Resonance-Interference Method for Determining $K\Sigma N$ Parity

RICHARD H. CAPPS
Northwestern University, Evanston, Illinois
(Received December 20, 1961)

A method is given for determining the orbital parity of the $\pi+\Sigma$ state of the 1520-Mev, strangeness (-1), resonance by measuring angular distributions and hyperon polarizations resulting from the process $K^-+p\rightarrow\pi+\Sigma$. The method depends on the observation of interference between the resonance state and at least one other state, and on the assumption that the relative phase of the resonant and nonresonant state *increases* with energy. The validity of this assumption is discussed. Since the orbital parity of the K^-+p state may be determined by continuing the measurements from the resonance energy down to the K^-+p threshold energy, this method may yield the intrinsic $K\Sigma N$ parity. The various ambiguities in the determination of partial wave amplitudes from experimental data are discussed. It is shown that the many-channel unitarity condition may be useful in resolving some of the ambiguities.

I. METHOD

We consider the elementary particle interactions,

$$K^-+p\rightarrow\pi^0+\Sigma^0, \tag{1a}$$

$$K^-+p\rightarrow\pi^-+\Sigma^+, \tag{1b}$$

and make the customary assumptions concerning the spins and isotopic spins of the particles. Time-reversal and space-reflection invariance are assumed. The asymmetry in the $\Sigma^+\rightarrow\pi^0+p$ or $\Sigma^0\rightarrow\Lambda^0\rightarrow\pi^-+p$ decay may be used to determine the sign and magnitude of the Σ polarization.[1] In this paper we present a method for determining the relative intrinsic parity $\mathcal{P}$ of the $\bar{K}N$ and $\pi\Sigma$ pairs by means of angular distribution and polarization measurements involving a resonance. The method is applied to the 1520-Mev, strangeness (-1), isotopic spin 0 resonance discovered by Ferro-Luzzi, Tripp, and Watson.[2] An advantage of the method is that no "special" measurements are required; even if $\mathcal{P}$ were known, it would be useful to measure the Σ polarization and angular distribution in order to investigate the resonance.

We denote the partial wave amplitudes for either process (1a) or (1b) by the symbol $\mathbf{f}_{j,l(K),l(\pi)}$, where j is the total angular momentum, and $l(K)$ and $l(\pi)$ are the values of the orbital angular momentum in the $\bar{K}+N$ and $\pi+\Sigma$ states. The amplitudes are normalized in terms of the corresponding elements of the unitary S matrix by the relation $\mathbf{f}_\alpha=S_\alpha/(2i)$.

It is well known that the assumptions of angular momentum and parity conservation, together with Σ polarization and angular distribution measurements, are *not* sufficient to determine $\mathcal{P}$; some other dynamical assumption is necessary. We illustrate this ambiguity by considering the 1520-Mev resonance. The angular distributions[2,3] in the various $\pi+\Sigma$ charge states of the $K^-+p\rightarrow\pi+\Sigma$ reactions, and in K^-+p elastic scattering, suggest that the resonance is of angular momentum $\frac{3}{2}$. The resonance in $\pi+\Sigma$ production may be associated with any one of the four possible $j=\frac{3}{2}$ amplitudes, i.e., the amplitudes $\mathbf{f}_{\frac{3}{2},1,1}$ and $\mathbf{f}_{\frac{3}{2},2,2}$ (if $\mathcal{P}=1$), or $\mathbf{f}_{\frac{3}{2},2,1}$ and $\mathbf{f}_{\frac{3}{2},1,2}$ (if $\mathcal{P}=-1$). If the resonance amplitude interferes with some other partial-wave amplitude, one can determine the relative parities of the two amplitudes, but the over-all fourfold ambiguity remains. This ambiguity is a generalization of the ambiguity pointed out by Minami in pion-nucleon scattering[4] and may be stated concisely in the following way. For any set of partial-wave amplitudes, there are three other sets that lead to the same angular distribution and hyperon-polarization, if the nucleon is unpolarized. The transformations between these equivalent sets of amplitudes are[5]

$$\begin{aligned}\mathbf{f}_{j,+,+}&\rightleftarrows\mathbf{f}_{j,-,-}^*\rightleftarrows\mathbf{f}_{j,-,+}\rightleftarrows\mathbf{f}_{j,+,-}^*,\\ \mathbf{f}_{j,-,-}&\rightleftarrows\mathbf{f}_{j,+,+}^*\rightleftarrows\mathbf{f}_{j,+,-}\rightleftarrows\mathbf{f}_{j,-,+}^*,\end{aligned}\tag{2}$$

where the transformation between any two columns is to be applied simultaneously to all amplitudes. The symbols $+$ and $-$ denote $j+\frac{1}{2}$ and $j-\frac{1}{2}$. The first two of these sets are possible if $\mathcal{P}=1$, and the last two are possible if $\mathcal{P}=-1$.

We propose to resolve this ambiguity by using two dynamical principles. The first principle concerns energy dependence near $\bar{K}+N$ threshold, and enables one to determine the $\bar{K}+N$ orbital angular momentum of any amplitude. The second principle concerns the interference between resonating and nonresonating amplitudes and may be used to determine $\pi+\Sigma$ orbital angular momentum. Simultaneous application of both principles enables one to determine the initial and final

[1] The signs and approximate magnitudes of the Σ^+ and Λ^0 decay asymmetry parameters are given by E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters **7**, 285 (1961).

[2] M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters **8**, 28 (1962).

[3] L. W. Alvarez, *Proceedings of the Ninth Annual International Conference on High-Energy Physics, Kiev, 1959* (Academy of Science, U.S.S.R., 1960) and University of California Radiation Laboratory Report UCRL-9354, 1960 (unpublished).

[4] S. Minami, Progr. Theoret. Phys. (Kyoto) **11**, 213 (1954); S. Hayakawa, M. Kawaguchi, and S. Minami, *ibid.*, **11**, 332 (1954).

[5] These equations may be derived from the equations given by Richard H. Capps, Phys. Rev. **115**, 736 (1959), [Eqs. (7) and p. 739]. The amplitudes T defined on page 739 of this reference are equal to twice the corresponding amplitudes $\mathbf{f}$ of the present work, except that $T_{l+1,l}=-2\mathbf{f}_{j,+,-}$. This change of sign is arbitrary, and may be made by changing the sign of all $\bar{K}+N$ states in which $l(K)=j+\frac{1}{2}$, if $\mathcal{P}=-1$.

orbital angular momentum of the 1520-Mev resonance, and hence the intrinsic $K\Sigma N$ parity as well.

A. Energy-Dependence Principle

The possibility of reducing the ambiguity by making use of the energy dependence of the amplitudes is well known and is included here for the sake of completeness. The crucial fact is that at energies close to the $\bar{K}+N$ threshold, the partial wave amplitude $\mathbf{f}_{j,l(K),l(\pi)}$ is proportional to the $l(K)+\frac{1}{2}$ power of the center-of-mass system $\bar{K}$ momentum, provided that there are no freakish singularities present, such as a pole in the amplitude exactly at the threshold energy. Because of this principle, it is known that the large cross section for $\pi+\Sigma$ production at low K momentum results from interactions in the S state of the $\bar{K}+N$ system. If one measures the Σ polarization and angular distribution throughout the energy region between $\bar{K}+N$ threshold and the resonance, one can determine the $\bar{K}+N$ orbital angular momentum of all important amplitudes by means of their interference with the S state and with each other. The application of this principle to the 1520-Mev resonance has been discussed previously by the author.[6]

B. The Resonance-Interference Principle

It is seen from Eq. (2) that the correspondence between two equivalent sets of amplitudes involves complex conjugation if and only if the two amplitudes differ in the orbital angular momentum of the $\pi+\Sigma$ state. (This special property of the $\pi+\Sigma$ state results from the fact that the equivalence refers to hyperon-polarization measurements, with the nucleon unpolarized.) Hence it may be possible to determine $\pi+\Sigma$ orbital parity by making use of the fact that the analytic properties of reaction amplitudes are not invariant to complex conjugation. This principle is crucial in the cusp method of measuring parities.[7] We will make use of a different form of the principle, namely that the energy derivative of the phase of an amplitude passing through a narrow resonance must be *positive*.[8] This principle results from the causality assumption and was proved by Wigner.[9] If only elastic processes are possible, the Wigner theorem is expressed in terms of the magnitude of the center-of-mass system momentum $\hbar\mathbf{k}$ and the "range" a of the interaction; if $ka\gg1$, the theorem is simply, $d\delta/dk>-a$.

[6] R. H. Capps, Phys. Rev. Letters **6**, 375 (1961).

[7] A thorough discussion of the cusp method as applied to associated production, with particular reference to the Minami ambiguity, is given by M. Nauenberg and A. Pais, Phys. Rev. **123**, 1058 (1961). Dr. Nauenberg has pointed out to the author that for elastic processes, the Minami transformation should be written in the form $\mathbf{f}_{j,+,+}\rightleftarrows-\mathbf{f}_{j,-,-}{}^*$, in order to be consistent with unitarity.

[8] The phase of an inelastic amplitude is fixed within an additive factor of π by the requirement that the S matrix be symmetric.

[9] Eugene P. Wigner, Phys. Rev. **98**, 145 (1955). In this reference Wigner proves the theorem under the assumption that only one channel is open, but points out that the theorem may be extended to the many-channel case.

If one assumes that the 1520-Mev resonance exists in some linear combination of the $\pi+\Sigma$ and $\bar{K}+N$ states, and that the resonance width is 20 Mev,[10] application of the Wigner criterion implies that the interaction range would have to be $\gtrsim 14$ f in order for the phase to be decreasing with energy at the resonance peak. Such a large range is unreasonable, so the phase must be increasing.

The Wigner theorem may be stated in modern dispersion language in the following way. The partial wave amplitudes for the process $\bar{K}+N\rightarrow\pi+\Sigma$ contain "right-hand" branch cuts starting at the energies where the $\bar{K}+N$, $\pi+\Sigma$, and other channels become open. The amplitudes also contain poles and "left-hand cuts" which represent the effective forces on the system; we call these the "force singularities." The force singularities are not necessarily all on the real axis. Suppose that a resonance "bump" is observed experimentally, centering at the energy E_0 on the right-hand cut. If this bump is very narrow compared to the distance from the nearest force singularity, some other singularity, closer than the force singularities, is implied. Because of the causality principle, this singularity cannot lie in the upper half-plane, so it must be reached by continuing down through the right-hand cut (and must be on an unphysical sheet). If this singularlty is a pole, or can be represented by several poles, the fact that it lies below the point E_0 implies that it leads to a positive energy-derivative of the phase of the amplitude at real energies near E_0.

In order to apply this principle to the process (1a) or (1b), one must observe the interference between the resonant amplitude and some other partial-wave amplitude at two different energies in the resonance region. One then assumes not only that the resonant phase is increasing with energy but that the relative phase of the resonant and nonresonant amplitudes is increasing. (This additional assumption is certainly reasonable, and is discussed more fully in Sec. IV.) This condition permits the determination of the $\pi+\Sigma$ orbital angular momentum of the resonance. A specific application is given in the next section.

II. APPLICATION TO THE 1520 MEV RESONANCE

We consider either process (1a) or (1b) and assume that the 1520-Mev resonance occurs in a state of $j=\frac{3}{2}$. We adopt a notation that allows us to write one set of angular distribution and polarization equations, valid for all four possibilities related by the generalized Minami transformation, Eq. (2). The resonant amplitude is denoted by $\mathbf{f}_{\frac{3}{2},a}$ where a represents both the (unknown) $\bar{K}+N$ and $\pi+\Sigma$ orbital angular momenta. Frequently the comma will be omitted and the angular momentum subscripts $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ will be abbreviated to

[10] The value of the width is given by R. D. Tripp, M. Ferro-Luzzi, and M. B. Watson, Bull. Am. Phys. Soc. **7**, 49 (1962). The author wishes to thank Dr. Tripp for sending this information prior to publication. More complete measurements have led to a width of 16 Mev (see reference 2).

1, 3, and 5, i.e., $\mathbf{f}_{\frac{3}{2},a}=\mathbf{f}_{3a}$. The other $j=\frac{3}{2}$ amplitude that is allowed by parity conservation is denoted by $\mathbf{f}_{3b}$. The amplitudes for any angular momentum j are denoted by $\mathbf{f}_{j,a}$ and $\mathbf{f}_{j,b}$, the correspondence being such that the difference in $\bar{K}+N$ (or $\pi+\Sigma$) orbital angular momenta between any two amplitudes with identical second subscripts is equal to the corresponding difference between the total angular momenta. (That is, if $\mathcal{P}=-1$ and the resonance occurs in a $P_{\frac{3}{2}}$ state of the $\pi+\Sigma$ system, then $\mathbf{f}_{j,a}=\mathbf{f}_{j,+,-}$ for any j; $\mathbf{f}_{j,b}=\mathbf{f}_{j,-,+}$ for any j.) The absolute magnitude of the amplitude $\mathbf{f}_i$ is denoted by f_i, and $\eta_{i,i'}$ represents the phase of $\mathbf{f}_i$ minus the phase of $\mathbf{f}_{i'}$. If all amplitudes of $j>\frac{3}{2}$ are equal to zero, the general expressions for the angular distribution and hyperon polarization are[5]

$$
\begin{aligned}
k_K^2\frac{d\sigma}{d\Omega}=&f_{1a}^2+f_{1b}^2+(f_{3a}^2+f_{3b}^2)(3\cos^2\theta+1)\\
&+(f_{1a}f_{1b}\cos\eta_{1a,1b}+2f_{3a}f_{1a}\cos\eta_{3a,1a}\\
&+2f_{3b}f_{1b}\cos\eta_{3b,1b})2\cos\theta\\
&+(f_{3a}f_{1b}\cos\eta_{3a,1b}+f_{3b}f_{1a}\cos\eta_{3b,1a})(6\cos^2\theta-2)\\
&\quad+f_{3a}f_{3b}\cos\eta_{3a,3b}(18\cos^3\theta-10\cos\theta), \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
k_K^2P\frac{d\sigma}{d\Omega}=&[\mp f_{1a}f_{1b}\sin\eta_{1a,1b}\\
&\mp f_{3a}f_{1a}\sin\eta_{3a,1a}\pm f_{3b}f_{1b}\sin\eta_{3b,1b}\\
&\mp(f_{3a}f_{1b}\sin\eta_{3a,1b}-f_{3b}f_{1a}\sin\eta_{3b,1a})3\cos\theta\\
&\quad\mp f_{3a}f_{3b}\sin\eta_{3a,3b}(9\cos^2\theta-1)]2\sin\theta. \qquad (4)
\end{aligned}
$$

The angle θ is defined by $\cos\theta=\mathbf{k}_K\cdot\mathbf{k}_\pi/(k_Kk_\pi)$, where $\hbar\mathbf{k}_K$ and $\hbar\mathbf{k}_\pi$ denote the momenta of the K and π particles in the center-of-mass system. The quantity P is the fractional Σ polarization, measured in the direction of $\mathbf{k}_K\times\mathbf{k}_\pi$. The upper signs apply if the $\pi+\Sigma$ orbital angular momentum of the resonance state is 1, and the lower signs apply if a $\pi+\Sigma$ D state is involved. The goal of the resonance-interference method is to determine which set of signs is correct.

It is well known that if all partial waves corresponding to angular momenta greater than some fixed angular momentum may be neglected, the number of terms in the $\cos\theta$ expansions of $d\sigma/d\Omega$ and $(Pd\sigma/d\Omega)/\sin\theta$ is equal to the number of unknown amplitude magnitudes and relative phases. Unfortunately, the equations are not linear, so that there are often multiple solutions, in which case ambiguities other than the generalized Minami ambiguity are present. A detailed discussion of these extra ambiguities is given in Sec. III. At present we ignore the extra ambiguities and present a simple illustration to show how the method might work.

Preliminary evidence concerning the process $K^-+p\rightarrow\pi^0+\Sigma^0$ indicates that the number of π^0's produced in the angular region $\cos\theta>0$ is approximately equal to the number produced in the region $\cos\theta<0$ at all K lab-system momenta in the range 300 Mev/c–500 Mev/c, (corresponding to the center-of-mass system energy W in the range 1485–1560 Mev).[2] However, the polar-equatorial ratio $R_{pe}=(p-e)/(p+e)$, (where p indicates the number of π^0's produced in the region $|\cos\theta|>0.5$, and e the number in the region $|\cos\theta|<0.5$) varies in this energy range. A simple interpretation of these data is that the only two important amplitudes are the resonant amplitude $\mathbf{f}_{3a}$ and the $j=\frac{1}{2}$ amplitude of the same parity, $\mathbf{f}_{1b}$. This interpretation can be tested by the polarization measurements, for it predicts that the $\cos\theta\sin\theta$ term of $Pd\sigma/d\Omega$ is appreciable at some energy in the resonance region, and that the $\sin\theta$ term of $Pd\sigma/d\Omega$ is small.[6] The polarization measurements have not yet been completed, so this interpretation, as well as the remarks about preliminary data made below, is presented here only to illustrate the application of the resonance-interference method.

The peak of the resonance occurs near 400-Mev/c K momentum, ($W\approx1520$ Mev) as observed from measurements of the total $I=0$ hyperon-production cross section.[2] The polar-equatorial ratio R_{pe} need not peak at the same energy though, since this ratio is very sensitive to the interference term involving $f_{3a}f_{1b}\cos\eta_{3a,1b}$ of Eq. (3). Preliminary data concerning the process $K^-+p\rightarrow\pi^0+\Sigma^0$ seem to indicate that R_{pe} is increasing at 400 Mev/c,[2] implying that $\cos\eta_{3a,1b}$ is increasing at this energy.[2] If we assume, from the causality principle of Sec. I(B), that $\eta_{3a,1b}$ is increasing in the resonance region, then this angle must lie in the range $-\pi<\eta_{3a,1b}<0$ at 400 Mev/c. If this is so, it is seen from Eq. (4) that a definite sign is predicted for the $\cos\theta\sin\theta$ term in $Pd\sigma/d\Omega$ at 400 Mev/c, and this sign is *opposite* for the two assumptions concerning the $\pi+\Sigma$ orbital parity of $\mathbf{f}_{3a}$. Hence, a measurement of this polarization correlation may determine the $\pi+\Sigma$ orbital parity of the resonance.

It should be pointed out that one cannot assume before measurements are made that the amplitudes $\mathbf{f}_{3a}$ and $\mathbf{f}_{1b}$ are approximately out of phase at the resonance peak, since there is no reliable way to predict the phase of $\mathbf{f}_{1b}$ at present. It may be that $\mathbf{f}_{3a}$ and $\mathbf{f}_{1b}$ are nearly in phase at 400 Mev/c, in which case the ratio R_{pe} should peak sharply as a function of energy and the $\cos\theta\sin\theta$ term in $Pd\sigma/d\Omega$ should change sign near the resonance peak. The direction of this sign change would then indicate the $\pi+\Sigma$ orbital parity of the resonance.

If the lack of interference between states of opposite parity actually occurs in the $K^-+p\rightarrow\pi^0+\Sigma^0$ process, and persists at all energies between the resonance and the K^-+p threshold, then the application of the energy-dependence principle of Sec. I(A) is simple and leads to the conclusion that the resonance results from a K^-+p D wave.[6]

III. AMBIGUITIES OTHER THAN MINAMI AMBIGUITIES

A. Generalized Yang Ambiguities

Ambiguities other than the generalized Minami ambiguity may exist because of the nonlinearity of the relations for the amplitudes, Eqs. (3) and (4). For example, if $\mathcal{P}=1$, so that the production amplitude may be written in the form $T=A+iB\boldsymbol{\sigma}\cdot\mathbf{k}_K\times\mathbf{k}_\pi$, the differential cross section and hyperon polarization are invariant to the simultaneous substitutions $A\rightarrow A^*$ and $B\rightarrow -B^*$. This is a generalization of the ambiguity pointed out by Yang in connection with pion-nucleon scattering.[11] If the above transformation is combined with the Minami transformation of Eq. (2), two generalized Yang ambiguities result. These are given below, in the notation of Sec. II.[5]

First generalized Yang ambiguity

$$\begin{aligned}\mathbf{f}_{j,a}&\rightleftarrows(2j)^{-1}[\mathbf{f}_{j,a}^*+(2j-1)\mathbf{f}_{j-1,b}^*],\quad(\text{all } j),\\ \mathbf{f}_{j-1,b}&\rightleftarrows(2j)^{-1}[(2j+1)\mathbf{f}_{j,a}^*-\mathbf{f}_{j-1,b}^*],\quad(\text{all } j>\tfrac{1}{2}).\end{aligned}\tag{5}$$

The second generalized Yang ambiguity is obtained by reversing the labels a and b in Eqs. (5).

These ambiguities are particularly troublesome in the present context because they involve complex conjugation, and hence complicate the use of the increasing-phase criterion to resolve the generalized Minami ambiguity. Fortunately, however, the generalized Yang transformations (unlike the Minami transformations) split a pure partial-wave amplitude into a linear combination of amplitudes, so that these transformations often lead to an implausible set of amplitudes.

B. Ambiguities Involving Only $j=\frac{1}{2}$ and $j=\frac{3}{2}$ Amplitudes

We now assume that only $j=\frac{1}{2}$ and $j=\frac{3}{2}$ amplitudes are present and further assume that the $\cos^3\theta$ term in the angular distribution and the $\sin\theta\cos^2\theta$ term in $Pd\sigma/d\Omega$ are both small, so that only one $j=\frac{3}{2}$ amplitude is appreciable. Hence, only the three amplitudes $\mathbf{f}_{3a}$, $\mathbf{f}_{1b}$, and $\mathbf{f}_{1a}$ need be considered. The determination of the three amplitude magnitudes and two relative phases is then formally equivalent to the problem considered by Crawford *et al.*[12] in connection with $K+\Lambda$ production, and leads in general to four solutions. Thus there is a new fourfold ambiguity in addition to the generalized Minami ambiguity.

It is expected that the criterion that only one amplitude varies rapidly near the resonance peak will be sufficient to resolve this new ambiguity, but the details of the resolution will depend on the observed behaviour of the amplitudes. We again illustrate with the ideal case in which the correct solution involves only the resonant amplitude $\mathbf{f}_{3a}$ and the slowly varying amplitude $\mathbf{f}_{1b}$. The $\cos\theta$ term in $d\sigma/d\Omega$ and the $\sin\theta$ term in $Pd\sigma/d\Omega$ then vanish. Two of the four solutions imply finite values for all three amplitudes $\mathbf{f}_{3a}$, $\mathbf{f}_{1b}$, and $\mathbf{f}_{1a}$, so aligned so that no interference between states of opposite parity is apparent. This requires a double accident at all energies; namely, if the phase of $\mathbf{f}_{1a}$ is chosen to be zero, then $\mathrm{Re}\mathbf{f}_{1b}=-2\,\mathrm{Re}\mathbf{f}_{3a}$ and $\mathrm{Im}\mathbf{f}_{1b}=\mathrm{Im}\mathbf{f}_{3a}$. Of course the alternate assumption that the complex amplitude $\mathbf{f}_{1a}$ is nearly zero may also be considered a double accident at all energies, but it is a much more plausible one.

The other two of the four solutions involve only the amplitudes $\mathbf{f}_{3a}$ and $\mathbf{f}_{1b}$ and are related by the first generalized Yang transformation, Eq. (5). In the incorrect solution, both amplitudes vary rapidly in the resonance region. If $\mathbf{f}_{3a}$ refers to the $\bar{K}+N$ $D_{\frac{3}{2}}$ state, and the measurements are continued down to the region of low $\bar{K}+N$ momentum, the incorrect solution produced by the first Yang transformation will imply that $\mathbf{f}_{3a}=-2\mathbf{f}_{1b}\sim k_K{}^{\frac{1}{2}}$ in the low-momentum region, in violation of the behavior expected for D waves. We conclude that the new fourfold ambiguity can be resolved, so that the methods of Sec. I can be used to resolve the remaining (generalized Minami) ambiguity.

C. Ambiguity Involving $j=\frac{5}{2}$ Amplitude

One cannot argue convincingly that 1520 Mev is such a low energy that no $j=\frac{5}{2}$ amplitude should be important. (This is especially true if $\mathcal{P}=1$, so that a $D_{\frac{5}{2}}\rightarrow D_{\frac{5}{2}}$ amplitude is possible.) In this section we relax our assumption concerning high angular-momentum states to the extent of allowing $j=\frac{5}{2}$ amplitudes to be present. The multiplicity of possible fits to the data is then doubled, because of the second generalized Yang ambiguity. Suppose that there actually are no $j=\frac{5}{2}$ amplitudes present, and that the resonant amplitude is $\mathbf{f}_{3a}$. The second Yang transformation implies that the data may equally well be fit by assuming that the resonance involves both $\mathbf{f}_{3a}$ and $\mathbf{f}_{5b}$ in the definite proportion $\mathbf{f}_{3a}=-\frac{1}{4}\mathbf{f}_{5b}$. Of course, such a proportionality relation between complex amplitudes is not likely to be satisfied throughout the resonance region. However, it is clear from these considerations that a pure $j=\frac{5}{2}$ resonance (with no $j=\frac{3}{2}$ amplitudes present) may be hard to distinguish experimentally from a $j=\frac{3}{2}$ resonance. The interference terms between $\mathbf{f}_{5b}$ and $\mathbf{f}_{1b}$ depend on θ in the same way as those between $\mathbf{f}_{3a}$ and $\mathbf{f}_{1b}$. The angular distribution for pure $j=\frac{5}{2}$ scattering is of the form $(15/4)\cos^4\theta-\frac{3}{2}\cos^2\theta+\frac{3}{4}$; if interference terms are present one can use angular distribution measurements to distinguish between a $j=\frac{3}{2}$ and a $j=\frac{5}{2}$ resonance only by

---

[11] See H. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. 2, pp. 72–75. The transformation between the Fermi and Yang amplitudes is $A\rightarrow A$, $B\rightarrow -e^{i\phi}B$. The transformation $A\rightarrow A^*$, $B\rightarrow -B^*$ of the present work usually would violate unitarity if only one channel were open. On the other hand, the phase factor ϕ in the Fermi-Yang transformation may be selected so that unitarity (for S and P waves) is preserved. However, because of this phase factor, the polarizations predicted by Fermi and Yang phase shifts are not simply related, in general.

[12] F. S. Crawford, Jr., *et al.*, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN* (CERN, Scientific Information Service, Geneva, 1958), p. 323.

determining the coefficient of the $\cos^4\theta$ term, which requires accurate measurements. Furthermore, if one assumes that the angular momentum of the resonance is $\frac{3}{2}$ when it is actually $\frac{5}{2}$, or vice versa, and applies the resonance-interference method of Sec. I(B), he will obtain the wrong answer concerning the $\pi+\Sigma$ orbital parity of the resonance. This can be seen from the fact that the confusion between $j=\frac{3}{2}$ and $j=\frac{5}{2}$ amplitudes is related to a generalized Yang transformation, which involves complex conjugation. Therefore, it is important to try to distinguish between a $j=\frac{3}{2}$ and a $j=\frac{5}{2}$ resonance not only by the angular distribution, but by the unitarity principle. This is done in Sec. V. First, though, we shall discuss the unitarity principle for many-channel reactions.

IV. MANY-CHANNEL UNITARY CONDITIONS

We consider that part of the unitary S matrix referring to a particular angular momentum and parity, and suppose that the number of open channels is n. Time-reversal invariance is assumed, so that we may choose the S matrix to be symmetric.[13] If channels consisting of more than two particles are open, the number of S-matrix channels is actually infinite, since some continuous variable (such as the energy of the di-pion in the $2\pi+\Lambda$ channel) must be used to parameterize the channels. We restrict ourselves to a finite number of channels for simplicity, although the formalism of this section is easily generalized to include continuous channel variables. The applications of the formalism in Sec. V would not be changed in an essential manner if an infinite number of $(2\pi+\Lambda)$ channels were included.

The symmetric unitary matrix S may be written in terms of a real symmetric matrix Q by the formula $S=(1+iQ)/(1-iQ)$.[14] The number of independent real parameters necessary to characterize S at a particular energy is the number necessary to characterize Q, $\frac{1}{2}n(n+1)$. A convenient choice of these parameters may be obtained in the following way. One may diagonalize the real Hermitian matrix Q with a real unitary (orthogonal) transformation. Such a transformation matrix may be thought of as a rotation in a real n-dimensional space. We choose as parameters the $\frac{1}{2}n(n-1)$ independent coefficients of the transformation matrix and the n phase shifts characterizing the diagonalized S or $\mathbf{f}$ matrix. These parameters will be continuous functions of energy. Since there are $\frac{1}{2}n(n+1)$ different elastic and inelastic processes possible, all of the elements of the S matrix could be determined experimentally if all the relevant cross sections could be measured.

The elements of the $\mathbf{f}$ matrix between the physical states α and β may be written in terms of the transformation coefficients and eigenphase shifts δ_i in the following way:

$$\mathbf{f}_{\alpha\beta}=\sum_i x_{\alpha i}x_{\beta i}e^{i\delta_i}\sin\delta_i, \tag{6}$$

where the transformation coefficient $x_{\alpha i}$ may be regarded as the cosine of the angle between the α axis and the i axis in the n-dimensional real space. Angular momentum and parity subscripts are suppressed. Certain sums of the squares of $\mathbf{f}_{\alpha\beta}$ satisfy very simple conditions, i.e.,

$$\mathrm{Im}\mathbf{f}_{\alpha\alpha}=\sum_\beta|\mathbf{f}_{\alpha\beta}|^2=\sum_i x_{\alpha i}^2\sin^2\delta_i, \tag{7}$$

and

$$\sum_{\alpha\beta}|\mathbf{f}_{\alpha\beta}|^2=\sum_i\sin^2\delta_i. \tag{8}$$

The theorem of Wigner discussed in Sec. I(B) implies that if the phase of one of the $\bar{K}+N\rightarrow\pi+\Sigma$ partial wave amplitudes is changing rapidly, it must be increasing. The question arises as to whether or not we are justified in assuming that this phase changes rapidly at all, however. The situation is quite different from the case in which only one channel is open. In the one-channel case, the partial-wave cross section is proportional to $\sin^2\delta$, so that a narrow bump in the energy dependence of the cross section requires a large $|d\delta/dE|$. On the other hand, the cross section for an inelastic process does not depend on the phase of the amplitude, since the phase and magnitude are independent. The assumption of a rapidly changing phase in the energy region of a bump in an inelastic cross section may be justified in two different ways. First, we may use the dispersion-theory argument of Sec. I(B) that a narrow bump implies the existence of a pole or poles close to the real axis on an unphysical sheet. In the second argument, we note from Eq. (8) that a rapid change in energy of the quantity $\sum_{\alpha\beta}|\mathbf{f}_{\alpha\beta}|^2$ implies a rapid change in one or more of the δ's. The quantity $\sum_{\alpha\beta}|\mathbf{f}_{\alpha\beta}|^2$ in the partial-wave state corresponding to the bump in the $\pi+\Sigma$ production cross section cannot be measured completely, but it almost certainly does contain a bump at an energy near 1520 Mev. If this bump is narrow enough and rapidly decreasing phases of the various amplitudes are to be avoided, then one or more of the eigenphase shifts must increase through 90° in the neighborhood of 1520 Mev. It may be seen from Eq. (6) that a rapidly increasing (resonant) eigenphase can lead to a nearly constant or decreasing phase in a particular amplitude $\mathbf{f}_{\alpha\beta}$, but only if the nonresonant contribution to $\mathbf{f}_{\alpha\beta}$ is as large or larger than the resonant contribution, and the two contributions interfere destructively. In such a case no peak would be observed in the particular process $\alpha\rightarrow\beta$ in the resonance region, so that such an anomalous behavior can be ruled out for the $K^-+p\rightarrow\pi+\Sigma$ process near the 1520-Mev resonance. Hence, the phase of the resonant amplitude must increase rapidly.

Since 1520 Mev is not close to the threshold of any strongly coupled two-body channel, a rapid change in the phase of an amplitude is likely only if the amplitude resonates (i.e., one of the eigenphases increases through 90°). We conclude that if only one partial wave is resonant near 1520 Mev in the $K^-+p\rightarrow\pi+\Sigma$ process, the relative phase of the resonant and any nonresonant amplitude must increase rapidly in the resonance region.

[13] F. Coester, Phys. Rev. 89, 619 (1953).
[14] See R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), page 296.

V. RESTRICTIONS IMPOSED BY UNITARITY ON SPIN OF RESONANCE

If only one channel were open the angular momentum of the resonance could be obtained easily by comparing the peak of the partial wave cross section with the maximum allowed by unitarity. If several channels are open this procedure may be generalized by using the formalism of Sec. IV. In this section we discuss the application of the unitarity principle to the I-spin zero, 1520-Mev resonance.

In one simple model, often applied to a narrow resonance involving more than one channel, it is assumed that the amplitudes referring to the partial-wave state of the resonance may be represented by simple Breit-Wigner formulas, i.e., $\mathbf{f}_{\alpha\beta}=\frac{1}{2}\Gamma_\alpha^{\frac{1}{2}}\Gamma_\beta^{\frac{1}{2}}/[(E_0-E)-i\frac{1}{2}\Gamma]$. The constant Γ_α is the partial width of channel α, and the total width Γ is equal to $\Gamma=\sum_\alpha\Gamma_\alpha$. This model corresponds to the assumptions that all the eigenphase shifts δ_i of Eq. (6) are zero except for one resonating phase shift δ_1. The partial widths are given in terms of the transformation coefficients by the relations $x_{\alpha 1}^2=\Gamma_\alpha/\Gamma$. One can show that in this simple model, the sizes of the resonance peaks in $\bar{K}+N$ elastic scattering, and in hyperon production[2] are consistent with the assignment $j=\frac{3}{2}$ for the resonance, but inconsistent with the assignment $j>\frac{3}{2}$. We will attempt to distinguish the possibilities $j=\frac{3}{2}$ and $j=\frac{5}{2}$ using a slightly different model, described below.

We assume that the $(2\pi+\Lambda)$ channels may be neglected, so that only the $\bar{K}+N$ and $\pi+\Sigma$ channels are open in the resonance state. In this two-channel case Eqs. (6) and (7) may be written in the form

$$|\mathbf{f}_{K\pi}|^2=\tfrac{1}{4}\sin^2(2\phi)\sin^2(\delta_1-\delta_2), \tag{9}$$

$$|\mathbf{f}_{KK}|^2+|\mathbf{f}_{K\pi}|^2=\sin^2\phi\,\sin^2\delta_1+\cos^2\phi\,\sin^2\delta_2, \tag{10}$$

where the subscripts K and π refer to the physical $\bar{K}+N$ and $\pi+\Sigma$ channels, and δ_1 and δ_2 are the eigenphase shifts. The diagonalizing matrix is equivalent to a rotation in a plane; we have chosen ϕ to be the angle between the $\pi+\Sigma$ axis and the δ_1 axis. The resonance is assumed to result from a rapid increase of δ_1 through 90°.

We denote the sum of the $K^-+p\to K^-+p$ and $K^-+p\to\bar{K}^0+n$ cross sections in the partial wave of the resonance by σ_K, and the corresponding $K^-+p\to\pi+\Sigma$ cross section by σ_π. It is seen from Eq. (9) that even if the nonresonant phase shift δ_2 is finite, the peak value of $|\mathbf{f}_{K\pi}|^2$ is given by $|\mathbf{f}_{K\pi}|^2_{\text{peak}}=\frac{1}{4}\sin^2(2\phi)$. The effect of the finite δ_2 is to shift the peaks of σ_K and σ_π in opposite directions. If we assume a $j=\frac{5}{2}$ resonance, and assume that the peak value of σ_π is in the range 6–18 mb $=(0.3\text{–}0.9)(\pi/k_K^2)$, then $0.2<4|\mathbf{f}_{K\pi}|^2_{\text{peak}}<0.6$. (We use the formula $\sigma_\pi=(2j+1)(\pi/k_K^2)|\mathbf{f}_{K\pi}|^2$, which applies for a pure $I=0$ state, since the K^-+p system has a probability $\frac{1}{2}$ of being in an $I=0$ state.) The inequality $4|\mathbf{f}_{K\pi}|^2_{\text{peak}}<0.6$ implies either that $|2\phi|<50°$ or $|\pi-2\phi|<50°$. These conditions imply that the resonance is much more strongly coupled to one channel than to the other. The possibility $|\pi-2\phi|<50°$, together with Eq. (10), implies that σ_K should have a peak of $\gtrsim 80$ mb. The possibility $|2\phi|<50°$ implies that the difference between σ_K at 400 Mev/c and at energies just outside the resonance should be less than 4 mb. Both alternatives contradict the data.[2]

On the other hand, if the resonance spin is $\frac{3}{2}$, then $0.3<4|\mathbf{f}_{K\pi}|^2_{\text{peak}}<0.9$. The possibility $4|\mathbf{f}_{K\pi}|^2_{\text{peak}}\sim 0.9$ implies that the two channels are coupled with approximately equal strength, and leads to the possibility that σ_K (peak) is about 10 mb or somewhat larger, in agreement with the data. We conclude that the cross-section measurements are strong evidence against the hypothesis of a $j=\frac{5}{2}$ resonance.

It is interesting to note that if only one channel is open, the alternate sets of amplitudes obtained by applying the generalized Yang transformations to the correct set generally violate the unitarity principle. Similarly, if only two channels are open, the alternate solutions obtainable by applying the Yang transformations simultaneously in two channels generally violate unitarity. For example, consider the condition $\mathbf{f}_{3a}=-\frac{1}{4}\mathbf{f}_{5b}$, [discussed in Sec. III(C)], which leads to the same experimental effects as a pure $j=\frac{3}{2}$ amplitude. In the one-channel case, in which the phase and magnitude of an amplitude are functions of only one parameter, such a condition can be satisfied only if both amplitudes vanish. Similarly, the simultaneous equalities $\mathbf{f}_{3a}=-\frac{1}{4}\mathbf{f}_{5b}$ for two different processes requires four real relations between the amplitudes; such relations can be satisfied only for particular values of the amplitudes if only two channels are open, since the amplitudes in any partial wave state are functions of only three real parameters in the two-channel case. Because of experimental limitations, this last consideration is not expected to be as useful in eliminating the possibility of a $j=\frac{5}{2}$ resonance as the considerations based on the sizes of the peaks in the various cross sections.

VI. CONCLUDING REMARKS

Our principle conclusion is that the $\pi+\Sigma$ orbital angular momentum of the 1520-Mev resonance can be determined by the resonance-interference method of Secs. I and II, provided that sufficiently accurate angular distribution and polarization measurements can be obtained. If the $\bar{K}+N$ orbital angular momentum of the resonance is determined also by continuing the measurements to the region of low K momentum, the intrinsic $K\Sigma N$ parity can be determined.

The resonance-interference method may also be applied to other resonances, such as any resonances at higher energy that may be discovered in the $\bar{K}+N\to\pi+Y$ processes. Of course such higher resonances would not be as favorable as the 1520-Mev resonance for determination of the $K\Sigma N$ intrinsic parity. However, the resonance-interference method provides a useful test for any tentative assignment concerning the amplitude re-

sponsible for a higher-energy resonance in the strangeness (-1) states. The method may also be applied to the process $\pi+N\rightarrow K+\Lambda$ in the region of center-of-mass system energy around 1730 Mev, since a $K+\Lambda$ resonance may exist in this energy region.[15] Since the $K+\Lambda$ state is of greater rest mass than the $\pi+N$ state, the energy-dependence and resonance-interference methods measure the same quantity, the $K+\Lambda$ orbital angular momentum of the "resonance," so that these methods cannot be combined to determine the relative intrinsic parities of the particles from associated-production data. However, the resonance-interference method may be useful in this case also as a test of any specific resonance model.

Note added in proof. Since this paper was written Tripp, Watson, and Ferro-Luzzi have obtained the result of odd $K\Sigma N$ parity by using this method [Phys. Rev. Letters 8, 175 (1962)]. The best polarization measurements are obtained for the $\Sigma^{+}+\pi^{-}$ events, and the situation is similar to that described near the end of Sec. 2, i.e., the amplitudes $\mathbf{f}_{3a}$ and $\mathbf{f}_{1b}$ are nearly in phase at 400 Mev/c and the change of sign of the polarization determines the parity.

ACKNOWLEDGMENT

The author would like to thank Dr. G. C. Wick for pointing out an ambiguity in the notation of the original manuscript.

[15] A resonance model of the $\Lambda+K$ production data is given by A. Kanazawa, Phys. Rev. 123, 997 (1961). References to the experimental work are given.

PHYSICAL REVIEW VOLUME 126, NUMBER 4 MAY 15, 1962

Lee-Yang Vector Field and Isotropy of the Universe*

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received December 29, 1961)

Lee and Yang suggested that, associated with heavy-particle conservation, there may exist an analog of the electromagnetic field, a field for which nucleons and antinucleons would serve as positive and negative "charges." It is shown that the null result from a recent repetition of the Eötvös experiment implies that, if it exists, the Lee-Yang interaction is at most only 10^{-7} of the gravitational interaction. This great weakness does not imply that the field does not exist. However, with the assumption of the isotropy of the average matter distribution of the universe, the Lee-Yang antisymmetric field tensor vanishes when averaged over sufficiently large volumes. This implies that, if the Lee-Yang field exists, nucleons and antinucleons are present in equal numbers in the universe, presumably gathered in matter and antimatter galaxies. However, it is found that the fact that a copious stream of γ rays is not present in the cosmic rays can be used to exclude such numbers of antimatter galaxies. It is concluded that the Lee-Yang field probably does not exist.

CONNECTED with baryon conservation, Lee and Yang[1] have suggested that there may exist a neutral, massless, gauge-invariant vector field analogous to the electromagnetic field. Nucleons and antinucleons would serve as positive and negative "charges," the sources of this field, and in motion constitute "currents." The tremendous circulating nucleon currents in the galaxy could result in the generation of the Lee-Yang analog of the magnetic field. The Lee-Yang analog of the electric field would lead to a repulsion between matter, tending to reduce the gravitational acceleration. It is evident that if it exists, the Lee-Yang interaction is weak, or matter would fall up, not down.

As shown by Lee and Yang, the null result of the Eötvös experiment[2] imposes a severe restriction upon the strength of the Lee-Yang field. The Eötvös experiment demonstrated with an accuracy of about 5 parts in 10^9 that gravitational accelerations are independent of the atomic weight of the falling body. More recently[3] it has been shown that the accelerations toward the sun of copper and lead are equal to an accuracy of one part in 10^{10}.

Consider the force exerted on an atom by the sun through the Lee-Yang analog of an electric field. This force is proportional to A, the nucleon number of the atom, and is independent of the motions of the nucleons inside the nucleus. Hence, the Lee-Yang force is independent of the binding energy of the nucleus, but this implies that the resulting acceleration of the atom depends upon the mass (and binding energy).

It is easily shown that the fractional difference in acceleration, toward the sun, of two different substances of different atomic weight is

$$\frac{\delta g}{g}\cong\gamma\left\langle\frac{A}{M}\right\rangle_s\left[\left(\frac{A}{M}\right)_2-\left(\frac{A}{M}\right)_1\right], \qquad (1)$$

* This work was supported in part by research contracts with the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] T. D. Lee and C. N. Yang, Phys. Rev. 98, 1501 (1955).

[2] R. v. Eötvös, D. Pekar, and E. Fekete, Ann. phys. 68, 11 (1922).

[3] R. H. Dicke, P. Roll, D. Currott, and R. Krotkov (to be published).