Coupling of Angular Momenta in Two-Particle States in Deformed Even-Even Nuclei

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The recent suggestion that low-lying intrinsic states in deformed nuclei are basically two-particle excitations is considered in more detail. The result of the strong-coupling model that the two-particle states are doubly degenerate with spins $\Omega=|\Omega_1\pm\Omega_2|$ is re-emphasized. It is suggested on the basis of experimental evidence that the degeneracy of the two states of each configuration is removed largely by residual spin-dependent forces, analogous to the splitting observed in deformed odd-odd nuclei. Coupling rules are given for the lower-lying state of each configuration based on this assumption. As an example of the application of the coupling rules to actual nuclei the decay schemes of the two Ta^{178} isomers to levels in Hf^{178} are considered in detail.

I. INTRODUCTION

THIS paper is one of a series in which the intrinsic energy levels of deformed even-mass nuclei are considered. In the first paper of this series (hereafter referred to as GM),[1] the particle states and coupling rules obtaining in deformed odd-odd nuclei were discussed. In the second (hereafter referred to as G60),[2] beta- and gamma-ray transition rates between intrinsic two-particle states in deformed even-mass nuclei were analyzed and classified. The present paper considers in more detail the two-particle configurations in deformed even-even nuclei and the coupling rules for them. As an example of the application of the coupling scheme proposed, we discuss in detail the levels in Hf^{178} observed in the decay of the two Ta^{178} isomers.

Section II contains a discussion of the coupling scheme. In Sec. III the relation of this scheme to the recently proposed independent-quasi-particle nuclear models is discussed. In the remaining sections we consider how well the coupling scheme explains the levels in Hf^{178}. Section IV consists of a summary of the intrinsic single-particle levels in the mass region around $Z=72$ and $N=106$. The way in which these single-particle states are coupled to form the excitations in the even-even nucleus Hf^{178} according to the scheme of Sec. II is discussed in Sec. V. The experimental data on the decays of the two Ta^{178} isomers are discussed in Sec. VI. These data are compared to the proposed energy levels and discussed in Sec. VII.

II. COUPLING SCHEME

The strong-coupling model of Bohr and Mottelson[3] has been very successful in explaining the spectrum of rotational states and the K-selection and K-intensity rules observed in both even- and odd-mass deformed nuclei.[4] Nilsson calculated wave functions for single-particle states in a deformed potential well to fit within the framework of this model,[5] and these wave functions have been shown to fit the experimental data very well.[6,7] In deformed odd-odd nuclei the odd-proton and odd-neutron states are now known to be just the states observed in the adjacent odd-mass nuclei, and hence can also be described by the Nilsson wave functions.[1,8] The double degeneracy of the proton-neutron configurations is now known to be split by a residual spin-dependent force.[1,8] The low-lying excitations in deformed odd-mass and odd-odd nuclei have therefore been accounted for qualitatively in detail.

No clear-cut analysis and classification of the low-lying ($\lesssim$2.5 Mev) intrinsic excitations of deformed even-even nuclei has as yet been made. It is known experimentally that the strong-coupling model applies to these nuclei because rotational states based on the ground and excited states have been observed. There has also been a great deal of discussion about collective vibrational states in these nuclei.[9] Recently, in an analysis of the beta-decay transition rates between deformed odd-odd and even-even nuclei (G60), we demonstrated that the experimental transition rates could be accounted for if we made the assumption that the low-lying intrinisc states of deformed even-even nuclei were two-particle states. We assumed that both particles in these states are described by the Nilsson wave functions, in analogy to the two-particle states in odd-odd nuclei, except that the two-particles in the even-even nucleus are either both protons or both neutrons. In the conceptually simple model suggested, the

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[1] C. J. Gallagher and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).

[2] C. J. Gallagher, Nuclear Phys. **16**, 215 (1960).

[3] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **26**, No. 14 (1952); A. Bohr and B. R. Mottelson, *ibid.*, **30**, No. 1, (1955).

[4] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

[5] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

[6] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter **1**, No. 8 (1959).

[7] F. S. Stephens, Jr., F. Asaro, and I. Perlman, Phys. Rev. **113**, 212 (1959).

[8] I. Perlman, *Proceedings of the International Conference on Nuclear Structure*, Kingston (University of Toronto Press, Toronto, 1960). Chap. 6, p. 547.

[9] For recent reviews of experimental data, see R. K. Sheline, Revs. Modern Phys. **32**, 1 (1960); I. Marklund, B. van Nooijen, and Z. Grabowski, Nuclear Phys. **15**, 533 (1960); K. P. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960); for a quantitative discussion see D. R. Bes, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **33**, No. 2 (1961).

excited state two-particle wave functions are written

$$\Psi=\left(\frac{2I+1}{16\pi^2}\right)^{\frac{1}{2}}\{\mathfrak{D}_{MK}^{I}(\vartheta_i)\chi_{\Omega_1}{}^{j_1}\chi_{\Omega_2}{}^{j_2}+(-)^{I-j_1-j_2}\mathfrak{D}_{M-K}^{I}(\vartheta_i)\chi_{-\Omega_1}{}^{j_1}\chi_{-\Omega_2}{}^{j_2}\}\Phi_{00},$$

where the wave functions for the rotational, intrinsic, and vibrational modes of motion have the same interpretation as discussed earlier,[2,3] and where particles 1 and 2, characterized by their projections along the nuclear symmetry axis Ω_1 and Ω_2 are either both neutrons or both protons.

The condition of strong coupling imposes the restriction that the state is only doubly degenerate, corresponding to the parallel or antiparallel orientation of the spins of the two particles, i.e., $\Omega=|\Omega_1\pm\Omega_2|$.

The primary purpose of the present paper is to emphasize this interpretation of the low-lying excited states in deformed even-even nuclei. A secondary purpose is to point out that the degeneracy of the doublet appears to be split, in analogy to odd-odd nuclei, largely by spin-dependent residual interactions.

The state of each $|\Omega_1\pm\Omega_2|$ configuration, which we expect to lie lower as a result of the spin-dependent force, can be seen most clearly if we consider the Nilsson states in their asymptotic-limit representation, i.e., in the limit of very large cylindrical deformation of the nucleus. The Nilsson states can then be represented by the quantum numbers N, n_z, Λ, and Σ. N is the principal quantum number, n_z is the number of nodes along the z axis, Λ is the number of nodes in the x, y plane, or the angular momentum along the symmetry axis, and $\Sigma=\pm\frac{1}{2}$ is the intrinsic spin projection on the symmetry axis, where we represent $\Sigma=\frac{1}{2}$ by $\uparrow$, and $\Sigma=-\frac{1}{2}$ by $\downarrow$. In this limit the spin of the nucleus Ω is equal to the sum of the angular momenta along the symmetry axis, $\Lambda=|\Lambda_1\pm\Lambda_2|$, and the intrinsic spin projection along the symmetry axis $\Sigma=|\Sigma_1\pm\Sigma_2|$, i.e., $\Omega=\Lambda+\Sigma$. We propose that the Ω of the lower-lying state of each configuration is

$$\Omega=|\Omega_1-\Omega_2|,\quad \text{if } \Omega_1=\Lambda_1\pm\tfrac{1}{2},\ \Omega_2=\Lambda_2\pm\tfrac{1}{2};$$

$$\Omega=\Omega_1+\Omega_2,\quad \text{if } \Omega_1=\Lambda_1\pm\tfrac{1}{2},\ \Omega_2=\Lambda_2\mp\tfrac{1}{2};$$

i.e., we propose that $\Sigma=\Sigma_{p_1,n_1}+\Sigma_{p_2,n_2}=0$ in the lower-lying state. This rule is an extension to two-like-particle states of the rule proposed by GM for odd-odd nuclei. However, we expect the $\Sigma=0$ state lies lower in the excited states of even-even nuclei, because the singlet interaction is known to be stronger than the triplet interaction in the case of like particles.

In summary, we are proposing that the lower-lying intrinsic states in deformed even-even nuclei are basically doubly degenerate two-particle states. The degeneracy of the two possible states appears to be split by residual interactions, and the residual interaction that at present seems to be most important in removing the degeneracy is the spin-dependent interaction known to split the degeneracy in deformed odd-odd nuclei.

To illustrate the application of the coupling scheme to an actual nucleus, we discuss below in some detail the energy levels of Hf^{178}.

III. RELATION TO THE SUPERFLUID MODEL

It should be noted at this point that while the strong-coupling model and Nilsson states have accounted for most details of the spectra of deformed nuclei, two physical quantities observed in this region which are not understood within the framework of the model are the absolute values of the moments of inertia and the energy gap of ≈ 1 Mev observed in the spectra of even-even nuclei.[10] These quantities have been successfully calculated recently by the addition of a pairing force to the effective nuclear Hamiltonian.[10–13] Furthermore, through these studies, theoretical justification for the assumption of two-particle states in even-even nuclei has emerged. And, by means of a recently proposed independent-quasi-particle or "superfluid" model of the nucleus, the calculation of many specific properties of individual nuclei, including spectra of two-particle states, has been carried out.[14] In a later paper, the analysis of all the spectra of deformed even-even nuclei in the $150<A<190$ region will be compared in detail with spectra calculated on the basis of this model.[15]

We focus attention on the simpler interpretation in terms of two-particle states described by Nilsson wave functions in the present paper primarily because it gives qualitatively the same answers to many questions as the more detailed theory, while remaining conceptually simpler. It is thus of more immediate use to the experimentalist.

IV. SINGLE-PARTICLE LEVELS

We take the following approach in attempting to catalogue the states expected in Hf^{178}. We first present all the available experimental data on the single-particle levels in adjacent odd-mass nuclei. We then show that the Nilsson diagrams give the level ordering observed. From this, and the detailed analysis of Mottelson and Nilsson,[6] we conclude that the Nilsson scheme gives a

[10] See B. R. Mottelson, *Nuclear Structure*, Cours de l'école d'été de physique théorique des houches 1958 (Dunod, Paris, 1959), p. 283.

[11] A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958); N. N. Bogolyubov, Doklady Akad. Nauk. SSSR **119**, 52 (1958).

[12] V. G. Solov'ev, J. Exptl. Theoret. Phys. SSSR **35**, 823 (1958); **36**, 1869 (1959); Nuclear Phys. **9**, 655 (1958/59); S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **31**, No. 11 (1959).

[13] J. Griffin and N. Rich, Phys. Rev. **118**, 850 (1960); O. Prior and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 16 (1960); A. B. Migdal, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 249 (1959); Yu. T. Grin, S. I. Drozdov, and D. F. Zaretsky, *ibid.* **38**, 1297 (1960).

[14] V. G. Solov'ev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter **1**, No. 11 (1961).

[15] C. J. Gallagher, Jr., and V. G. Solov'ev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter **2**, No. 2 (1962).

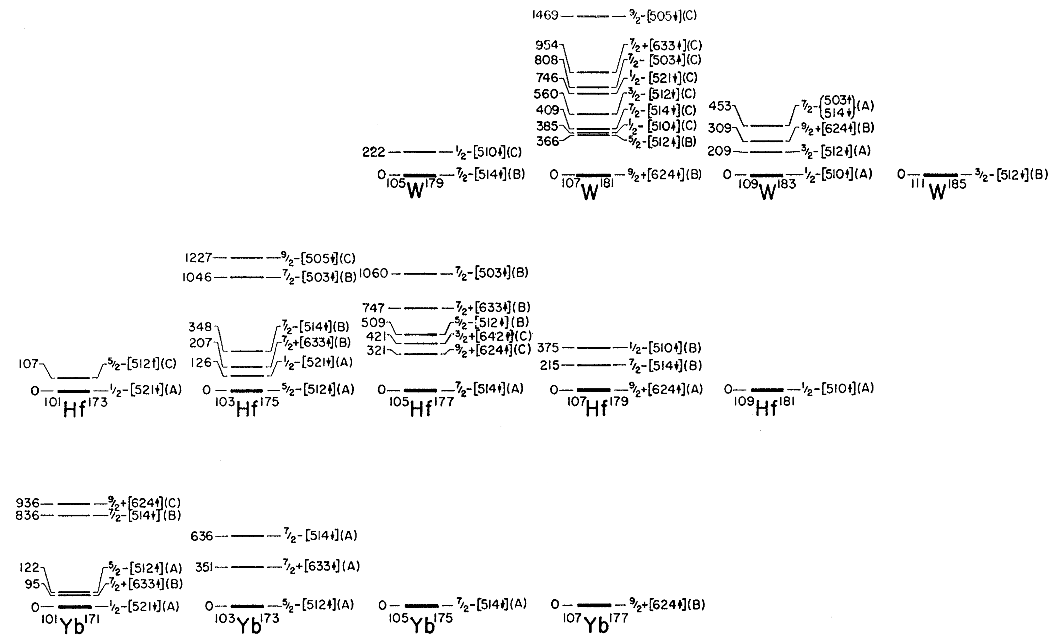

FIG. 1. Intrinsic energy levels observed in odd-N isotopes $N=101$ through $N=111$. References to experimental data are given in Table I. Energy levels are characterized by excitation energy (in kev), experimentally determined spin and parity, and assignment in terms of the asymptotic-limit quantum number notation $[Nn\Lambda\Sigma]$ (see text for discussion) of the Nilsson wave functions. Following Mottelson and Nilsson (reference 6) each level is assigned an arbitrary grade which refers to the certainty with which the level is experimentally established.

valid characterization of the experimental situation. We then conclude that we can use the Nilsson scheme to account for all two-particle excitations in the spectra without missing any.

To order the spectrum correctly we should have a method for determining the relative energies of the two-particle states. There is no way of doing this within the framework of the present discussion. We assume, therefore, that the level order of the two-particle states follows the level order of the single-particle states in odd-mass nuclei.

A special type of two-particle state occurs when $\Omega_1=\Omega_2$. In this case the state $\Omega_1-\Omega_2$ will have $\Omega=0$. In odd-odd nuclei, residual forces are known to displace the odd- and even-spin rotational states with respect to each other in such states.[16–18] Newby has pointed out that in odd-odd nuclei there seems to be no *a priori* way of deciding which band will lie lower, at least on the basis of present data.[19] We assume residual forces can displace the odd and even bands of $\Omega=0$ states in even-even nuclei also. Determining the order remains an experimental question.

A problem to which the present scheme offers no solution is in deciding whether an excited 2+ state is a two-particle state or a collective gamma vibration.[3,9] Another aspect of essentially the same problem is our inability to decide whether an excited 0+ state is a collective beta vibration[3,9] or a pair excitation. Such basically quantitative questions show clearly the qualitative nature of the present proposals.

The odd-neutron spectra for neutron numbers $N=101$–111 are illustrated in Fig. 1. In this figure the spectra are so arranged that isotones lie below each other. References to the original experimental data are given in Table I. The odd-proton spectra for proton numbers $Z=69$–75 are illustrated in Fig. 2. References to the original experimental data on the odd-proton states are given in Table II.

The systematic way in which the neutron levels are filled is clearly shown in Fig. 1. From left to right, the ground states for the successive neutron numbers 101–111 are seen to be 1/2−, 5/2−, 7/2−, 9/2+, 1/2−, and 3/2−. These levels appear as the ground states for a given N^{ir} regardless of Z. Furthermore, the energy of a level varies as a function of neutron number in a systematic way, in that it first appears as a high-lying state, decreases in energy with successive neutron numbers until it becomes the ground state, then increases in energy again as more neutrons are added until its energy is so high it disappears from the spectrum.

The proton levels are shown in Fig. 2. Several isotopes of a given Z are usually known, and we show them horizontally as a function of increasing N. In this case

[16] F. Asaro, I. Perlman, J. O. Rasmussen, and S. G. Thompson, Phys. Rev. **120**, 934 (1960).
[17] R. G. Helmer and S. B. Burson, Phys. Rev. **119**, 788 (1960); J. S. Geiger, R. L. Graham, and G. T. Ewan, Bull. Am. Phys. Soc. **5**, 255 (1960).
[18] D. A. Varshalovich and L. K. Peker, Izvest. Akad. Nauk. SSSR **25**, 287 (1961).
[19] N. D. Newby, Jr., Phys. Rev. **125**, 2063 (1962).

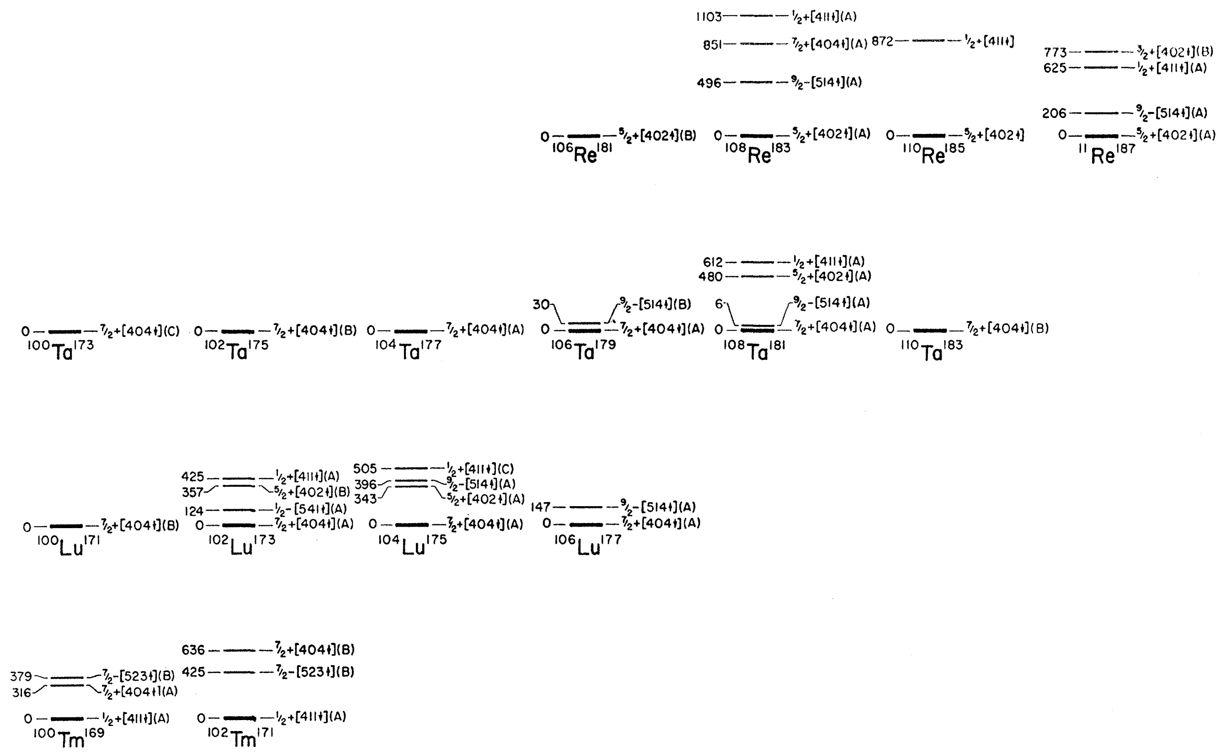

FIG. 2. Intrinsic energy levels observed in odd-Z isotopes $Z=69$ through $Z=75$. References to experimental data are given in Table II. The notation used in the figure is identical to that of Fig. 1.

ground states are counted from top to bottom, and are, for $Z=69$–75, successively 1/2+, 7/2+, (9/2−), and 5/2+. The 9/2− state is in parenthesis because, although it never becomes the ground state for any Z, its systematic appearance indicates it is only accidental that it is not the tantalum ground state. The 124-kev 1/2− level has only been observed in Lu^{173}.[20]

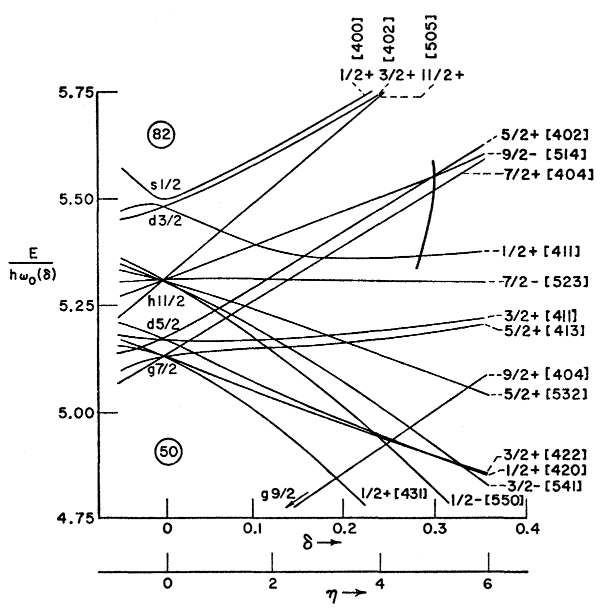

FIG. 3. Nilsson diagram for the odd-proton states $50<A<82$. This is the adjusted diagram which appears in reference 6. The region of the levels under consideration is marked by a heavy line.

The Nilsson diagrams for the $50<Z<82$ and $82<N<126$ regions are shown in Figs. 3 and 4.[6] The regions of interest to the present discussion are indicated. As can be seen, the ordering of levels predicted is as experimentally observed for both protons and neutrons. The asymptotic limit assignments (in the $Nn_z\Lambda\Sigma$ notation, are successively for the neutron numbers 101–111) [521↓] (1/2−), [512↑] (5/2−), [514↓] (7/2−), [624↑] (9/2+), [510↑] (1/2−), and [512↓] (3/2−), and for the proton numbers 69 –75,[411↓] (1/2+), [404↓] (7/2+), [514↑] (9/2−), and [402↑] (5/2+).

Mottelson and Nilsson have demonstrated that these Nilsson states account satisfactorily for most of the detailed properties of the levels in the mass region under discussion.[6] Although the present compilation includes more data on the levels in these nuclei than that of Mottelson and Nilsson, the conclusions we reach on the basis of the additional data only reaffirm Mottelson and Nilsson's conclusions. We believe that a repeti-

[20] This $\frac{1}{2}-$ level is the first clear cut example in the $50<Z<82$ region of an intrinsic proton level from the next proton shell. [See J. Valentin, D. J. Horen, and J. M. Hollander, University of California Radiation Laboratory Report UCRL-9731 1961 (unpublished)]. The coupling of such states as this to the levels of the $50<Z<82$ shell will produce two-particle states in the spectra of even-even nuclei in this region in addition to the ones we consider here.

TABLE I. References to experimental data on the odd-neutron levels shown in Fig. 1. This table contains references only to data which have appeared since Mottelson and Nilsson's compilation (reference 6) which we refer to as MN in the table. The reference MN alone in the table means no new data have appeared since MN, and the original references can be found there. We have only included references to papers which establish new levels or redetermine spins or parities of previously known levels. For references to other types of data see, for example, the Nuclear Data Sheets.

Nucleus	References
$_{70}Yb^{171}$	MN,[a] HHM59[b]
Yb^{173}	MN, BMH[c]
Yb^{175}	MN
Yb^{177}	MN
$_{72}Hf^{173}$	HHM60,[d] VHH[e]
Hf^{175}	HHM60
Hf^{177}	MN, HHM60, WMN[f]
Hf^{179}	MN, HHM60
Hf^{181}	MN
$_{74}W^{179}$	MN, HHM60
W^{181}	MN, HHM60
W^{183}	MN, GN[g]
W^{185}	MN

[a] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter Vid. Selsk 1, No. 8 (1959).
[b] B. Harmatz, T. H. Handley, and J. W. Milhelich, Phys. Rev. **114**, 1082 (1959).
[c] J. W. Bichard, J. W. Milhelich, and B. Harmatz, Phys. Rev. **116**, 720 (1959).
[d] B. Harmatz, T. H. Handley, and J. W. Milhelich, Phys. Rev. **119**, 1345 (1960).
[e] J. Valentin, D. J. Horen, and J. M. Hollander, University of California Radiation Laboratory Report UCRL-9731, June, 1961 (unpublished)
[f] H. I. West, Jr., L. G. Mann, and R. J. Nagle, Phys. Rev. **124**, 527 (1961).
[g] C. J. Gallagher, Jr., and H. L. Nielsen, Nuclear Phys. **24**, 422 (1961).

tion of these results here is therefore not justified, and we refer the reader to the orginal analysis for a detailed discussion of the intrinsic properties of the states.

It should be remarked that in both Figs. 1 and 2 some levels appear to be missing. However, it can easily be shown that the missing levels are just those to which beta- and/or gamma-ray transitions are forbidden on the basis of the Nilsson wave functions, and hence their nonappearance is explained in a consistent way by the Nilsson wave functions.

We believe these figures demonstrate clearly the simplicity introduced into a discussion of the experimental data by the use of the Nilsson scheme.

V. TWO-PARTICLE STATES IN Hf^{178}

In Hf^{178} the 7/2− [514↓] neutron pair is the last filled orbital, and the 9/2+ [624↑] orbital is the first unoccupied orbital. 8− $(\Sigma=0)$ and 1− $(\Sigma=1)$ states are expected from coupling these two states. The next state appearing in the neutron spectrum is the 5/2− hole state [512↑]. Coupling this state to the [514↓] state produces a 6+ $(\Sigma=0)$ state and a 1+ $(\Sigma=1)$ state. Its coupling to the [624↑] state produces a 2− $(\Sigma=0)$ state and a 7− $(\Sigma=1)$ state.

Either the 9/2− [514↑] or the 7/2+ [404↓] proton state is the last occupied state, and the other is the first unoccupied state. These couple to produce an 8− $(\Sigma=0)$ and a 1− $(\Sigma=1)$ state. The odd-mass level

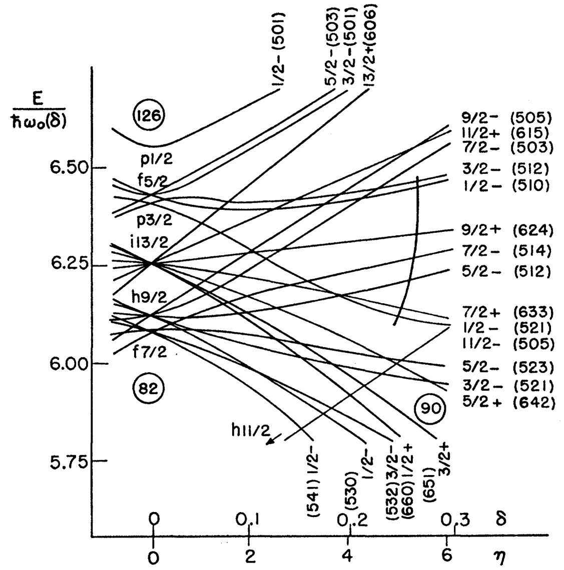

FIG. 4. Nilsson diagram for the odd-neutron states $82<A<126$. This is the adjusted diagram which appears in reference 6. The region of the levels under consideration is marked by a heavy line.

energies indicate that the 5/2+ [402↑] state is next at a somewhat higher energy. Coupling the [402↑] state to the [404↓] state produces a 6+ $(\Sigma=0)$ state and a 1+ $(\Sigma=1)$ state. Coupling the [402↑] state to the [514↑] state produces a 2− $(\Sigma=0)$ state and a 7− $(\Sigma=1)$ state.

Higher-lying levels produced by coupling high-lying single-particle states to these levels and to each other can be determined from the Nilsson diagram. For the

TABLE II. References to experimental data on the odd-proton levels in Fig. 2. The organization of this Table is identical to that of Table I. (See caption of Table I for further details.)

Nucleus	References
$_{69}Tm^{169}$	MN[a]
Tm^{171}	MN
$_{71}Lu^{173}$	VHH[b]
Lu^{175}	MN
Lu^{177}	MN
$_{73}Ta^{173}$	HHM60[c]
Ta^{175}	HHM60
Ta^{177}	MN, HHM60
Ta^{179}	MN, HHM60
Ta^{181}	MN, MB[d]
Ta^{183}	MN
$_{75}Re^{181}$	MN, HHM60
Re^{183}	N[e]
Re^{185}	MN
Re^{187}	GEM[f], BOO[g]

[a] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter 1, No. 8 (1959).
[b] J. Valentin, D. J. Horen, and J. M. Hollander, University of California Radiation Laboratory Report UCRL-9731, 1961 (unpublished).
[c] B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **119**, 1345 (1960).
[d] A. H. Muir and F. Boehm, Phys. Rev. **122**, 1564 (1961).
[e] J. O. Newton, Phys. Rev. **117**, 1520 (1960).
[f] C. J. Gallagher, Jr., W. F. Edwards and G. Manning, Nuclear Phys. **19**, 18 (1960).
[g] K. Maack Bisgaard, K. Olesen and P. Ostergaard, Report, 1961, Physical Institute, University of Aarhus, Denmark (unpublished).

sake of simplifying the discussion and the illustration we limit ourselves to the 6 neutron and 6 proton states formed by coupling the 3 proton and 3 neutron single-particle states discussed above.

In Fig. 5(b) these 12 levels are illustrated. The two-neutron and two-proton states are shown in different columns. In addition, the three lowest-lying states of each group have $\Sigma=0$. The energy scale is, of course, arbitrary. States that could be populated by allowed or first-forbidden beta decay from the two Ta^{178} isomers (discussed below) are indicated by an asterisk. The selection rules on which the assignments of the asterisks are based are discussed in G 60.

VI. ANALYSIS OF THE LEVEL SPECTRUM

In Fig. 6 the decay scheme of the 9.3-min isomer of Ta^{178} is shown. The decay scheme of 2.1-hr Ta^{178} is shown in Fig. 1 of the preceding paper.[21] The decay scheme of 9.3-min Ta^{178} is as previously reported.[22,23] Intrinsic levels of Hf^{178} populated in these decays are shown schematically in Fig. 5(a).

Levels Populated by 2.1-hr Ta^{178}

In the preceding paper we pointed out that the 1148-kev level has $K\pi=8-$ and the 1480-kev level probably has $K\pi=8-$ also. We proposed that the 1148-kev level is the two-proton configuration [514↑+404↓]. The

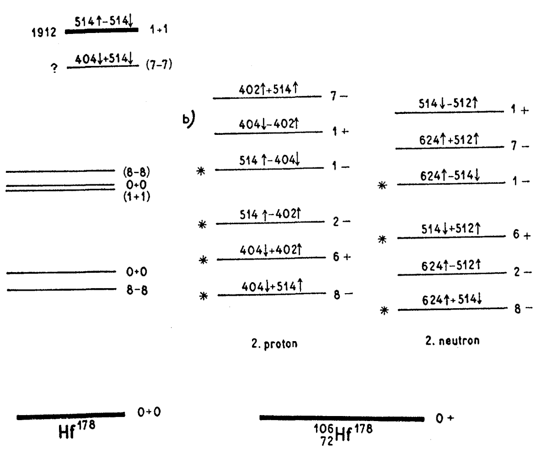

FIG. 5(a). Intrinsic energy levels of Hf^{178} observed in the electron-capture decay of 9.3-min Ta^{178} and 2.1-hr Ta^{178}. The analysis into intrinsic states is as proposed in references 21 and 22. (b). Two-particle states expected in Hf^{178} at low energies according to the discussion of Sec. V. The notation is discussed in the text. The three lower-lying states of the neutron and proton configurations have $\Sigma=0$; the three upper states have $\Sigma=1$. The energy scale is arbitrary. Levels marked by an asterisk are theoretically expected to be populated by allowed (*au* or *ah*) or first-forbidden (1*u* or 1*h*), according to the selection rules proposed in G60 (reference 2).

[21] C. J. Gallagher and H. L. Nielsen, preceding paper [Phys. Rev. **126**, 1520 (1962)].

[22] C. J. Gallagher, H. L. Nielsen, and O. B. Nielsen, Phys. Rev. **122**, 1590 (1961).

[23] U. Bertelsen, J. Borgreen, and O. Nathan, Phys. Rev. **123**, 564 (1961).

1480-kev level was assigned as the two-neutron state [514↓+624↑]. As can be seen from Fig. 5(b), these are just the two lowest-lying $\Sigma=0$ states expected on the basis of the discussion of Sec. IV. The beta-decay log *ft* and *M*1 transition between the two levels indicate that the two states are actually linear combinations of the two configurations.

Levels Populated by the 9.3-min Ta^{178}

The 0+ level in Hf^{178} at 1440 kev has been assigned as a state which contains a large amplitude of the [514↓ −514↓] neutron pair, which also has a large amplitude in the Hf^{178} ground state.[22] The approach discussed here does not qualititatively account for such a mixing of levels. We discussed this level in more detail earlier.[22] We previously assigned the 1197-kev level as a beta vibrational state, but we now have reason to believe this description may be wrong.[24] The 1+ level at 1430 kev is not as well established experimentally as the 0+ states and it is also difficult to interpret. It could be either of the $1+\Sigma=1$ states shown in Fig. 5(b), but neither of these states explains the log *ft* ≈ 4.4 observed for the transition. We do not know how to interpret this level. The remaining levels observed in Hf^{178} following 9.3-min Ta^{178} decay have been assigned as rotational levels.

VII. CONCLUSIONS

The Ta^{178}-Hf^{178} System

The discussion presented above shows that the assumptions discussed in the text can account for the two 8− states which appear in the Hf^{178} spectrum and predict that these states should be the lowest-lying two-neutron and two-proton states. They also explain the fast beta decay branch to *one* of the 8− states. On the other hand, the mixing of the 8− states that must be assumed in order to account for the fast beta branches to both states and the *M*1 transition between them is not predicted. They predict pair excitations, but not the mixing of the 1440-kev level with the Hf^{178} ground state

[24] We think now that it is a type of pair excitation. The specific type of pair excitation we have in mind is the two-quasi-particle state defined by the wave function (cf. reference 14)

$$\Psi(\nu_1,\nu_1)=(U_{\nu 1}a^{+}{}_{\nu 1}a_{-\nu 1}-V_{\nu 1})\cdot\prod_{\nu\neq\nu_1}(U_\nu+V_\nu a^{+}{}_\nu a^{+}{}_{-\nu})\Psi_0$$

refered to a ground-state wave function

$$\Psi=\prod_\nu(U_\nu+V_\nu a_\nu{}^{+}a_{-\nu}{}^{+})\Psi_0,$$

where Ψ_0 is the vacuum state. The energy calculated (reference 15) for the 1440-kev 0 state, which we identify by the fast beta transition to it as the state in which the probabilities $U_{\nu 1}$, $V_{\nu 1}$ are for the state $\nu_1=[514\downarrow]$ or $\nu_1=[624\uparrow]$, is 1.7 ± 0.34 Mev. The calculated energy therefore seems to fit the data, and we gain confidence in the results. The energy calculated for the 0 proton state in which the probabilities $U_{\nu 1}$, $V_{\nu 1}$ are for the state $\nu_1=[404\downarrow]$ or $[514\uparrow]$ is 1.2 ± 0.24, again fitting the experimental data, if we assume this is the 1197-kev 0 state. On the basis of these results we believe the 1197-kev state should be reassigned as the proton-pair excitation. What constitutes a beta vibration in this nucleus is not clear to us.

that also follows from a consistent interpretation of the beta-decay branches of 9.3-minute Ta^{178} (however, see reference 24).

The assumptions are also supported by some negative evidence, namely, that no levels other than those predicted are observed. In fact, all of the levels observed (perhaps even the 1430-kev level) appear to be observed because they contain some amplitude of the [514↑] (proton) or [514↓] (neutron) states, between which the allowed unhindered transition [514↑]⇄[514↓] is possible. By virtue of its large matrix element this transition dominates all other decay modes of the Ta^{178} isomers. Thus we believe that the unobserved levels marked with asterisks in Fig. 5(b) have not been observed either because the experimental techniques used to date have not been adequate to find the weak branches, which are expected to populate these levels, or because the energies of the levels are greater than the decay energy of the isomers.

Although the present results do not prove our assumptions about the residual spin-dependent force, they are consistent, in that both 8− states have $\Sigma=0$. Further studies of the 9.3-min Ta^{178} decay scheme should reveal weak branches to the pair of 1− states produced by recoupling the 8− configurations, and should thus provide a definite experimental answer to this question.

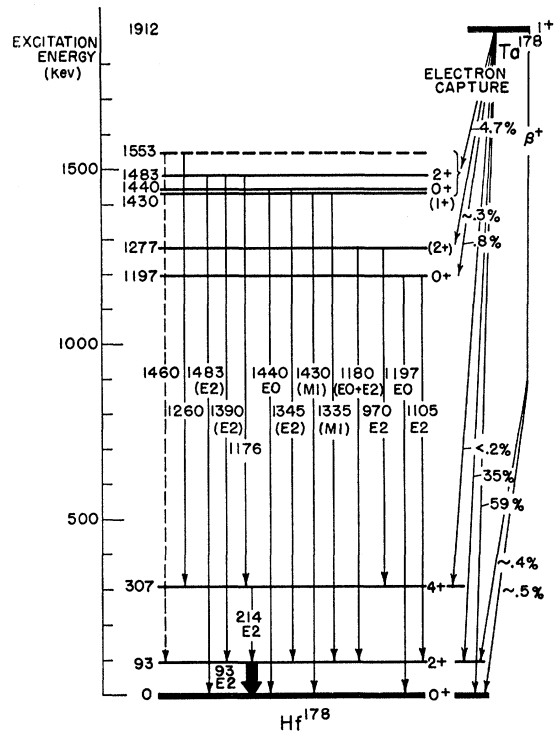

FIG. 6. The decay scheme of 9.3-min Ta^{178} as reported in reference 22.

The Coupling Scheme

As mentioned earlier, we have completed a detailed analysis of all the spectra of deformed even-even nuclei in the $150<A<190$ region and compared the results to predictions based on the assumptions of the text. This analysis leads basically to the same conclusions as the analysis of the Hf^{178} spectrum, namely, the gross aspects of the spectra seem to be described by the assumptions discussed in the text, but there is evidence that, in addition to the spin-dependent interaction which appears to split the $|\Omega_1 \pm \Omega_2|$ doublets, there are also interactions between configurations.

We state above that this approach is a simplified description of the results expected on the basis of independent-quasi-particle models. Because the results of the extensive analysis of the experimental spectra also provide insight into the general theory of pairing correlations, the general analysis will be published elsewhere[15] and compared to detailed calculations of spectra based on an independent-quasi-particle, or "superfluid" nuclear model which takes into account blocking effects.[14]

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