Monte Carlo Calculation of Transport of Fast Electrons

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Transmission, backscattering, and spatial distribution of ionization energy loss of electrons in thick absorbers have been calculated by a random-sampling method by use of an IBM-7090 digital computer. Results are presented for 0.4–4.0 Mev electrons normally incident on C, Al, and Cu absorbers. Ratios of extrapolated range of monoenergetic electrons to total range at incident energy were found to vary from 50 to 90%, to depend only weakly on incident energy, and to decrease markedly with increasing atomic number of absorber. Back-scattering coefficients were found to decrease with increasing energy in this range, and are in agreement with the recent experimental data of Frank; values of $(4.2\pm0.9)\%$ and $(16.6\pm1.8)\%$ were obtained for 2-Mev electrons incident on Al and Cu.

INTRODUCTION

THE penetration of fast electrons through matter is characterized by multiple scattering in the Coulomb fields of absorber atoms and statistical variations in rate of ionization energy loss, these processes having a more pronounced effect than in the case of penetration of heavier charged particles. The transport properties are of interest in connection with numerous experimental applications involving electron beams, and have received considerable attention in the literature. An excellent review of this topic has been given by Birkhoff.[1] The scattering and straggling that occur in absorbers, thin compared to the range of incident electrons, have been treated analytically with results that agree satisfactorily with experiment. The transport problem becomes more complicated when the absorber thickness reaches a value such that mean scattering angle is comparable to a radian. Birkhoff comments that for this case the problem becomes so difficult theoretically that, in general, recourse only to experimental data can be made. One aspect of the thick absorber problem, namely spatial distribution of ionization energy loss, has been treated by Spencer[2] by use of the moments method. The computational method was described in Birkhoff's review article[1]; tabulations of computed results have been published subsequently.[2]

The thick-absorber problem is amenable to a Monte Carlo calculation, though the required volume of computation is a deterrent unless a very fast computer is available. The number of elastic scatterings undergone by a fast electron in slowing down is, of course, quite large so that simulation of the individual collisions in a Monte Carlo calculation is not feasible. Instead the trajectory may be subdivided into a relatively small number (of the order of 10–50) of short segments along each of which the net angular deflection is fairly small (say less than 20 degrees on the average), so that the results of analytical treatments give satisfactory approximations to the probability distribution of the net angular deflection. The problem may then be treated as if the electron suffered a discrete energy loss and angular deflection in a single collision at the end of each of the trajectory segments. The choice of segment lengths is a compromise between the conflicting requirements of acceptably small amount of numerical computation and the relatively small deflection angle necessitated by approximations involved in the analytical treatments of angular distributions. The effects of range straggling, i.e., statistical fluctuations in rate of ionization energy loss, may be readily included in calculations, though this effect is of relatively minor significance compared to the multiple Coulomb scattering.

Monte Carlo calculations of electron transport have been performed by Hebbard and Wilson,[3] Leiss, Penner, and Robinson,[4] Sidei and Higasimura,[5] and MacCallum.[6] Hebbard and Wilson[3] calculated the energy spectrum of electrons emerging from various thicknesses of Al and Au foils irradiated by normally incident 1-Mev electrons. Both scattering and range straggling were included. Leiss, Penner, and Robinson[4] calculated transmission curves for electrons of energies of 5–55 Mev normally incident on carbon. They assumed a Gaussian scattering distribution and included the effects of range straggling. They also took into account the effects of radiative energy losses, which are appreciable at these high electron energies. Sidei and Higasimura's[5] calculations, which treated the effects of Coulomb scattering only on the penetration of 2-Mev electrons through Al, were performed without the aid of an automatic computer. By suitable processing of their data they were able to extract from their 100 sample trajectories the transmission curves for lower incident energies, and also the dependence of the 2-Mev back-scattering coefficient on incident angle. MacCallum[6] has performed calculations of back-scattering coefficients in the energy range 10–500 kev and reported a relativistic decrease in this coefficient. The back-scattering problem

[1] R. D. Birkhoff, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 34.
[2] L. V. Spencer, National Bureau of Standards Monograph No. 1, 1959 (unpublished).
[3] D. F. Hebbard and P. R. Wilson, Australian J. Phys. **8**, 90 (1955).
[4] J. E. Leiss, S. Penner, and C. S. Robinson, Phys. Rev. **107**, 1544 (1957).
[5] T. Sidei, T. Higasimura, and K. Kinosita, Mem. Fac. Eng., Kyoto Univ. **19**, 220 (1957).
[6] C. MacCallum, Bull. Am. Phys. Soc. **5**, 379 (1960).

has also been discussed in recent papers by Everhart[7] and Archard,[8] but there do not appear to have been any detailed calculations at energies above 500 kev other than the case treated by Sidei.

Although there have been several Monte Carlo calculations of electron transport, as noted above, it is clear that there has not been any systematic coverage of a wide range of absorber materials and incident electron energies comparable to Spencer's coverage of the problem of energy dissipation. This paper presents results of recent Monte Carlo calculations which combine features of the earlier work by other groups and extend the range of cases studied. Results presented here include transmission and back-scattering fractions for 0.4- to 4-Mev electrons normally incident on C, Al, and Cu absorbers, dependence of extrapolated range on incident electron energy and atomic number of absorber, and a comparison of calculated spatial distribution of ionization energy loss with similar results obtained by Spencer.[2]

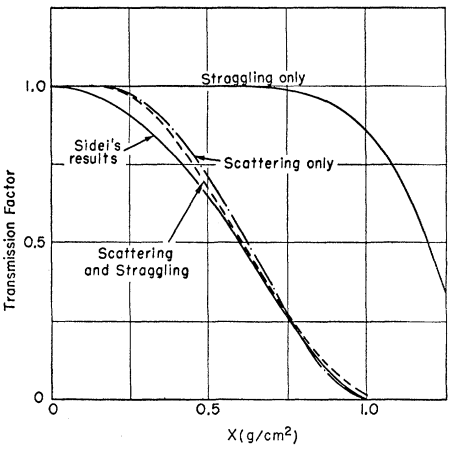

FIG. 1. Calculated transmission curves for 2-Mev electrons normally incident on aluminum. Results of calculations which include scattering and straggling separately and in combination are shown for comparison. Also shown are results of calculations by Sidei.

COMPUTATIONAL METHOD

The computational procedure is a straightforward application of the Monte Carlo method,[9,10] except that, as mentioned above, the effects of multiple collisions along each of several track segments are treated as individual collisions with energy loss and angular deflection distributions given by the appropriate analytical treatments. Typical electron trajectories are constructed by sampling from these distributions, with each electron being incident at a specified angle θ_0 and energy E_0. Each electron is assumed to be stopped when its energy is reduced below a specified value $E_{\min}$ such that the residual range is a small fraction (5% or less) of the range at the incident energy. In constructing the trajectories, no boundary effects are taken into account, i.e., the absorber is assumed to extend in all directions from the source a distance greater than the total electron range. The effects of boundaries are taken into account in abstracting backscattering and transmission fractions from the calculated trajectories; e.g., in determining the transmission fraction for a given absorber thickness, the passage of a particular electron across the exit boundary is counted only once. In this way several absorber thicknesses are treated in a single computer run.

The form of the distribution of effective polar scattering angles was taken from the work of Molière[11]; the tabulations of scattering functions calculated by Bethe[12] were used since these are more complete than the original tables given by Molière. The satisfactory accuracy of Molière's formulation has been borne out by results of various experiments,[13] though refinements have been suggested by Nigam and Sundaresan[14] and by Blunck and Leisegang.[15] The average rate of ionization energy loss was taken from the tables of Nelms.[16] In treating the range straggling, Landau's[1] straggling function was used, with the amount of energy loss normalized so as to give an average loss in agreement with the Bethe-Bloch[17] formula; numerical values of the latter were taken from Nelms' tables. The present treatment of straggling differs slightly from that of Leiss, Penner, and Robinson[4] in that the large-energy-loss tail was approximated by the expression given by Landau, rather than a $1/(\Delta E-\Delta E_p)^2$ formula.

The length of trajectory segments are determined by a specified value of σ/E which was usually chosen of the order of 0.02, where σ is the segment length in g/cm^2 and E is in Mev. Sampling from the various probability distributions, $f(x)$, is accomplished by generation of pseudorandom numbers, R (uniformly distributed on the interval from 0–1), and solution of the equation $\int_0^x f(x)dx=R$. For the azimuthal angles, this leads to $\phi=2\pi R$. For polar scattering angles and for energy losses the equation is solved by interpolation in tables of normalized cumulative probabilities.

The calculations were performed by use of an IBM-7090 digital computer. The computer program was prepared in two stages; the initial version of the program treated only multiple scattering, i.e., energy

[7] T. E. Everhart, J. Appl. Phys. **31**, 1483 (1960).
[8] G. D. Archard, J. Appl. Phys. **32**, 1505 (1961).
[9] G. Goertzel and H. Kalos, *Progress in Nuclear Energy* (Pergamon Press, New York, 1958), Series I, Vol. II.
[10] U. Fano, L. V. Spencer, and M. J. Berger, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 38.
[11] G. Molière, Z. Naturforsch **3a**, 78 (1948).
[12] H. A. Bethe, Phys. Rev. **89**, 1256 (1953).
[13] A. O. Hanson, L. H. Lanzl, E. M. Lyman, and M. B. Scott, Phys. Rev. **84**, 634 (1951).
[14] B. P. Nigam, M. K. Sundaresan, and T. Y. Wu, Phys. Rev. **115**, 491 (1959).
[15] I. Blunck and S. Leisegang, Z. Physik **128**, 500 (1950).
[16] A. T. Nelms, *Energy Loss and Range of Electrons and Positrons*, National Bureau of Standards Circular No. 577 (U. S. Government Printing Office, Washington, D. C., 1956).
[17] H. A. Bethe, Ann. Physik **5**, 325 (1930).

losses were calculated on the basis of average rates of loss as a function of energy. This program was used to test various portions of the calculational scheme and, in particular, to obtain results for comparison with the work of Sidei.[5] The calculated transmission fractions as a function of thickness of Al absorber were in satisfactory agreement with Sidei except for very thin absorbers. However, the calculated back-scattering coefficient for 2-Mev electrons normally incident on Al was about 3% as compared to Sidei's value of 15%. In an effort to resolve this discrepancy the problem was recalculated with various values of σ/E, sample size, and pseudorandom-number sequence. A low value of back-scattering coefficient (3% to 5%) was obtained from all these calculations as well as from calculations made with the later version of the program.

The second version of the program, with which results reported here were obtained, is arranged so that the user can select inclusion of the effects of multiple scattering and range straggling separately or in combination. Calculated results include optional combinations of transmission and back-scattering fractions, energy spectra of transmitted and back-scattered electrons, and spatial distribution of ionization energy loss. Transmission fractions are calculated for ten absorber thicknesses in a single application of the program. It may be of interest also to extract from the computation, data pertaining to the angular distribution of electrons as a function of absorber depth[18]; this, however, has not been provided for in the current version of the program.

RESULTS

Figure 1 shows calculated values of the fraction of electrons transmitted through various thicknesses of aluminum for the case of normal incidence of 2-Mev electrons. The effects of scattering and range straggling have been treated separately as well as in combination. As expected, the inclusion of range straggling has only a relatively minor effect on the results, principally resulting in a higher tail on the transmission curve at the greater thicknesses. Also shown for comparison is the result of a calculation by Sidei.[5]

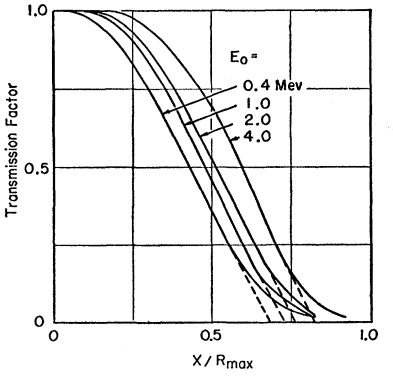

FIG. 2. Calculated transmission curves for electrons of various energies normally incident on aluminum.

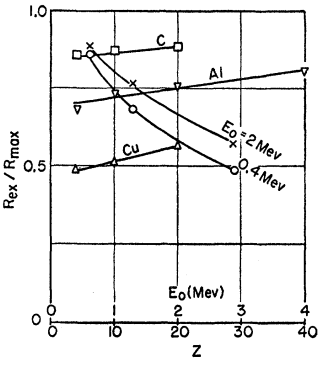

FIG. 3. Dependence of R_{ex}/R_{max} on incident energy and on atomic number of absorber.

Transmission curves for electrons of various energies normally incident on aluminum are shown in Fig. 2. In this figure and elsewhere except when specifically mentioned, both scattering and range straggling have been included. For simplicity the abscissa in Fig. 2 is the ratio of absorber thickness to total range at the incident energy. It is found, in agreement with experimental data, that the transmission curves have relatively straight portions which permit of meaningful extrapolation to obtain an "extrapolated range" R_{ex}. Calculations have also been performed for C and Cu absorbers and R_{ex}/R_{max} obtained, where R_{max} is total range at incident energy as obtained from the tables of Nelms.

The dependence of R_{ex}/R_{max} on energy of incident electrons and atomic number of absorber material is depicted in Fig. 3. These values are only weakly dependent on incident energy but decrease markedly with increasing atomic number of absorber. The values of R_{ex}/R_{max} for aluminum are in satisfactory agreement with experimental data as summarized by Katz and Penfold.[19]

Calculated backscattering coefficients and results of a few measurements[20–23] are shown in Fig. 4. The plotted experimental points, with the exception of Frank's data, are for orientation purposes only. These were taken from the original papers[20–22] without a careful study of their normalization, which was apparently somewhat arbitrary in some of the earlier measurements. Although such a trend is not evident from the experimental points shown in Fig. 4, it has been frequently stated in the literature[7–8] that, at least for incident electron energies below a few hundred kev, back-scattering coefficients are essentially independent of energy. The present computational results, which of course pertain to relativistic energies, are found to

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[19] L. Katz and A. S. Penfold, Revs. Modern Phys. **24**, 28 (1952).
[20] V. H. Frank, Z. Naturforsch **14a**, 247 (1959).
[21] H. Kanter, Ann. Physik **20**, 9 (1957).
[22] J. G. Trump and R. J. Van de Graaff, Phys. Rev. **75**, 44 (1949).
[23] H. Kulenkampff and W. Spyra, Z. Physik **137**, 416 (1954).

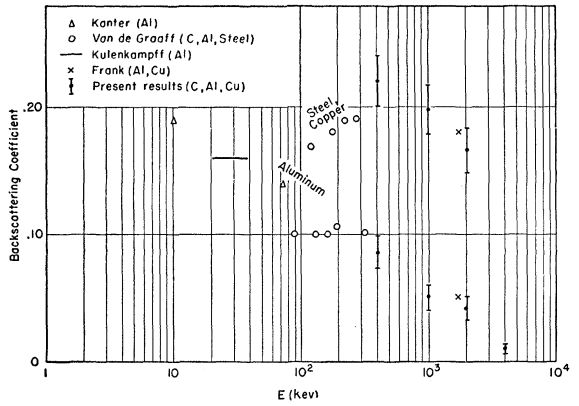

FIG. 4. Calculated backscattering fractions compared to results of various experiments. The only available experimental data above 1 Mev are those of Frank.

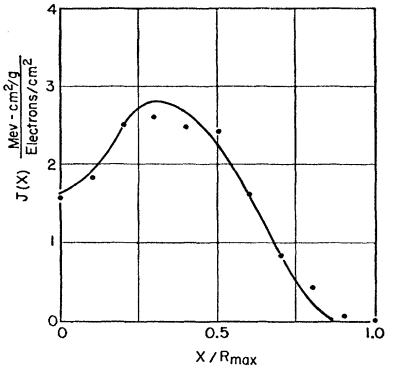

FIG. 5. Spatial distribution of ionization energy loss of 4-Mev electrons in aluminum. Plotted points are results of present Monte Carlo calculations. Curve shows results of Spencer's moments method calculations.

decrease markedly with increasing incident energy. The present results are in satisfactory agreement with results of recent experiments by Frank,[20] which represent the only available experimental data in this energy range.[20a]

Figure 5 shows calculated values of the spatial distribution of ionization energy loss. The weighting factor sec θ which is applied in "scoring the game" for rate-of-energy-loss results in a statistical uncertainty which is greater than that of the corresponding transmission fraction, and the calculated points in Fig. 5 show a noticeable scatter. There is at least qualitative agreement with the results of moments methods calculations, which are also shown in Fig. 5. Both calculations indicate a maximum rate of loss at an absorber depth of $\frac{1}{3}$ to $\frac{1}{2}$ of the total electron range, both for this case and other combinations of electron energy and absorber material. The higher tail on the distribution at great depths in the case of the present results presumably arises from the effects of range straggling, which was not included in the moments methods calculations.

[20a] *Note added in proof*. Results of a thorough experimental study of backscattering of 1-, 2-, and 3-Mev electrons from a number of materials have very recently been published [K. A. Wright and J. G. Trump, J. Appl. Phys. **33**, 687 (1962)]. Calculated results presented here are in good agreement with these experimental data.

ACKNOWLEDGMENTS

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