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Virtual Particles*

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We define particles or resonances which do not correspond to the usual Breit-Wigner type poles in the partial wave amplitudes. They are described in terms of the trajectory of the poles of the S matrix in the complex angular-momentum plane as a function of energy, when this trajectory does not quite reach a physical value of J. Range and scattering parameters are used to determine the trajectories near the threshold, which in turn are related to high-energy cross sections for processes in which these particles are exchanged in the cross channels. The trajectories for the two definitely known examples, namely the $I=0$, π-π, S-wave "virtual state", and the single n-p S-wave "virtual state", as well as the triplet n-p trajectory, are determined.

IN this note we use Regge's continuation to complex angular momentum[1,2] in order to define and describe particles or resonances that do not correspond to a usual Breit-Wigner type pole of the partial wave amplitudes. From the point of view of the analytic structure of the S matrix regarded as a simultaneous function of angular momentum and energy, however, there is only a quantitative difference between these and ordinary particles. There are at least two definite examples of such resonances, namely, the $I=0$, π-π, S-wave "virtual state"[3] and the well-known singlet n-p, S-wave "virtual state,"[4] both near the threshold. The general situation is described here, which is valid for any value of angular momentum and energy, and it is suggested that some of the higher spin resonances recently observed in strong interactions might belong to this category. Furthermore, we determine the connection between range and scattering parameters and the trajectory in the complex angular-momentum plane of poles of the S matrix that corresponds to virtual particles. This trajectory in turn is related to high-energy cross sections for processes in which these particles are exchanged in the cross channel.[2,5]

Our considerations are based on the fact that the total two-body elastic-scattering amplitude can be written, using a Watson-Sommerfeld transformation in the complex l plane,[1] as a sum of—in general few—pole terms plus a regular remainder. The pole terms control the asymptotic behavior of the amplitude, and also the bound states and resonances in the partial wave amplitudes. One can thus separate explicitly the singular parts of the amplitudes. This procedure is a substitute for the use of subtractions in the dispersion relations. For a given set of quantum numbers, the poles are separated by more than one unit of angular momentum. Therefore, in the vicinity of a bound state or resonance, the total amplitude may be approximated by a Regge-pole term of the form[6]

$$A(q,\cos\theta)=\beta(q)P_{\alpha(q)}(-\cos\theta)/\sin\pi\alpha(q), \qquad (1)$$

where $\alpha(q)$ is the position of the pole of the S matrix in the complex l plane as a function of the center of mass momentum q. The partial wave projections of Eq. (1) are

$$A(q,l)=(1/\pi)\beta(q)/\{[\alpha(q)-l][\alpha(q)+l+1]\}, \qquad (2)$$

which clearly shows the pole in the l plane at $l=\alpha(q)$. For $q^2<0$, $\alpha(q)$ is a real and increasing function of q; for $q^2>0$, $\alpha(q)$ has a positive imaginary part.

We first discuss the threshold behavior of $\alpha(q)$. It will be shown that the behavior of phase shifts near threshold is consistent with a square-root singularity of $\alpha(q^2)$ at $q^2=0$. Therefore, near $q^2=0$, $\alpha(q^2)$ can be written[7]

$$\alpha(q)=\alpha_R(0)+(-q^2)^{\frac{1}{2}}(d\alpha_I/dq)_0+\tfrac{1}{2}(d^2\alpha_R/dq^2)_0q^2. \qquad (3)$$

Thus, for small $q^2>0$, the imaginary and real parts of α are given, respectively, by

$$\alpha_I=q(d\alpha_I/dq)_0$$

and

$$\alpha_R=\alpha_R(0)+\tfrac{1}{2}(d^2\alpha_R/dq^2)_0q^2. \qquad (4a)$$

These expressions have also been verified numerically and for the triplet n-p scattering and give exactly the deuteron binding energy in terms of the scattering parameters.[8] Once the three parameters in Eq. (3) are determined, the behavior of *all* partial wave amplitudes due to the pole term (1) and near $q^2=0$ are obtained by inserting Eqs. (4) into Eq. (2). We do not write the

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[7] This equation is a special case for $\alpha_R(0)\approx 0$ of the exact threshold behavior of the poles discussed elsewhere; [A. O. Barut, D. E. Zwanziger, Phys. Rev. (to be published)].
[8] The deuteron case shows that, in the analytic continuation of Eq. (3) below threshold, we have to take the negative sign of the square root.

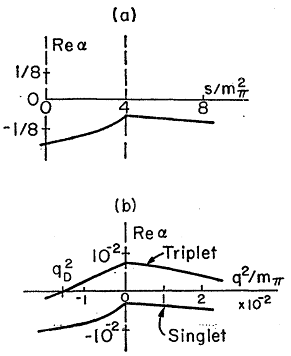

FIG. 1. Trajectory of the Regge poles near threshold. The real part of α is plotted against (a) $s=4(q^2+1)$ in the case of $I=0$, for a π-π virtual particle, and (b) q^2 in the case of n-p singlet and triplet poles.

TABLE I. Parameters of the Regge poles $\alpha=\alpha(q^2)$ near threshold $q^2=0$. Above threshold the imaginary part of α behaves as $\alpha_I=(d\alpha_I/dq)_0 q$, and the real part as $\alpha_R=\alpha_R(0)+\frac{1}{2}(d^2\alpha_R/dq^2)_0 q^2$. The residue of the pole is essentially given by $\beta(0)$, and $\alpha(s=0)$ will determine the power of the total cross section in the crossed channels.

Pole	$\alpha_R(q=0)$	$(d\alpha_I/dq)_0$ (m_π^{-1})	$(d^2\alpha_R/dq^2)_0$ (m_π^{-2})	$\beta(0)$ (m_π^{-1})	$\alpha(s=0)$
π-π, $I=0$[a]	$-1/16$	$1/8$	$-1/32$	$-\pi/4$	$-3/16$
n-p, $^1S_0$[b]	-2.5×10^{-2}	0.42	-0.39	-2.57	...
n-p, $^3S_1$[b]	6×10^{-2}	0.36	-0.18	-2.27	...

[a] Evaluated from Eq. (7) on the basis of a scattering length $a\approx 2m_\pi^{-1}$ (see reference 2) and an assumed range $r_0\approx r\approx\frac{1}{2}m_\pi^{-1}$. Here $\alpha(s=0)$ is evaluated from Eq. (9), $s=4(q^2+1)$.

[b] Evaluated from range and scattering parameters fitted by a square-well potential (see reference 3). For the triplet state, the approximate expression for the position of the trajectory at the threshold is $\alpha_R(q=0)\approx(\frac{1}{3})E_B^{\frac{1}{2}}r_0$, where E_B is the binding energy and r_0 the range of the forces (see reference 7). The calculations for this case are nonrelativistic.

general expression here, but if $l=0$ and we have $q^2(d^2\alpha_R/dq^2)_0\ll 1$, we obtain

$$A(q,l0)\cong(\beta/\pi)/[\alpha_R(0)+cq^2+iq(d\alpha_I/dq)_0], \tag{5}$$

where $c=\frac{1}{2}(d^2\alpha_R/dq^2)_0-(d\alpha_I/dq)_0^2$. Equation (5) is precisely the amplitude corresponding to the effective-range approximation $q\cot\delta=a^{-1}+rq^2/2$, and we obtain by comparison

$$[\alpha_R(0)/(d\alpha_I/dq)_0]=-1/a,$$
$$\beta(0)/\alpha_R(0)=a\pi,$$

and

$$(d^2\alpha_R/dq^2)_0/(d\alpha_I/dq)_0-2(d\alpha_I/dq)_0=-r.$$

If we neglect the curvature $(d^2\alpha_R/dq^2)_0$ for the time being, we find

$$\begin{aligned}(d\alpha_I/dq)_0&=r/2;\\ \alpha_R(0)&=-\tfrac{1}{2}(r/a);\\ \beta(0)&=-(\pi/2)r,\end{aligned} \tag{6}$$

where a is the scattering length and r is the effective range.

We give now a quite independent calculation of the parameters of Eq. (6), using the range of the forces involved. If we take as a model a short-range potential, a square well of range r_0 and strength $V_0=(\pi/2-\epsilon)^2/r_0^2$ not quite strong enough to make an S-wave bound state, the position of the pole in the l plane is given by $\alpha_R(0)\cong-\pi\epsilon/4$. The scattering length a is related to ϵ by $a=2r_0/\pi\epsilon$. Hence we have

$$\begin{aligned}\alpha_R(0)&=-\tfrac{1}{4}r_0/a;\\ (d\alpha_I/dq)_0&=\tfrac{1}{4}r_0;\\ \beta(0)&=-(\pi/2)r_0.\end{aligned} \tag{7}$$

The two estimates agree roughly. Their difference gives us the curvature of the trajectory

$$(d^2\alpha_R/dq^2)_0=\tfrac{1}{4}r_0(r_0/2-r). \tag{8}$$

Both for singlet and triplet n-p Regge poles as well as for the $I=0$, π-π pole, the curvature as determined from the effective range formula is negative, i.e., the trajectories turn at the threshold. This behavior expresses the fact that there are no S-wave resonances without an S-wave bound state. For trajectories near $l=1$ (or higher), the situation is different. Here we can have a P-wave resonance without a P-wave bound state, and in this case we expect the curvature to be positive. The real part of α as a function of s or q^2 is shown in Fig. 1, and the parameters are given in Table I. It is important to note that the parameters of the cusp depend only on the range of the forces and not on scattering length. Below the threshold, q^2 is less than zero, and $\alpha(q)$ is real and is given by[7]

$$\alpha\equiv\alpha_R(q)=\alpha_R(0)-(-q^2)^{\frac{1}{2}}(d\alpha_I/dq)_0+\tfrac{1}{2}(d^2\alpha_R/dq^2)_0q^2. \tag{9}$$

This expression allows us to extrapolate α to the point $s=0$ or $q^2=m_\pi^2$ in the case of the π-π, $I=0$ pole (or the so-called ABC pole). The quantity $\alpha_{\mathrm{ABC}}(s=0)$ is also shown in Table I. The exchange of the $I=0$, π-π system in the crossed channel results in a total cross section in the forward direction which varies as $E^{-[1-\alpha(0)]}$, where E is the laboratory energy.[2,5]

Using the above method, we can also discuss the threshold behavior of a Regge trajectory very close to an integer l. Again, if we assume that the single pole dominates the lth partial wave in question, we can compare the amplitude (2) with that corresponding to the effective-range formula $q^{2l+1}\cos\delta=a^{-1}+rq^2/2$. Then near $q^2=0$ we find that[7]

$$\alpha_R=\alpha_R(0)+Aq^2,$$
$$\alpha_I=Bq^{2l+1},$$

or

$$\alpha=\alpha_R(0)+B(-q^2)^{(2l+1)/2}+Aq^2.$$

This discontinuity is superimposed upon a generally smooth trajectory at the threshold.

We now discuss the general situation where a resonance is observed without the trajectory of the pole crossing an integer value of l or J. First, the threshold can occur, in principle, close to an integer $l\neq 0$. This case may be expected to be qualitatively the same as the case $l=0$ discussed above. More interesting is the following situation. In the case of resonances, the function $\alpha_R(E)$ is an increasing function of E even above the threshold, except possibly for a small cusp at the

threshold. When $\alpha_R(E)$ becomes equal to an integer, Eq. (2) gives a Breit-Wigner resonance if $\alpha_R(E)$ is assumed to vary linearly with E locally near E_r.[5] We consider the case where the $\alpha_R(E)$ curve turns very close to a physical integer.[9] In this case the expansion of $\alpha_R(E)$ is of the form

$$\alpha_R(E)=\alpha_R(E_r)+\tfrac{1}{2}(E-E_r)^2(d^2\alpha_R/dE^2)+\cdots. \quad (10)$$

The partial wave amplitude is given by

$$A(E,l)\simeq\frac{(2/\pi)\beta(E)/[(2l+1)(d^2\alpha_R/dE^2)]}{(E-E_r)^2+i(\Gamma/2)+C}, \quad (11)$$

where $\Gamma/2\cong 2\alpha_I/(d^2\alpha_R/dE^2)_0$, and C is a very small constant. This amplitude corresponds to two energy poles $E=E_r\mp(\Gamma^{\frac{1}{2}}/2)(1-i)$. Therefore, if in Eq. (11) a single-pole term is taken literally as the total amplitude, one obtains a resonance cross section that is approximately given by $1/[(E-E_r)^4+(\Gamma/2)^2]$. If we take one of the energy poles only, the resonance shape becomes $1/[(E-E_r-\Gamma^{\frac{1}{2}}/2)^2+\Gamma/4]$. Such a resonance can be of importance only if the curvature at the turning point is small. No examples of resonances of this type are known at present. At any rate, the virtual particles, although somewhat different in character, are special manifestations of the poles of the S matrix in the complex angular-momentum plane.

[9] If exchange potential is assumed, only alternate values of l give physical resonances.

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Long-Range Scalar Interaction*

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The long-range scalar field, associated with a neutral, massless, boson, has been generally considered to be nonexistent. This belief is based on the lack of overt effects, observed in the laboratory, from such a field. It is shown that if this long-range interaction were to exist, it would of necessity be weak. The physical reason for this is the large contribution, having its origin in the enormous amount of matter at great distance in the universe, to the magnitude of the scalar. By comparison, the contribution of local matter is miniscule, leading to a weak interaction of about the same strength as gravitation. Furthermore, it is shown that such an interaction, in its effects, would be very similar to gravitation and could be distinguished only with difficulty. It is concluded that there is not yet a compelling observation which could be used to exclude the long-range scalar interaction.

THE neutral, massless, boson fields play a uniquely important role in the universe. They are the sources of the quasi-static long-range interactions by which distant parts of the universe make their presence felt in the laboratory. If it be assumed that nature abhors a field more complicated than tensor, i.e., an elementary particle spin greater than 2, then only three types of fields require consideration, scalar, vector, and tensor.

Examples of vector and tensor fields are known, in the form of electromagnetism and gravitation. While electromagnetism apparently plays an important role in the dynamics of the galaxy, there is little reason to believe that, aside from radiation effects it is important for cosmology. In fact, with the usual assumption that the universe is uniform and isotropic when averaged over large volumes, the average electric charge density must be zero, for the isotropy of space requires the vanishing of electric and magnetic fields, implying in turn the vanishing of average charge and current densities.

The metric tensor field associated with inertial and and gravitational forces is presumably an instrument through which the mass distribution of the universe makes its presence felt in the laboratory. It is presumed that, in accordance with Mach's Principle, the local inertial coordinate systems are determined by the mass distribution of the universe. However, the appropriate boundary conditions upon the metric field, which would exhibit generally this unique dependence upon the mass distribution, have not yet been formulated. According to general relativity this is the sole local influence of distant matter, and, in accordance with the equivalence principle, in a freely falling, nonrotating laboratory there are no observable gravitational effects having their origin in distant matter (aside from, generally weak, tidal effects).

If one were to have some small faith in the proposition that nature is not capricious, that the physical world is

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