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Connection between Gauge Invariance and Mass

DAVID G. BOULWARE* AND WALTER GILBERT†
Department of Physics, Harvard University, Cambridge, Massachusetts
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The limit, as the bare mass vanishes, of a theory of a massive neutral vector meson interacting with charged fields is investigated. A redefinition of the charged-field operators is exhibited so that the original theory, involving a positive definite metric, goes over smoothly to a radiation gauge theory. The spectral forms for the boson two-point function are exhibited to show that the limit to a gauge-invariant theory does not restrict the interacting mass of the vector particle. A soluble example is given in which these limits can be studied in detail and in which the gauge-invariant limit describes a massive vector particle.

I. INTRODUCTION

OUR purpose is to explore the connection between a theory of a massive neutral vector meson and electrodynamics; between mass and gauge invariance. We shall investigate the limit, as the bare mass vanishes, of a vector particle to show that the physical implications of the theory go over smoothly to those of radiation gauge electrodynamics. Our original quantization of the vector field has three degrees of freedom, a positive definite metric, and manifest Lorentz covariance. In order to exhibit a smooth limit in which the longitudinal modes of the vector field decouple completely, we find it necessary to redefine the charged field operators and to give up *manifest* covariance.

We shall then explore the structure of the two-point functions to show that the existence of this smooth limit for the operator structure of the theory does not restrict the mass of the vector particle described by the theory. We find no connection between the gauge-invariant structure of the limit theory and the mass spectrum. Johnson[1] has recently argued that the mass of a vector meson is always lowered by its interactions. We shall show that the assumptions that he made were too severe. Schwinger[2] has also made this observation and has inferred that there is no necessary connection between gauge invariance and the existence of a massless particle.

To illustrate these considerations we exhibit a soluble theory in which we can explicitly follow the operator structure as the bare mass vanishes. In this model the gauge-invariant limit describes a massive particle. We can then construct a class of interacting theories which may be viewed alternatively as describing either a gauge-invariant or a massive vector field; the charged fields involved have, respectively, either radiation gauge or Lorentz covariant transformation properties.

The vanishing mass limit of the vector meson has been considered previously by Coester,[3] who has argued that the limit of an indefinite metric theory of the vector meson is a Lorentz gauge quantization of electrodynamics. Glauber[4] has shown in a similar context, using a theory due to Stueckelberg[5] involving five degrees of freedom and supplementary conditions, that a gauge-invariant theory of a massive, neutral vector meson can be developed.

II. LIMIT AS THE BARE MASS VANISHES

Let us consider a spin-one meson coupled to other fields. The Lagrangian density[6] may be written

$$\mathcal{L}=-\tfrac{1}{2}[F^{\mu\nu}(\partial_\mu A_\nu-\partial_\nu A_\mu)-\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu}+m_0{}^2A^\mu A_\mu]+j^\mu A_\mu+\mathcal{L}_{\text{part}}.$$

We do not need to specify the Lagrangian density for the other fields, but we may think of it in the form

$$\mathcal{L}_{\text{part}}=-\tfrac{1}{2}\chi\alpha^\mu(1/i)\partial_\mu\chi-\tfrac{1}{2}\chi M\chi-\mathcal{H},\qquad(1)$$

and then we would take $j^\mu=\frac{1}{2}e_0\chi\alpha^\mu q\chi$. We shall not assume in general that $\partial_\mu j^\mu=0$. The equations of motion for the vector meson are

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu,\quad \partial_\nu F^{\mu\nu}+m_0{}^2A^\mu=j^\mu,$$

and the nonvanishing commutation relations for the independent degrees of freedom are

$$\delta(x^0-x'^0)[F^{0k}(x),A^l(x')]=i\delta^{kl}\delta(x-x').\qquad(2)$$

A^0 is a dependent variable; we assume that j^0 and A^k commute at equal times, and thus

$$\delta(x^0-x'^0)[A^0(x),A^k(x')]=-i(\partial^k/m_0)\delta(x-x').\qquad(3)$$

A^0 commutes with F^{0k} but fails to commute with the χ fields, having commutation relations determined by those of j^0.

In order to exhibit the limit of the theory as the bare mass m_0 goes to zero, we pick a specific frame and divide the field into three-dimensionally transverse and longitudinal parts. We separate out a term that becomes, in the limit, the Coulomb interaction. Thus we use as

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[1] K. Johnson, Nuclear Phys. **25**, 435 (1961).
[2] J. Schwinger, Phys. Rev. **125**, 397 (1962).
[3] F. Coester, Phys. Rev. **83**, 798 (1951).
[4] R. J. Glauber, Progr. Theoret. Phys. (Kyoto) **9**, 295 (1953). See also H. Umezawa, Progr. Theoret. Phys. (Kyoto) **7**, 551 (1952).
[5] E. C. G. Stueckelberg, Helv. Phys. Acta. **11**, 299 (1938).
[6] We use $\hbar=c=1$. Greek indices run from 0 to 3, Latin indices from 1 to 3, and the metric is spacelike $(-1,1,1;1)$. All fields are Hermitian and the charge matrix q is Hermitian and antisymmetric. For further explanation, see J. Schwinger, Phys. Rev. **115**, 721 (1959).

variables

$$
\begin{aligned}
&A^k = {}^TA^k + \partial^k\varphi/m_0,\\
&F^{0k} = E^k + \partial^k[\{1/(\nabla^2 - m_0^2)\}j^0 - (m_0/\nabla^2)\Pi],\\
&A^0 = \Pi/m_0 - \{1/(\nabla^2 - m_0^2)\}j^0,\\
&H^k = \tfrac{1}{2}\epsilon^{klm}(\partial_l{}^TA_m - \partial_m{}^TA_l).
\end{aligned}
\tag{4}
$$

The field φ carries the degree of freedom associated with the longitudinal part of A^k. Π carries the conjugate degree associated with the longitudinal part of F^{0k} or, equivalently by the equation of constraint, with A^0. The commutation relations are now

$$
\begin{aligned}
\delta(x^0 - x'^0)[E^k(x), {}^TA^l(x')] &= i(\delta^{kl}\delta(x-x'))^T\\
&= i\left[\delta^{kl}\delta(x-x') - \partial^k\partial'^l\frac{\delta(x^0-x'^0)}{4\pi|\mathbf{r}-\mathbf{r}'|}\right]
\end{aligned}
\tag{5}
$$

and

$$
\delta(x^0 - x'^0)[\Pi(x), \varphi(x')] = i\delta(x-x'). \tag{6}
$$

The other equal-time commutators of these meson fields with each other vanish. φ commutes with the χ fields; Π does not, but has commutators related to those of j^0

$$
[\Pi, \chi] = \{\nabla^2/(\nabla^2 - m_0^2)\}[j^0, \chi]. \tag{7}
$$

The equations of motion for these variables are

$$
\begin{aligned}
&\partial^0\,{}^TA^k = E^k,\\
&\partial_0 E^k = -(\nabla^2 - m_0^2)\,{}^TA^k - {}^Tj^k,
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
&\partial^0\varphi = (1 - m_0^2/\nabla^2)\Pi,\\
&\partial_0\Pi = -\nabla^2\varphi + \frac{\partial_\mu j^\mu}{m_0} + \frac{m_0}{\nabla^2 - m_0^2}\partial_0 j^0,
\end{aligned}
\tag{9}
$$

or

$$
\partial^2\varphi = m_0^2\varphi + \left(1 - \frac{m_0^2}{\nabla^2}\right)\frac{\partial_\mu j^\mu}{m_0} + \frac{m_0}{\nabla^2}\partial_0 j^0.
$$

Equations (5) and (8) for the transverse degrees are clearly well behaved in the limit as m_0 vanishes. Equations (6) and (9) are well behaved in the limit provided that $\partial_\mu j^\mu/m_0$ remains bounded. The current must be divergenceless in the limit. If, further, $\partial_\mu j^\mu/m_0$ and $m_0\partial_0 j^0$ vanish, the longitudinal modes become uncoupled in the limit. However, if these modes are to have no effects in the limit, they must commute with the other degrees of freedom. The commutator of Π with the χ fields becomes singular in the limit, requiring that we redefine the fields. Before we do this, let us examine the energy-momentum tensor and the Lagrangian density. The relevant part of the energy-momentum tensor is the total energy of the system.

$$
\begin{aligned}
P^0 = \int d\tau \Big\{ &\tfrac{1}{2}(E^2 + H^2) + \tfrac{1}{2}m_0^2\,{}^TA^l\,{}^TA_l - {}^Tj^k\,{}^TA_k + T_{\text{part}}{}^{00}\\
&- j^0\frac{1}{\nabla^2 - m_0^2}j^0 + \frac{1}{2}\left[\Pi^2 + (\partial_k\varphi)^2 - m_0^2\Pi\frac{1}{\nabla^2}\Pi\right] - j^k\frac{\partial_k\varphi}{m_0}\Big\},
\end{aligned}
$$

in terms of the independent variables of the meson field. We assume that the structure of $T_{\text{part}}{}^{00}$, has the form

$$
\tfrac{1}{2}\chi\mathfrak{A}^k(1/i)\partial_k\chi + \bar{T}^{00},
$$

where $\bar{T}^{00}$ does not contain any explicit derivatives. The Lagrangian density, in the independent variables of the meson field, is

$$
\begin{aligned}
\mathfrak{L} = &-E^k\partial_0\,{}^TA_k - \tfrac{1}{2}(E^2 + H^2) - \tfrac{1}{2}m_0^2\,{}^TA^k\,{}^TA_k + {}^Tj^k\,{}^TA_k\\
&+ \mathfrak{L}_{\text{part}} + \tfrac{1}{2}j^0\frac{1}{\nabla^2 - m_0^2}j^0 - \Pi\partial_0\varphi - \tfrac{1}{2}\Pi^2 - \tfrac{1}{2}(\partial_k\varphi)^2\\
&+ \tfrac{1}{2}m_0^2\Pi\frac{1}{\nabla^2}\Pi + m_0 j^0\frac{1}{\nabla^2 - m_0^2}\partial_0\varphi + j^\mu\frac{\partial_\mu\varphi}{m_0},
\end{aligned}
$$

where $\mathfrak{L}_{\text{part}}$ is given by Eq. (1). The term that becomes the Coulomb energy in the limit appears explicitly in these expressions. We observe that the longitudinal mode decouples with the exception of the terms $j^k\partial_k\varphi/m_0$ and $j^\mu\partial_\mu\varphi/m_0$, respectively. The same result holds for the generator of Lorentz transformations.

In order to obtain a limit in which the fields decouple smoothly we assume that the current can be constructed using the gauge transformation properties of the χ fields. Then if we define new field variables

$$
\chi = e^{i\Lambda}\chi', \quad \Lambda = \frac{e_0 q}{m_0}\frac{\nabla^2}{\nabla^2 - m_0^2}\varphi \tag{10}
$$

(φ and χ commute at equal times), the effect of this redefinition is to render Π and χ' independent; their commutator now vanishes. This follows from Eqs. (6) and (7) and the fact that j^0 is the generator of gauge transformations

$$
\delta(x^0 - x'^0)[\chi(x), j^0(x')] = e_0 q\chi(x)\delta(x-x').
$$

This redefinition removes from the Lagrangian and energy the terms that become singular in the limit. In the Lagrangian density the interaction terms involving φ become

$$
-m_0 j^k\frac{1}{\nabla^2 - m_0^2}\partial_k\varphi - \mathcal{H}(e^{i\Lambda}\chi').
$$

The structure of the Hamiltonian makes it clear that we cannot simply drop the term $j^\mu\partial_\mu\varphi/m_0$ in the Lagrangian by virtue of current conservation. Dropping such a term involves an implicit redefinition of the fields which we have exhibited. If the current is conserved, $\mathcal{H}$ is gauge invariant and hence does not become a function of φ. If the current is not conserved, before the limit is taken, the dependence of $\mathcal{H}$ on φ is just that needed to produce the interaction terms in the equation of motion for Π, Eq. (9), since

$$
(1/m_0)\{\nabla^2/(\nabla^2 - m_0^2)\}\partial_\mu j^\mu = \delta\mathcal{H}/\delta\varphi. \tag{11}
$$

Equation (11) follows because we have constructed the

current using the action principle applied to variations of the operators χ of the form $\delta\chi = ie_0 q\delta\lambda(x)\chi(x)$. The condition that $\partial_\mu j^\mu/m_0$ vanish, so that φ and Π decouple in the limit, is equivalent to the condition that $\mathcal{H}$ become independent of φ.

We have now explicitly exhibited a set of fields ${}^T\mathbf{A}$, $\mathbf{E}$, φ, Π, and χ' which behave smoothly in the limit as the bare mass vanishes. If $\partial_\mu j^\mu/m_0$ and $m_0\partial_0 j^0$ vanish in this limit, φ and Π become free fields. The new "charged" field variables χ' are no longer manifestly covariant but depend on the specific Lorentz frame in which the original reduction was made. The generator of Lorentz transformations is well behaved in the limit and will transform ${}^T\mathbf{A}$ and χ' into the radiation-gauge fields in a new frame. The operator gauge transformation that accompanies an infinitesimal Lorentz transformation can be easily derived by comparing the zero-mass limits in two different Lorentz frames and using the manifest covariance of the χ field to find the relation between the χ' fields.

III. SPECTRAL FORMS AND PHYSICAL MASSES

Does this transition to the radiation gauge, when the bare mass vanishes, constrain in any way the mass spectrum associated with the vector field? We shall examine the spectral forms of the two-point functions for the field to answer this question. We have in general

$$\langle A^\mu(\xi)A^\nu(0)\rangle = \int_0^\infty d\kappa^2\left[\left(g^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{\kappa^2}\right)\sigma(\kappa^2) - \partial^\mu\partial^\nu\delta(\kappa^2)c - \frac{\partial^\mu\partial^\nu}{\kappa^2}\rho(\kappa^2)\right]\Delta^{(+)}(\xi,\kappa^2),$$

$$\langle F^{\mu\lambda}(\xi)A^\nu(0)\rangle = \int_0^\infty d\kappa^2(\partial^\mu g^{\lambda\nu}-\partial^\lambda g^{\mu\nu})\sigma(\kappa^2)\Delta^{(+)}(\xi,\kappa^2),$$

where

$$\Delta^{(+)}(\xi,\kappa^2) = \int\frac{dk}{(2\pi)^3}\theta(k^0)\delta(k^2+\kappa^2)e^{ik\xi}.$$

The weight functions are all positive functions of m_0.

$$\sigma(\kappa^2)\geq 0,\quad c\geq 0,\quad \rho(\kappa^2)\geq 0.$$

We have explicitly isolated, in the constant c, any δ-function singularity at $\kappa^2=0$ in σ/κ^2 or ρ/κ^2. The canonical commutation relations [Eq. (2)] require

$$\int_0^\infty d\kappa^2\,\sigma(\kappa^2)=1$$

and the commutator of A^0 and A^k, Eq. (3) implies that

$$\frac{1}{m_0^2} = c + \int_0^\infty\frac{d\kappa^2}{\kappa^2}[\sigma(\kappa^2)+\rho(\kappa^2)]. \tag{12}$$

We have already seen that the physical modes, as we proceed to the limit, are ${}^TA$ and φ. The spectral forms for these modes are

$$\langle {}^TA^k(\xi)\ {}^TA^l(0)\rangle = \left[\delta^{kl}-\frac{\partial^k\partial^l}{\nabla^2}\right]\int_0^\infty d\kappa^2\,\sigma(\kappa^2)\Delta^{(+)}(\xi,\kappa^2)$$

and

$$\langle\varphi(\xi)\varphi(0)\rangle = m_0^2\int_0^\infty d\kappa^2\left[\frac{\sigma(\kappa^2)+\rho(\kappa^2)}{\kappa^2}-\frac{\sigma(\kappa^2)}{\nabla^2}\right]\Delta^{(+)}(\xi,\kappa^2) + cm_0^2\Delta^{(+)}(\xi,0).$$

The mass spectrum associated with the transverse modes is determined by σ. Equation (12), however, can always be satisfied by choices of $\rho(\kappa^2)$ and c and does not restrict the interacting mass. If $\partial_\mu j^\mu/m_0$ and $m_0\partial_0 j^0$ vanish, we have

$$\lim_{m_0\to 0}\langle\varphi(\xi)\varphi(0)\rangle = \Delta^{(+)}(\xi,0) \tag{13}$$

since φ becomes a noninteracting field. The integral of σ is bounded, hence

$$\lim_{m_0\to 0} m_0^2\sigma = 0.$$

Then Eq. (13) becomes

$$\lim_{m_0\to 0}\frac{m_0^2}{\kappa^2}[\sigma(\kappa^2)+\rho(\kappa^2)] = (1-\lambda)\delta(\kappa^2), \tag{14}$$

where

$$\lambda = \lim_{m_0\to 0} m_0^2 c.$$

Equation (12) is the integral of Eq. (14).

If ρ is nonvanishing, σ is constrained only if

$$\lim_{m_0\to 0}\frac{m_0^2}{\kappa^2}\rho(\kappa^2)\neq(1-\lambda)\delta(\kappa^2).$$

This clearly does not permit any general statement, even though ρ must satisfy $\lim_{m_0\to 0} m_0^2\rho = 0$ as a consequence of

$$\lim_{m_0\to 0}\left\langle\frac{\partial_\mu j^\mu}{m_0}\phi\right\rangle = 0.$$

If we assume $\partial_\mu j^\mu = 0$ and thus $\rho = 0$; we could observe the following types of behavior, among others;

Case (1). If $\lambda = 1$ there is no restriction on σ in the limit. This corresponds physically to the existence of a zero-mass particle in the φ spectrum.

Case (2). If $\lambda=0$, we must have

$$\lim_{m_0\to 0}\frac{m_0^2}{\kappa^2}\sigma(\kappa^2)=\delta(\kappa^2).$$

If $\sigma(\kappa^2)=D(m_0)\delta(\kappa^2-D(m_0)m_0^2)+\sigma'$, where

$$\lim_{m_0\to 0}\frac{m_0^2}{\kappa^2}\sigma'=0,$$

then the limit condition, Eq. (14), is satisfied for any $D(m_0)$, including one that vanishes in the limit. This corresponds to the existence of a particle in the $^T\mathbf{A}$ spectrum with mass lower than the bare mass of the vector meson. In the limit this particle disappears (has vanishing normalization). We can only demonstrate the existence of a zero-mass particle if we beg the question.

IV. AN EXAMPLE

We shall now exhibit a soluble-field theory in which we can carry out the limit as the bare mass vanishes but in which the vector particle remains massive. Consider the Lagrangian density

$$\mathcal{L}=-\tfrac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu-\partial_\nu A_\mu)+\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}-\tfrac{1}{2}m_0^2A^\mu A_\mu +gA^\lambda B_\lambda-B^\lambda\partial_\lambda B+\tfrac{1}{2}B^\lambda B_\lambda,$$

describing the interaction between a vector field A^λ and a massless scalar field B. gB^λ plays the role of the current j^λ. Our field equations are

$$\partial_\lambda B^\lambda=0,\qquad F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu,$$
$$B^\lambda=\partial^\lambda B-gA^\lambda,\quad \partial_\lambda F^{\mu\lambda}=-m_0^2A^\mu+gB^\mu.$$

The canonical commutation relations for the scalar field are

$$\delta(x^0-x'^0)[B^0(x),B(x')]=i\delta(x-x'),$$

with spectral forms

$$\langle A^\mu(\xi)A^\nu(0)\rangle=\left(g^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{m_0^2+g^2}\right)\Delta^{(+)}(\xi,\,m_0^2+g^2) -\frac{g^2\partial^\mu\partial^\nu}{m_0^2(m_0^2+g^2)}\Delta^{(+)}(\xi^2,0),$$

$$\langle B(\xi)B(0)\rangle=(1+g^2/m_0^2)\Delta^{(+)}(\xi,0).$$

In order to make the zero-mass limit well behaved we introduce the fields ^TA and φ of Eq. (4) and perform the gauge transformation analogous to Eq. (10),

$$B'=B-\frac{g\varphi}{m_0}-\frac{gm_0}{\nabla^2-m_0^2}\varphi.$$

Then the two-point functions for the new fields are

$$\langle {}^TA^k(\xi)\,{}^TA^l(0)\rangle=(\delta^{kl}-\partial^k\partial^l/\nabla^2)\Delta^{(+)}(\xi,\,m_0^2+g^2),$$

$$\langle\varphi(\xi)\varphi(0)\rangle=\left(\frac{m_0^2}{m_0^2+g^2}-\frac{m_0^2}{\nabla^2}\right)\Delta^{(+)}(\xi,\,m_0^2+g^2) +\frac{g^2}{m_0^2+g^2}\Delta^{(+)}(\xi,0),$$

$$\langle B'(\xi)B'(0)\rangle=\left(1-\frac{g^2}{\nabla^2-m_0^2}\right)\left(\frac{m_0^2}{m_0^2+g^2}\right)\Delta^{(+)}(\xi,0) +g^2\left(\frac{\nabla^2}{\nabla^2-m_0^2}\right)\left[\frac{1}{m_0^2+g^2}-\frac{1}{\nabla^2}\right] \times\Delta^{(+)}(\xi,\,m_0^2+g^2).$$

Now in the limit, the transverse modes have the mass g and the original longitudinal modes become a free field of zero mass. The transformed scalar boson has a mass g and a noncovariant vacuum expectation value

$$(1-g^2/\nabla^2)\Delta^{(+)}(\xi,g^2).$$

If the scalar particle is given a mass μ_0, the limit is only well behaved if the bare masses of the scalar and vector particle vanish together. The conditions that the fields decouple require that

$$\lim_{m_0\to 0}\frac{\mu_0}{m_0}=0.$$

Let us solve this theory for $m_0=0$ in the radiation gauge from the start. The Lagrangian has the gauge invariance

$$A^\lambda\to A^\lambda+\partial^\lambda\Lambda,$$
$$B\to B+g\Lambda,\tag{15}$$
$$B^\lambda\to B^\lambda.$$

The equations of motion for the independent degrees of freedom are

$$\partial^2\,{}^TA^k=-g\,{}^TB^k=g^2\,{}^TA^k$$

and

$$\partial^2B=g^2B.$$

Using the constraints

$$B^k=\partial^kB-g\,{}^TA^k,$$
$$A^0=-(g/\nabla^2)B^0,$$

we thus find immediately

$$\langle {}^TA^k(\xi)\,{}^TA^l(0)\rangle=(\delta^{kl}-\partial^k\partial^l/\nabla^2)\Delta^{(+)}(\xi,g^2),$$
$$\langle B(\xi)B(0)\rangle=(1-g^2/\nabla^2)\Delta^{(+)}(\xi,g^2).$$

Let us consider the implications of this model for a fully interacting theory: a massless vector meson interacting linearly with a scalar meson and also with a divergenceless current arising from charged fields. The Lagrangian will be gauge invariant under the trans-

formation (15) if the charged fields undergo the appropriate gauge transformation. In the radiation gauge quantization of this theory there are two degrees of freedom in the vector field and one in the scalar field. The charged fields are not manifestly covariant. If we write the Lagrangian in terms of the independent degrees of freedom for the vector field and introduce new operators, $\bar{F}^{\mu\nu}$, $\bar{A}^{\mu}$ whose transverse parts are the corresponding operators for the vector field and whose longitudinal parts are defined in terms of the scalar field:

$$^{L}\bar{F}^{0k}=(1/\nabla^2)\partial^k[gB^0+j^0],$$
$$^{L}\bar{A}^{k}=-\partial^k B/g,$$

and

$$\bar{A}^0=-B^0/g,$$

and if the charged field variables are redefined

$$\chi=e^{ie_0qB/g}\bar{\chi}=\exp[-ie_0q(1/\nabla^2)\partial_k\bar{A}^k]\bar{\chi},$$

then the interaction of the new variables $\bar{F}^{\mu\nu}$, $\bar{A}^{\nu}$, and $\bar{\chi}$ are those described by a Lorentz-invariant Lagrangian for a vector particle of mass g.

Note added in proof. We would like to thank Professor B. Zumino for drawing our attention to a treatment by Steuckelberg [Helv. Phys. Acta **30**, 209 (1957)] of the uncoupling of the longitudinal modes of a vector meson.

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μ Capture with Six Nucleon Form Factors*

J. BARCLAY ADAMS†
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received October 26, 1961; revised manuscript received February 2, 1962)

The matrix element for μ capture ($\mu^-+p\rightarrow n+\nu$) including not only the effects of vector, axial-vector, weak-magnetism, and induced-pseudoscalar couplings but also two "second-class" couplings has a small dependency on these hitherto undetected couplings. Capture in μ-mesonic hydrogen from the S states with both $F=1$ and $F=0$ is computed, and angular distributions of recoil neutrons and capture rates are given as functions of the six coupling constants. It is found that the second-class terms may contribute fully as much as weak-magnetism and induced-pseudoscalar terms.

INTRODUCTION

STUDY of μ-meson capture in hydrogen ($\mu^-+p\rightarrow n+\nu$) might allow detection of some as yet unobserved terms in the interaction Hamiltonian. The capture rates and angular distributions of recoil neutrons are affected not only by known vector and axial-vector couplings, but also by the presumed induced-pseudoscalar[1] and weak-magnetism[2] couplings and two hypothesized "second-class" couplings.[3] The weak-magnetism and induced-pseudoscalar effects are predicted by definite theories. The second-class interactions are allowed by the invariance principles known to govern the weak interactions. They would be expected if one accepts the principle of the renormalizibility of first-order weak interactions. On the other hand, complete absence of second-class interactions would indicate a relationship between the weak interactions and isotopic spin such as the conserved vector current theory, which predicts that the vector interactions are all first class. A simple theory that has vector = axial-vector coupling and no others has the peculiar feature that no capture takes place in a μ-mesonic atom in the hyperfine triplet state.[4] For this reason, the $F=1$ capture rate is a good measure of the deviation from this simple theory due to inequality of vector and axial-vector coupling constants and (or) the presence of any other couplings. Capture by individual protons provides a clearer test of the theory than capture by more complex nuclei, since in analysis of capture on the complex nuclei one is beset by uncertainties in computing the μ-meson wave function and nuclear wave function for the initial state and the nuclear wave function for the final state. Moreover, if one employs a nuclear model with a core of nucleons of zero total angular momentum with one proton in orbit around it, the spin of the core protons is correlated neither to the spin of the nucleus nor to the spin of the meson. This means that, although the probability of capture by the one outermost proton may be highly sensitive to the hyperfine configuration of the meson, the probability of capture by any of the many core protons is completely insensitive to the hyperfine configuration, and captures by the core largely obscure the hyperfine effect.[5] On the other hand, for hydrogen there is no uncertainty of nuclear structure. Muon wave func-

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† National Science Foundation Predoctoral Fellow.
[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[3] S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[4] J. Bernstein, T. D. Lee, C. N. Yang, and H. Primakoff, Phys. Rev. **111**, 313 (1958).
[5] The Pauli exclusion of neutrons tends to inhibit core capture, however; see the calculation of H. Uberall, Phys. Rev. **121**, 1219 (1961).