{ Mev}.\end{aligned} \quad (38)$$

We can see that $M_D>M_\Xi-M_N$. This is consistent with the stability of Ξ in strong interactions. Note that M_D as estimated is very close to the nucleon mass M_N. On the other hand, $M_D<2M_K$ and, of course, $M_D<2M_K+M_\pi$. It guarantees the stability of D in strong interactions. Further, $M_R<M_K+M_\pi$ and $M_R<M_K+2M_\pi$. Hence R is also stable in strong interactions. Singly-strange mesons R^0 and R^- would, therefore, decay by electrodynamic process $R\to\bar{K}+\gamma$, whereas R^+ and R^{--} and doubly-strange mesons D would decay only by weak processes, e.g., $R\to 2\pi$ and $D\to\bar{K}+\pi$. Similarly, for antiparticles $\bar{R}^0$ and $\bar{R}^-$ we would have $\bar{R}\to K+\gamma$, whereas for $\bar{R}^+$ and $\bar{R}^{--}$ and $\bar{\rho}$ we would have only weak processes.

All the results concerning the hypothetical R and ρ mesons are valid if our estimation of masses is not completely wrong and, of course, if the extension of the formalism to mesons is correct. Especially, if $M_R>M_K+2M_\pi$, R is not stable in strong interactions because of $R\to\bar{K}+2\pi$. The considerations about π^*, K^*, R^*, and D^* are reasonable only under the additional assumption that $P_1\ T_F=1$ excited mesons exist, and that π^* and $\bar{K}^*$ correspond to the experimental $\pi\pi$ and $\pi\bar{K}$ resonances, respectively. The existence of excited

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mesons is, from the theoretical point of view, a dynamical problem depending on the symmetries of the strong interactions. Note that the excited mesons π^* and $\bar{K}^*$ would be precisely some of the vector mesons postulated as elementary particles by certain recent theories.[15,16] Excited mesons other than P_1 $T_F=1$ may *a priori* also exist, e.g., vector mesons P_1 $T_F=0$. The elementary meson $J=0$, $T=0$, denoted usually by π_0^0, is, on the contrary, excluded by the present formalism.

IV. UNIVERSAL YUKAWA STRONG INTERACTION

In this section we should like to consider the possibility that Yukawa strong interactions between baryons and mesons π, K, R, and D follow from a "universal Yukawa strong interaction" (UYSI), which establishes a symmetry between four of those Yukawa strong interactions. We will propose the UYSI in a form which shall contain baryon-pion and baryon-K-meson interactions given by (15) and (21). It will relate the functions $g_{F,S}{}^{\pi}(\lambda)$ and $g_{F,S}{}^{K}(\lambda)$.

To this end we introduce the notation

$$\xi_{(\alpha)}=\begin{cases}1\\ \xi_\alpha\\ (1/\sqrt{2})\xi_{\alpha_1}\xi_{\alpha_2}\end{cases},\quad M_{i(\alpha)}=\begin{cases}M_i\\ M_{i\alpha}\\ M_{i\alpha_1\alpha_2}\end{cases},\tag{39}$$

and write tentatively the following interaction:

$$H^{BM}=\tfrac{1}{2}i\bar{B}\gamma_5\{g_F(\xi_\alpha{}^*\xi_\alpha)\tau_i\xi_{(\alpha)}{}^*+g_S(\xi_\alpha{}^*\xi_\alpha)\times[\xi_\gamma{}^*\tau_{i\gamma\delta}\xi_\delta,\xi_{(\alpha)}{}^*]\}BM_{i(\alpha)}+\text{H.c.},\tag{40}$$

where $g_{F,S}(\lambda)$ are arbitrary real functions. The summation index (α) runs here over the three possibilities indicated in (39).

This Yukawa strong interaction (without derivatives) obeys the IEP given by (B) and (M), and has the following symmetries higher than the charge independence (CI), i.e., an invariance under transformations generated by $\mathbf{T}=\mathbf{T}^B+\mathbf{T}^M$.

(i) The "strangeness independence" (SI) defined as an invariance under rotations in the "intrinsic configurational space." This space is by definition a linear space spanned on basis vectors:

$$|(\alpha)\rangle=\xi_{(\alpha)}{}^*|0\rangle\quad\text{or}\quad\begin{cases}|0\rangle\\ |\alpha\rangle=\xi_\alpha{}^*|0\rangle\\ |\alpha_1\alpha_2\rangle=(1/\sqrt{2})\xi_{\alpha_2}{}^*\xi_{\alpha_1}{}^*|0\rangle\end{cases},\tag{41}$$

where $|0\rangle$ is the "vacuum" vector in the algebra given by (3).

(ii) The interactions $H^{B\pi}$ and H^{BD} following from (40) have the DS, i.e., an invariance under transformations generated by $\mathbf{T}_F{}^B+\mathbf{T}^\pi+\mathbf{T}^D$. If $g_S(\lambda)=0$, the whole interaction (40) has the GDS defined as an invariance under transformations generated by

$$\mathbf{T}_F=\mathbf{T}_F{}^B+\mathbf{T}_F{}^M=\mathbf{T}_F{}^B+\mathbf{T}^\pi+\mathbf{T}_F{}^K+\mathbf{T}_F{}^R+\mathbf{T}^D.$$

The first part of statement (ii) follows from a calculation, which leads to the formula

$$H^{BM}=H^{B\pi}+H^{BK}+H^{BR}+H^{BD},\tag{42}$$

where $H^{B\pi}$ is given by (19) with $g_F{}^\pi(\lambda)=g_F(\lambda)$ and

$$\begin{aligned}H^{BD}&=\tfrac{1}{2}i\bar{B}\gamma_5g_F(\xi_\alpha{}^*\xi_\alpha)\tau_i(1/\sqrt{2})\xi_{\alpha_2}{}^*\xi_{\alpha_1}{}^*BM_{i\alpha_1\alpha_2}+\text{H.c.}\\&=\tfrac{1}{2}ig_F(2)\bar{\Xi}\gamma_5\boldsymbol{\tau}N\cdot\mathbf{D}+\text{H.c.}\end{aligned}\tag{43}$$

The interaction H^{BK} is here given by (21) with $g_{F,S}{}^K(\lambda)=g_{F,S}(\lambda)$. If $g_S(\lambda)=0$, we get the formula

$$H^{BK}+H^{BR}=\tfrac{1}{2}i\bar{B}\gamma_5g_F(\xi_\alpha{}^*\xi_\alpha)\tau_i\xi_\alpha{}^*BM_{i\alpha}+\text{H.c.},\tag{44}$$

which demonstrates the GDS for interaction $H^{BK}+H^{BR}$.

As it was mentioned in Sec. I. there is an experimental support for $g_F(\lambda)\approx 3g_S(\lambda)$ rather than for $g_S(\lambda)=0$. If the relation $g_F(\lambda)\approx 3g_S(\lambda)$ is true and $g_F(\lambda)=\text{const}$, we obtain from (18) and (24) [where, in the case of (40), $g_F{}^\pi(\lambda)=g_F{}^K(\lambda)=g_F(\lambda)$, $g_S{}^\pi(\lambda)=0$, and $g_S{}^K(\lambda)=g_S(\lambda)$] the following formulas:

$$g^{N\Lambda K}\approx -g^{N\Sigma K}\approx\frac{1}{\sqrt{6}}g^\pi,\quad g^{\Xi\Lambda K}\approx-\frac{2}{\sqrt{6}}g^\pi,\quad |g^{\Xi\Lambda K}|\gg|g^{\Xi\Sigma K}|.\tag{45}$$

Here $g^\pi\equiv g^{N\pi}=g^{Y\pi}=g^{\Xi\pi}$.

If the GDS were true for (40), i.e., if $g_S(\lambda)=0$, then the K and R interactions taken together, $H^{BK}+H^{BR}$, would not destroy the DS of π and D interactions, $H^{B\pi}$ and H^{BD}. Hence, processes without real K and R mesons would display the DS, provided the mass differences between Λ and Σ as well as K and R would be neglected. On the other hand, processes with real K or R mesons would violate the DS. Thus, we would have in this case the situation desired for K mesons by Pais.[8] If $g_F(\lambda)=\text{const}$, we would have here

$$\begin{aligned}g^{N\Lambda K}&=\frac{3}{2\sqrt{6}}g^\pi,\quad g^{N\Sigma K}=-\frac{1}{2\sqrt{6}}g^\pi,\\ g^{\Xi\Lambda K}&=-\frac{3}{2\sqrt{6}}g^\pi,\quad g^{\Xi\Sigma K}=\frac{1}{2\sqrt{6}}g^\pi.\end{aligned}\tag{46}$$

If $|g_F(\lambda)|>3|g_S(\lambda)|$, the second term in (40) violating the GDS is smaller than the first one preserving the GDS, and we may have here the former situation as an approximation. If $\frac{1}{3}g_F(\lambda)>g_S(\lambda)\geqslant 0$ and $g_F(\lambda)=\text{const}$, $g_S(\lambda)=\text{const}$, we obtain

$$\begin{aligned}&\frac{1}{\sqrt{6}}g^\pi<g^{N\Lambda K}\leqslant\frac{3}{2\sqrt{6}}g^\pi,\quad \frac{1}{\sqrt{6}}g^\pi>-g^{N\Sigma K}\geqslant\frac{1}{2\sqrt{6}}g^\pi,\\ &\frac{2}{\sqrt{6}}g^\pi>-g^{\Xi\Lambda K}\geqslant\frac{3}{2\sqrt{6}}g^\pi,\quad 0<g^{\Xi\Sigma K}\leqslant\frac{1}{2\sqrt{6}}g^\pi.\end{aligned}\tag{47}$$

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The baryon–K-meson coupling constants are here smaller than the baryon-pion coupling constant by reasonable factors.

CONCLUDING REMARK

The formalism described in Secs. I and II is a quite general concise formulation of the usual baryon-pion and baryon-kaon interactions, provided the property (B) is true.

One may say that the experimentally suggested property (B) can be *understood* in terms of this formalism. The extension of the formalism to mesons, discussed in Secs. III and IV, leads to the property (M), which may be verified only by experiment. If both (B) and (M) were true, the formalism presented would be a general tool to write down and discuss all baryon-meson interactions. The assumption of equal parities and spins of all elementary baryons and all elementary mesons plays an essential role in the presented formulation of the formalism.

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Regge Poles in π-π Scattering*

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The connection between Regge poles, bound states and resonances, and asymptotic behavior in momentum transfer is reviewed within the framework of the analytically continued S matrix, and a convergent iteration procedure is given for calculating the position and residue of a Regge pole in terms of a given (generalized) potential. By examining the long-range potential in the $\pi\pi$ system, it is inferred that Regge poles should appear in the $I=0$ and $I=1$ states, and that the latter pole may be responsible for the ρ meson while the former may well dominate *high-energy* behavior at low-momentum transfer in the crossed channels. The connection of this possibility with forward coherent (diffraction) scattering in general is explored, and a number of experimental predictions are emphasized. Finally it is shown that the short-range forces due to exchange of 4, 6, $\cdots$ pions are likely to be repulsive and must be included in some form if a consistent solution is to be achieved.

I. INTRODUCTION

In the S-matrix theory of strong interactions, dynamical resonances and bound states have been easily and naturally handled insofar as partial-wave (one-variable) dispersion relations are concerned, but they have been a source of confusion with respect to double-dispersion relations. Froissart[1] showed that partial waves with $J>1$ are completely determined by the double-spectral functions; at the same time, as emphasized in the original paper by Mandelstam,[2] resonances or bound states require subtractions in the double-spectral integrals if the usual convergence criteria are applied. The resolution of this dilemma was given by Regge for nonrelativistic potential scattering, where in fact all partial waves are determined by the double-spectral function (even though in the absence of a "crossed" channel, the considerations of Froissart are inapplicable). Regge's explanation is based on the occurrence of poles in the complex angular momentum plane and the association of such poles with resonances and bound states.[3]

The point at issue is essentially the asymptotic behavior of the scattering amplitude as $\cos\theta$ approaches infinity and the energy is kept fixed. This is a highly unphysical region but, as it is here that the double spectral function fails to vanish, the question is of interest to us. The number of subtractions in $\cos\theta$ which it is necessary to perform depends on the asymptotic behavior. As subtraction terms in $\cos\theta$ are just polynomials in this variable, they correspond to low partial waves, so that the number of partial waves which are undetermined by the double-spectral function depends on the number of subtractions necessary.

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