K^* and K Meson Pole Contributions to Leptonic Λ Decay*

RICHARD E. NORTON
University of California, Los Angeles, California

(Received September 18, 1961; revised manuscript received January 17, 1962)

An attempt is made to calculate the leptonic Λ decay rate on the basis of the known K_{e3} and $K_{\mu 2}$ decay rates. Unsubtracted dispersion relations are written for the relevant matrix elements of the strangeness-changing vector weak current and the absorptive parts are approximated by retaining only the contribution from the K^*. In this way the vector current contribution to the Λ decay is related to the known K_{e3} rate, the known effective π-K-K^* coupling constant, and the as yet unknown Λ-p-K^* coupling constant. The latter coupling constant will hopefully be determined soon from extrapolation of the associated production data. The axial vector contribution to Λ decay is related to the $K_{\mu 2}$ decay rate by writing unsubtracted dispersion relations for the divergence of this current and approximating the absorptive parts by keeping only the contribution of the K meson. The numerical consequences of this calculation can only be evaluated once the Λ-p-K^* is determined, but a crude estimate suggested by "unitary symmetry" yields a leptonic Λ decay rate a few times smaller than is now suggested experimentally.

I. INTRODUCTION

IN a series of publications[1,2] a method was discussed whereby the $\pi \to \mu + \bar{\nu}$ decay rate could be determined from the known axial vector coupling constant in β decay, G_A, and the renormalized pion-nucleon coupling constant, g_π. The relation obtained there[2] for the π decay rate is (M_N = nucleon mass)

$$\Gamma_\pi = \frac{G_A^2}{16\pi}\left(\frac{2M_N}{g_\pi}\right)^2 m_\mu^2 m_\pi \left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2, \tag{1}$$

and this yields

$$\Gamma_\pi(\text{calc}) = 3.7\times 10^7 \text{ sec}^{-1}$$

compared to the experimental value

$$\Gamma_\pi(\text{exp}) = 3.9\times 10^7 \text{ sec}^{-1}.$$

The expression for Γ_π in Eq. (1) was also obtained by Goldberger and Treiman[3] by a different technique, and it was in fact the questionable validity of the approximations employed in this work that motivated the attempt in reference 2 for a more plausible derivation of this successful relationship.

In references 1 and 2 the expression for Γ_π in Eq. (1) follows from two assumptions: first, that unsubtracted dispersion relations are satisfied for the divergence of the axial vector β-decay matrix element; and second, that the one pion contribution to the absorptive part dominates at low momentum transfer. The extremely good agreement with experiment that follows then serves as strong evidence that these assumptions are valid.

On the basis of this success one is led to attempt a similar calculation relating the amplitude for $K \to \mu + \bar{\nu}$ and the axial vector contribution to the leptonic Λ decay, $\Lambda \to p + e + \bar{\nu}$. As has already been noted,[1] however, such a comparison yields an extremely small value for the strangeness-changing pseudovector coupling constant—at least an order of magnitude less than the universal weak-coupling constant effective in nuclear β decay. The axial vector contribution to the leptonic Λ decay would then be more than one hundred times smaller than that implied by a universal weak interaction. On the other hand, the rate $\Lambda \to p + e + \bar{\nu}$ seems to be no less than one tenth[4] that which follows from universality. It must be concluded, therefore, that either the procedure so successful in comparing pion decay to nuclear β decay cannot be extended to these strangeness-changing processes, or that the leptonic Λ decay proceeds predominantly through the vector interaction.

It is the purpose of this paper to investigate the latter possibility. In particular, we attempt to relate the decay $K^- \to \pi^0 + e + \bar{\nu}$ (which proceeds only through the vector, strangeness-changing current)[5] to the effective vector coupling constant operative in $\Lambda \to p + e + \bar{\nu}$. We write unsubtracted dispersion relations for both the K_{e3} and $\Lambda \to p + e + \bar{\nu}$ matrix elements of the strangeness changing vector current and then approximate the absorptive parts of these relations by retaining only the contributions from the K^* "pole." With these approximations, the strangeness-changing vector coupling constant can then be obtained in terms of the $K^- \to \pi^0 + e + \bar{\nu}$ decay rate and the ratio of coupling constants $\gamma_{\Lambda p K^*}/\gamma_{\pi K K^*}$, where $\gamma_{\pi K K^*}$ and $\gamma_{\Lambda p K^*}$ are the effective coupling strengths for the $\pi K K^*$ and $\Lambda p K^*$ interactions, respectively. The former of these coupling constants can

* Supported in part by the National Science Foundation and the Air Force Office of Scientific Research, Physical Sciences Directorate.

[1] Y. Nambu, Phys. Rev. Letters, 4, 380 (1960).

[2] M. Gell-Mann and M. Levy, Nuovo cimento 16, 705 (1960); J. Bernstein, M. Gell-Mann, and L. Michel, Nuovo cimento 16, 560 (1960); and J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo cimento 17, 757 (1960).

[3] M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958); see also M. L. Goldberger, Rev. Mod. Phys. 31, 797 (1959).

[4] W. Humphrey, J. Kirz, A. Rosenfeld, J. Leitner, and Y. I. Rhee, Phys. Rev. Letters 6, 478 (1961).

[5] Of course, it is the axial vector current which contributes to the K_{e3} decay if the K-π relative parity is odd. To simplify the discussion, all the remarks in Sec. I are made on the assumption of an odd Λ-nucleon-K parity (i.e., even K-π parity providing the Λ-nucleon parity is defined to be even). Secs. II and III contain a discussion of how the results are altered in the case that the K is scalar.

be obtained from the known linewidth of the K^* resonance.[6] It is hoped the latter will soon be determined from extrapolation of the associated production data $\pi^- + p \to K^0 + \Lambda$ to the K^* "pole."

Since this extrapolation has not as yet been accomplished, we cannot give here the numerical consequences of our calculation. One can, however, make a crude guess and set the coupling constant ratio equal to unity.[7] In this case a vector strangeness changing weak coupling results for the leptonic Λ decay which is about one fourth that which would be obtained by a universal interaction [see Eq. (15) in Sec. II]. Although this result is probably too small, it is roughly in the correct range. A more definitive conclusion regarding the usefulness of the calculation presented here must await a determination of $\gamma_{\Lambda p K^*}$ from the data on associated production.

There are of course some questions concerning the theoretical validity of the approximations employed. Perhaps foremost in the minds of some readers is a doubt concerning the applicability of unsubtracted dispersion relations. It has already been observed,[8] for example, that the high-momentum behavior in perturbation theory implies a need for subtractions in some form factors of the weak vector current. On the other hand, such an argument cannot be considered conclusive. It is an attractive possibility that subtractions are never needed.

An interesting, although perhaps not very convincing, argument for no subtractions was presented in reference 2. The possibility was suggested that the β-decay matrix element of the divergence of the vector weak current is a finite multiple of the same matrix element of the pion field operator. That this matrix element goes to zero for infinite momentum transfer then becomes a necessary requirement for the existence of the pion propagator.

In the climate of today, when serious doubts have arisen concerning concepts based upon conventional field theory, such an argument is not very effective. We only point out that analagous remarks can be made in support of the unsubtracted dispersion relations employed here. In particular, if the matrix elements of the vector strangness-changing weak current and of the "field operator" for the K^* (we assume here, as in the remainder of this paper, that the K^* has spin one)[9] are finite multiples of each other, then the existence of the K^* propagator implies the vanishing of the weak current matrix elements at infinite momentum transfer. In any case, even if these arguments lend no support whatever, we feel that in lieu of a conclusive argument to the contrary, the unsubtracted dispersion relations employed here should be investigated for their physical implications.

Concerning the reliability of retaining only the K^* pole terms in the absorptive parts of the dispersion relations, we feel that such a procedure should represent quite well the contribution from the K-π intermediate state in the low mass region. Since there are no contributing intermediate states of mass less than $m_K + m_\pi$, it seems reasonable to hope that the effect of the K^* dominates at low momentum transfer.

In Sec. II the essential features of the calculation are presented, and in Sec. III the results summarized.

II. THE CALCULATION

For the decay processes of interest, $\Lambda \to p + e + \bar{\nu}$, $K^- \to \pi^0 + e + \bar{\nu}$, and $K^- \to \mu + \bar{\nu}$, we assume the effective part of the weak-interaction Hamiltonian H_W to have the form[10]

$$H_W = \frac{G}{\sqrt{2}} \int d^3x \, [s_\mu{}^V(x) + s_\mu{}^A(x)][l_\mu{}^V(x) + l_\mu{}^A(x)] + \text{H.c.}, \tag{2}$$

where $s_\mu{}^V$ ($s_\mu{}^A$) is the vector (axial vector) strangeness-changing current, and $l_\mu{}^V$ and $l_\mu{}^A$ are the corresponding leptonic currents,

$$l_\mu{}^V + l_\mu{}^A = \bar{\nu}\gamma_\mu(1+\gamma_5)e + \bar{\nu}\gamma_\mu(1+\gamma_5)\mu. \tag{3}$$

The weak-coupling constant G equals $1.01 \times 10^{-5} M_N{}^{-2}$. The leptonic currents in Eq. (3) can be treated as free fields (ignoring electromagnetic corrections and effects higher order in G) to trivially eliminate all reference to leptons in the matrix elements of H_W.

Some of the results depend upon the choice of parity assignments. As has become conventional, we define the Λ-nucleon relative parity to be even, and then examine separately the consequences implied by both a pseudoscalar and scalar K meson. We outline the calculations and state the results first assuming a pseudoscalar K meson (i.e., even K-π parity) and then it will be a simple matter to indicate how the results are altered if the K is scalar.

After elimination of the leptonic wave functions and explicit numerical factors appearing in H_W, the transition amplitudes for the two decays $\Lambda \to p + e + \bar{\nu}$ and $K^- \to \pi^0 + e + \bar{\nu}$ are determined by the three matrix

[6] M. Alston, L. Alvarez, P. Eberhard, M. Good, W. Graziano, H. Ticho, and S. Wojcicki, Phys. Rev. Letters **6**, 300 (1961).

[7] M. Gell-Mann, California Institute of Technology Synchotron Laboratory Report CTSL-20, 1961 (unpublished). The model of strong-interaction symmetry discussed in this work suggests that the coupling constant ratio $\gamma_{\Lambda p K^*}/\gamma_{\pi K K^*}$ is in the neighborhood of unity.

[8] S. W. MacDowell, Phys. Rev. **116**, 1047 (1959).

[9] A procedure analogous to that employed here could be followed for the case of a spin-zero K^*; except then, by comparing with the treatment of the axial vector currents, it would appear more appropriate to write unsubtracted dispersion relations for the divergence of the vector current. See, for example, S. Weinberg and J. Bernstein, Phys. Rev. Letters **5**, 481 (1960).

[10] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

elements ($k\equiv p_\Lambda - p_p$ and $q\equiv p_K - p_\pi$):

$$\langle p_pS'|s_\mu{}^V(0)|p_\Lambda S\rangle=\frac{1}{(2\pi)^3}\left(\frac{M_pM_\Lambda}{E_pE_\Lambda}\right)^{\frac{1}{2}}\bar{u}(p_pS')\left[\gamma_\mu\frac{S_V}{G}F_1(k^2)+i\sigma_{\mu\nu}k_\nu F_2(k^2)+k_\mu F_3(k_j)\right]u(p_\Lambda S), \tag{4}$$

$$\langle p_pS'|s_\mu{}^A(0)|p_\Lambda S\rangle=\frac{1}{(2\pi)^3}\left(\frac{M_pM_\Lambda{}^{\frac{1}{2}}}{E_pE_\Lambda}\right)\bar{u}(p_pS')\left[\gamma_\mu\gamma_5\frac{S_A}{G}G_1(k^2)+i\sigma_{\mu\nu}k_\nu\gamma_5G_2(k^2)+k_\mu\gamma_5G_3(k^2)\right]u(p_\Lambda S), \tag{5}$$

and

$$\langle p_\pi|s_\mu{}^V(0)|p_K\rangle=\frac{(2\pi)^{-3}}{(4\omega_K\omega_\pi)^{\frac{1}{2}}}[f_1(q^2)p_{K\mu}+f_2(q^2)q_\mu]. \tag{6}$$

The right-hand sides of these equations, expressed in terms of the invariant functions $f_i(q^2)$, $F_j(k^2)$, and $G_k(k^2)$, are the most general forms consistent with the requirements of invariance. $F_1(0)$ and $G_1(0)$ are defined to be unity so that S_V and S_A are the effective vector and axial vector coupling constants which determine the rate of leptonic Λ decay.

As discussed in Sec. I the procedure is to now write unsubtracted dispersion relations for all the form factors $F_i(k^2)$ and $f_j(q^2)$ appearing in Eqs. (4) and (6); for example,

$$f_i(q^2)=\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{\mathrm{Im}f_i(-m^2)dm^2}{q^2+m^2-i\epsilon}, \tag{7}$$

and then to approximate the imaginary parts by including only the contributions from the K^* "pole." For simplicity in carrying out the calculations, we treat the K^* as if it were stable and as if there existed an eigenstate $|p_{K^*}\lambda\rangle$ of the energy momentum operator describing a K^* of momentum p_{K^*} and polarization λ. Such a procedure is of course not correct, nor necessary,[8] but because of the small K^* line width it must yield essentially correct results for the range of momentum transfer which contribute to the decay processes considered here.

The problem of evaluating the explicit contribution of the K^* intermediate state is standard but entails a considerable amount of algebra to account for the kinematics of a vector particle. These steps will not be repeated here, but rather we write the results immediately:

$$\frac{S_V}{G}F_1(k^2)=\frac{\sqrt{3}c\gamma_{\Lambda pK^*}}{k^2+M_{K^*}{}^2},\quad F_2(k^2)=\frac{cR_2}{k^2+M_{K^*}{}^2},$$
$$F_3(k^2)=-\frac{\sqrt{3}c\gamma_{\Lambda pK^*}(M_\Lambda-M_p)}{M_{K^*}{}^2(k^2+M_{K^*}{}^2)}, \tag{8}$$

and

$$f_1(q^2)=-\frac{2c\gamma_{\pi KK^*}}{q^2+M_{K^*}{}^2},$$
$$f_2(q^2)=\frac{c\gamma_{\pi KK^*}(M_{K^*}{}^2+m_K{}^2-m_\pi{}^2)}{M_{K^*}{}^2(q^2+M_{K^*}{}^2)}. \tag{9}$$

The as yet undefined constants appearing in Eqs. (8) and (9) are determined from the following relations:

$$\langle 0|s_\mu{}^V(0)|p_{K^*}\lambda\rangle=\frac{c}{(2\pi)^{\frac{3}{2}}(2\omega_{K^*})^{\frac{1}{2}}}\epsilon_\mu{}^\lambda(p_{K^*}), \tag{10}$$

$$\langle p_{K^*}\lambda|J_p(0)|p_\Lambda S\rangle$$
$$=\frac{1}{(2\pi)^3}\left(\frac{M_\Lambda}{2\omega_{K^*}E_\Lambda}\right)^{\frac{1}{2}}\epsilon_\mu{}^\lambda(p_{K^*})$$
$$\times[\gamma_\mu\sqrt{3}\gamma_{\Lambda pK^*}+i\sigma_{\mu\nu}p_{K^*\nu}R_2+p_{K^*\mu}R_3]u(p_\Lambda S), \tag{11}$$

$$\langle p_{K^*}\lambda|J_\pi(0)|p_K\rangle=\frac{2\epsilon^\lambda(p_{K^*})\cdot p_K\gamma_{\pi KK^*}}{(2\pi)^3(4\omega_K\omega_{K^*})^{\frac{1}{2}}}, \tag{12}$$

where, in Eqs. (11) and (12), the matrix elements are to be evaluated at the momentum transfers $(p_\Lambda-p_{K^*})^2=-M_p{}^2$ and $(p_K-p_{K^*})^2=-m_\pi{}^2$, respectively. The current operators $J_p(0)$ and $J_\pi(0)$ are the source currents for the renormalized proton and pion fields, $\psi_p{}^R(x)$ and $\phi_\pi{}^R(x)$:

$$(-i\gamma_\mu\partial_\mu+M_p)\psi_p{}^R(x)\equiv J_p(x),$$
$$(\square^2-m_\pi{}^2)\phi_\pi{}^R(x)\equiv J_\pi(x). \tag{13}$$

In computing the decay rate $K^-\to\pi^0+e+\bar{\nu}$, the second form factor f_2 appearing in Eqs. (6) and (9) can be completely ignored since its relative contribution is $(m_e/m_K)^2\sim 10^{-6}$ that of $f_1(q^2)$. By appealing to the known rate of K_{e3} decay (3.4×10^6 sec^{-1}), and the expression for $f_1(q^2)$ in Eq. (9), we can then determine the constant $|2c\gamma_{\pi KK^*}M_{K^*}{}^{-2}|$ with the result

$$|2c\gamma_{\pi KK^*}/M_{K^*}{}^2|=0.097. \tag{14}$$

The coupling constant S_V in Eqs. (4) and (8) then becomes

$$\frac{S_V}{G}=\frac{\gamma_{\Lambda pK^*}}{\gamma_{\pi KK^*}}\times 0.26,\quad (K\text{ pseudoscalar}). \tag{15}$$

We now turn our attention to the axial vector coupling constant S_A appearing in Eq. (5). As in references 1, 2

we take the divergence of Eq. (5):

$$\langle p_p S'|-i\partial_\mu s_\mu{}^A(0)|p_\Lambda S\rangle$$
$$=\frac{1}{(2\pi)^3}\left(\frac{M_pM_\Lambda}{E_pE_\Lambda}\right)^{\frac{1}{2}}\bar{u}(p_pS')\gamma_5 u(p_\Lambda S)$$
$$\times\left[(M_\Lambda+M_p)\frac{S_A}{G}G_1(k^2)+k^2G_3(k^2)\equiv D(k^2)\right], \quad (16)$$

write unsubtracted dispersion relations for $D(k^2)$, and approximate the absorptive part by including only the contribution from the K-meson pole. The result of such a procedure is

$$D(k^2)=(M_\Lambda+M_p)\frac{S_A}{G}G_1(k^2)+k^2G_3(k^2)=\frac{g_Kc'M_{K^*}{}^2}{k^2+M_{K^*}{}^2}, \quad (17)$$

where

$$\langle 0|s_\mu{}^A(0)|p_K\rangle=\frac{ic'}{(2\pi)^{\frac{3}{2}}(2\omega_K)^{\frac{1}{2}}}p_{K\mu}, \quad (18)$$

and g_K is the K-Λ-nucleon coupling constant defined by

$$\langle p_K|J_p(0)|p_\Lambda S\rangle=\frac{-ig_K}{(2\pi)^3}\left(\frac{M_\Lambda}{2\omega_KE_\Lambda}\right)^{\frac{1}{2}}\gamma_5u(p_\Lambda S) \quad (19)$$

evaluated at a momentum transfer $(p_\Lambda-p_K)^2=-M_p{}^2$.

The matrix element appearing in Eq. (18) completely determines the decay $K^-\rightarrow\mu+\bar{\nu}$,

$$\Gamma_{K\rightarrow\mu\bar{\nu}}=\frac{G^2c'^2}{8\pi}\frac{m_\mu{}^2}{m_K{}^3}(m_K{}^2-m_\mu{}^2)^2, \quad (20)$$

and therefore the constant c' can be determined from the known decay rate for this process (4.8×10^7 sec^{-1}). If this result, together with a value of g_K[11] ($g_K{}^2/4\pi=2.2$), are substituted into Eq. (17), the axial vector coupling constant S_A is determined to be

$$S_A/G=0.089, \quad (K\text{ pseudoscalar}). \quad (21)$$

We have assumed in these calculations that the K is pseudoscalar. On the other hand if the K has even parity, $K^-\rightarrow\pi^0+e+\bar{\nu}$ proceeds through $s_\mu{}^A$, instead of $s_\mu{}^V$, and this decay should be related to the axial vector coupling constant effective in the leptonic Λ decay. Rather than Eq. (15), we then have

$$S_A/G=\gamma_{\Lambda pK^*}/\gamma_{\pi KK^*}\times0.26, \quad (K\text{ scalar}), \quad (22)$$

and of course for this case the K^* must be considered a pseudovector. For the determination of S_V when the K is assumed to be scalar, we follow the procedure discussed above for the calculation of S_A. Eqs. (16)–(21) should be altered in this case by replacing $s_\mu{}^A$, S_A, and $G_i(k^2)$ by $s_\mu{}^V$, S_V, and the $F_i(k^2)$ and also by removing the explicit appearance of all γ_5's. All these changes are essentially changes of notation. Effects of numerical consequence do arise however since, without the γ_5, the divergence in Eq. (16) brings in the factor (M_p-M_Λ) instead of (M_p+M_Λ). If this same replacement is made in Eq. (17), and if g_K appearing there is reinterpreted to describe the strength of a scalar Λ-nucleon-K interaction ($g_K{}^2/4\pi=0.063$),[11] then we obtain in analogy to Eq. (21)

$$\frac{S_V}{G}=0.089\left(\frac{M_\Lambda+M_p}{M_\Lambda-M_p}\right)^2\frac{0.063}{2.2}=0.18, \quad (K\text{ scalar}). \quad (23)$$

[11] B. McDaniel, A. Silverman, R. Wilson, G. Cortellessa, Phys. Rev. **115**, 1039 (1959).

III. CONCLUSIONS

Because of the small momentum transfer in the leptonic Λ decay, $-(M_\Lambda-M_p)^2\leq(p_\Lambda-p_p)^2\leq-m_e{}^2$, it is very reasonable that the dominant contributions to this process arise from the "charge" from factors $F_1(k^2)$ and $G_1(k^2)$ in Eqs. (4) and (5); and that throughout this range these form factors do not vary appreciably from the value unity. Such an assumption is consistent with the results of our calculation expressed in Eqs. (8) and (17). The decay rate $\Lambda\rightarrow p+e+\bar{\nu}$ can then easily be computed in terms of the two coupling constants S_V and S_A, and due to the low momentum transfers involved the contribution of $S_A{}^2$ is roughly three times that of $S_V{}^2$. We can then define an effective strangeness-changing weak-coupling constant S_W,

$$S_W{}^2\equiv(S_V{}^2+3S_A{}^2)/4, \quad (24)$$

which is a convenient measure for comparing the Λ and nuclear β decays. The experimental evidence[4] presently seems to indicate a value of $S_W{}^2/G^2\approx\frac{1}{8}$.

From Eqs. (15) and (21) we obtain here

$$S_W{}^2/G^2=0.017(\gamma_{\Lambda pK^*}{}^2/\gamma_{\pi KK^*}{}^2)+0.006, \quad (K\text{ pseudoscalar}) \quad (25)$$

and from Eqs. (21) and (22)

$$S_W{}^2/G^2=0.008+0.052(\gamma_{\Lambda pK^*}{}^2/\gamma_{\pi KK^*}{}^2), \quad (K\text{ scalar}). \quad (26)$$

For either parity assignment of the K the consistency of these results with the present experimental situation requires that $\gamma_{\Lambda pK^*}$ be greater than $\gamma_{\pi KK^*}$; for a scalar K by a factor of about 2 and for a pseudoscalar K by a factor around 7. In the first case $\Lambda\rightarrow p+e+\bar{\nu}$ would proceed almost completely through the axial vector weak current, while a pseudoscalar K would require the vector contribution to dominate. These predictions will soon be amenable to experimental study. Finally we point out again the need for a determination of $\gamma_{\Lambda pK^*}$ from the associated production data. Such an evaluation will allow a definitive conclusion regarding these results.

ACKNOWLEDGMENTS

The author would like to express his gratitude for enlightening and stimulating conversations with Professor Robert Finkelstein, Professor Murray Gell-Mann, and Professor Luigi Radicati; and also with Dr. Herbert Fried. He also expresses thanks to Mrs. Judy Scott for considerable aid with the calculations.