Nuclear Reactions Induced by Neutrinos of Intermediate Energy*

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Nuclear reactions with electron emission leading to all final nuclear states, induced by neutrinos of definite energy (e.g., from a source of stopped μ^-), are discussed on the basis of a Fermi gas model of the nucleus. It is shown that the cross section is not reduced very much below the sum of the single free nuclear cross sections, and that the emitted electron may be identified by kinematics. This offers the possibility for demonstrating the identity of "electron" and "muon" neutrinos.

THE question of a possible identity of "electron" and "muon" neutrinos, ν_e and ν_μ, can be solved experimentally by demonstrating that electrons may be produced in reactions initiated by a ν_μ, such as

$$\nu_\mu + n \rightarrow p + e^-. \quad (1)$$

This requires neutrino beams of high intensity because of the smallness of the cross sections involved: For the reaction (1), the cross section reaches at most 0.75×10^{-38} cm^2 for high neutrino energy, $\gtrsim 1$ Bev (depending on the types of weak interaction form factors assumed[1]). For neutrinos of intermediate energy, such as given off by stopped pions and muons,[2] the cross section is at least an order of magnitude smaller. Nevertheless, use of such neutrinos may present the following distinct advantages[2–4]:

(1) They may be available with higher intensity;
(2) Their source has a well-defined spatial location;
(3) They possess a well-defined energy (e.g., ν_μ's from nuclear capture of negative muons have a fairly narrow spectrum around[2] 90 Mev).

Using these last two properties, electrons from the reaction (1) or its analog in a complex nucleus,

$$\nu_\mu + A_Z \rightarrow A_{Z+1}^* + e^-, \quad (2)$$

may possibly be identified with reasonable certainty by their kinematics. This was shown to be the case[4] for the reaction

$$\nu_\mu + \mathrm{C}^{12} \rightarrow \mathrm{N}^{12}{}_{\text{ground}} + e^- \quad (3)$$

(the ground state of N^{12} being identified by its delayed positron decay), but the total cross section was found to be no more than $\sim 4 \times 10^{-41}$ cm^2. We shall show here on the basis of a Fermi gas model[5] of the nucleus that the reaction (2), leading to all possible final states A_{Z+1}^*, offers a larger cross section to the neutrinos—down from the sum of cross sections of the individual neutrons, due to the action of the exclusion principle in the nucleus, only by a factor $\sim \frac{1}{2}$ (for 90-Mev neutrino energy)—, and at the same time still permits to some extent a kinematical interpretation. The Fermi-gas picture is admittedly quite a rough model; the cross sections involved, however, are so small that few details in them will be distinguishable anyway in the foreseeable future. The model will thus be sufficient to give us an idea of the gross features.

We use the nonrelativistic weak-interaction Hamiltonian,

$$H = g(\phi_p^\dagger \psi_e^\dagger [1 + X\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}^N][(1+\gamma_5)/\sqrt{2}]\psi_\nu \phi_n), \quad (4)$$

of sufficient accuracy for intermediate energies, and obtain the cross section of reaction (1) with a moving neutron:

$$d\sigma = \frac{p^2 dp}{\pi} \frac{d\Omega}{4\pi} g^2 (1+3X^2)(1+\xi w)\delta(\nu - p - \Delta). \quad (5)$$

Here, p is the magnitude of the electron momentum $\mathbf{p}$ (its rest mass being neglected), ν is the neutrino momentum (all in the laboratory), $w = \mathbf{p}\cdot\boldsymbol{\nu}/p\nu$, $\xi = (1-X^2)(1+3X^2)^{-1}$, g is the universal Fermi coupling constant $g = 10^{-5} M^{-2}$, M = proton mass, and $X = -1.21$, the ratio of Gamow-Teller and Fermi coupling constants. Kinematics gives

$$\begin{aligned} \Delta &\equiv E_p - E_n = \nu - p, \\ -\mathbf{q} &\equiv \mathbf{p}_p - \mathbf{p}_n = \boldsymbol{\nu} - \mathbf{p}, \end{aligned} \quad (6)$$

with neutron and proton energies and momenta E_n, $\mathbf{p}_n$; E_p, $\mathbf{p}_p$. We decompose $\mathbf{p}_n$ into two components $\mathbf{p}_{\parallel}$ and $\mathbf{p}_{\perp}$, parallel and perpendicular to $-\mathbf{q}$; this gives

$$2M\Delta = 2p_{\parallel}q + q^2 \quad \text{or} \quad p_{\parallel} = q^{-1}[M(\nu - p) - \tfrac{1}{2}q^2]. \quad (7)$$

We multiply (5) by $2\pi(4\pi P^3/3)^{-1} dp_{\parallel} p_{\perp} dp_{\perp}$ and integrate over all momenta $\mathbf{p}_n$ subject to (6) and to the condition that $\mathbf{p}_n$ lie inside the neutron Fermi sphere of radius $P = 250$ Mev, and $\mathbf{p}_p$ outside the proton sphere because of the Pauli principle. The $dp_{\parallel}$ integration removes the energy δ function. This gives for the cross section per neutron (assuming $\frac{1}{2}A$ neutrons):

$$\frac{2}{A} d\sigma = g^2(1+3X^2) \frac{3M}{8\pi P^3} \frac{p_{\perp\max}^2 - p_{\perp\min}^2}{q} \times (1+\xi w) p^2 dp dw, \quad (8)$$

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[1] T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960); N. Cabibbo and R. Gatto, Nuovo cimento **15**, 304 (1960).
[2] H. Überall, Nuovo cimento (to be published).
[3] B. Pontecorvo, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 617.
[4] H. Überall, Phys. Rev. **126**, 1572 (1962).
[5] For its use in the related process of μ meson capture, cf. J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949); H. Überall, Nuovo cimento **6**, 533 (1957).

where for $P \geqslant p_{\parallel} \geqslant P-q$, the limits on $p_{\perp}^2$ are:

$$p_{\perp\max}^2 = P^2 - p_{\parallel}^2, \quad p_{\perp\min}^2 = 0, \tag{9a}$$

and for $P-q \geqslant p_{\parallel} \geqslant -\frac{1}{2}q$:

$$p_{\perp\max}^2 = P^2 - p_{\parallel}^2, \quad p_{\perp\min}^2 = P^2 - (p_{\parallel}+q)^2. \tag{9b}$$

The differential cross section (8) has been evaluated numerically for an incident neutrino energy $\nu = 90$ Mev, and is presented in Fig. 1 as a function of $x = p/\nu$, for various values of the cosine of the emission angle, w. It is seen that the spectrum of the emitted electron is more or less concentrated closely below the incident neutrino energy, depending on the emission angle. This confirms our initial statement that even for neutrino-initiated transitions to all final nuclear states, emitted electrons might still be identifiable kinematically. It is to be hoped that these predictions of the Fermi-gas model should not be modified too strongly in an actual nucleus which could contain neutrons with higher momenta.

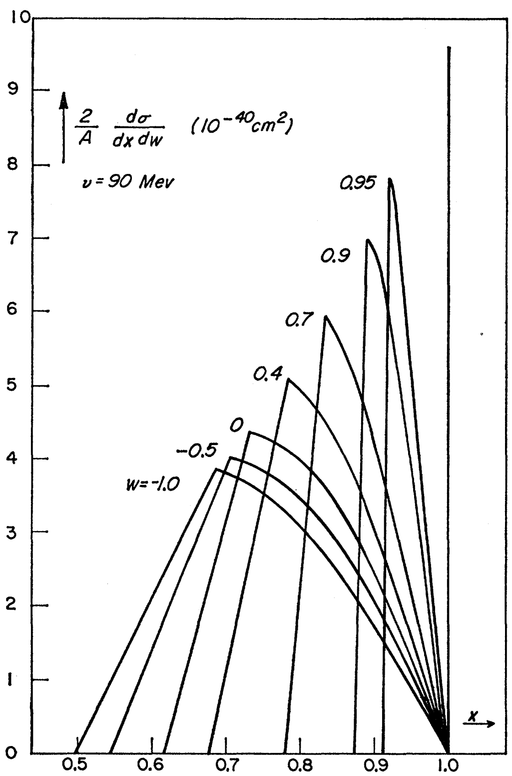

FIG. 1. Electron spectra from $\nu + A_Z \rightarrow A_{Z+1}^* + e^-$ at various emission angles, for 90-Mev neutrinos, on the basis of a Fermi-gas model.

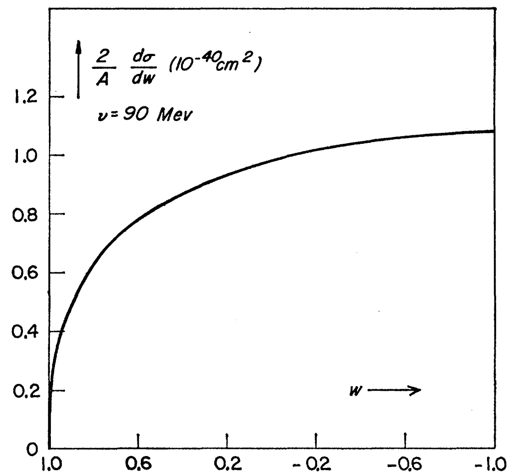

FIG. 2. Angular distribution of the electrons, for 90-Mev neutrinos.

A numerical integration of the curves of Fig. 1 leads to the angular distribution shown in Fig. 2, which is fairly isotropic. Its integration gives for the total cross section per neutron:

$$(2/A)\sigma = 1.8 \times 10^{-40} \text{ cm}^2. \tag{10}$$

This is to be compared to the cross section for a single neutron, which is essentially (neglecting details of recoil effects):

$$\sigma_n \cong \bar{x}^2 \sigma_0, \quad \sigma_0 = \pi^{-1}\nu^2 g^2 (1+3X^2) = 7.0 \times 10^{-40} \text{ cm}^2. \tag{11}$$

With an average electron momentum $\bar{x} \sim 0.75$, we find a ratio

$$(2/A)\sigma/\sigma_n \sim 0.46; \tag{12}$$

this shows that the exclusion principle in the nucleus does not reduce the reaction probability by a substantial amount below the single-nucleon cross section. One can thus conclude that nuclear reactions induced by neutrinos of a definite energy in the intermediate region may permit a demonstration of the identity of ν_μ and ν_e due to the not unreasonably small cross sections, and to the possibility of a kinematic identification of emitted electrons.