threshold. When $\alpha_R(E)$ becomes equal to an integer, Eq. (2) gives a Breit-Wigner resonance if $\alpha_R(E)$ is assumed to vary linearly with E locally near E_r.[5] We consider the case where the $\alpha_R(E)$ curve turns very close to a physical integer.[9] In this case the expansion of $\alpha_R(E)$ is of the form

$$\alpha_R(E)=\alpha_R(E_r)+\tfrac{1}{2}(E-E_r)^2(d^2\alpha_R/dE^2)+\cdots. \quad (10)$$

The partial wave amplitude is given by

$$A(E,l)\simeq\frac{(2/\pi)\beta(E)/[(2l+1)(d^2\alpha_R/dE^2)]}{(E-E_r)^2+i(\Gamma/2)+C}, \quad (11)$$

where $\Gamma/2\cong 2\alpha_I/(d^2\alpha_R/dE^2)_0$, and C is a very small constant. This amplitude corresponds to two energy poles $E=E_r\mp(\Gamma^{\frac{1}{2}}/2)(1-i)$. Therefore, if in Eq. (11) a single-pole term is taken literally as the total amplitude, one obtains a resonance cross section that is approximately given by $1/[(E-E_r)^4+(\Gamma/2)^2]$. If we take one of the energy poles only, the resonance shape becomes $1/[(E-E_r-\Gamma^{\frac{1}{2}}/2)^2+\Gamma/4]$. Such a resonance can be of importance only if the curvature at the turning point is small. No examples of resonances of this type are known at present. At any rate, the virtual particles, although somewhat different in character, are special manifestations of the poles of the S matrix in the complex angular-momentum plane.

[9] If exchange potential is assumed, only alternate values of l give physical resonances.

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Long-Range Scalar Interaction*

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The long-range scalar field, associated with a neutral, massless, boson, has been generally considered to be nonexistent. This belief is based on the lack of overt effects, observed in the laboratory, from such a field. It is shown that if this long-range interaction were to exist, it would of necessity be weak. The physical reason for this is the large contribution, having its origin in the enormous amount of matter at great distance in the universe, to the magnitude of the scalar. By comparison, the contribution of local matter is miniscule, leading to a weak interaction of about the same strength as gravitation. Furthermore, it is shown that such an interaction, in its effects, would be very similar to gravitation and could be distinguished only with difficulty. It is concluded that there is not yet a compelling observation which could be used to exclude the long-range scalar interaction.

THE neutral, massless, boson fields play a uniquely important role in the universe. They are the sources of the quasi-static long-range interactions by which distant parts of the universe make their presence felt in the laboratory. If it be assumed that nature abhors a field more complicated than tensor, i.e., an elementary particle spin greater than 2, then only three types of fields require consideration, scalar, vector, and tensor.

Examples of vector and tensor fields are known, in the form of electromagnetism and gravitation. While electromagnetism apparently plays an important role in the dynamics of the galaxy, there is little reason to believe that, aside from radiation effects it is important for cosmology. In fact, with the usual assumption that the universe is uniform and isotropic when averaged over large volumes, the average electric charge density must be zero, for the isotropy of space requires the vanishing of electric and magnetic fields, implying in turn the vanishing of average charge and current densities.

The metric tensor field associated with inertial and and gravitational forces is presumably an instrument through which the mass distribution of the universe makes its presence felt in the laboratory. It is presumed that, in accordance with Mach's Principle, the local inertial coordinate systems are determined by the mass distribution of the universe. However, the appropriate boundary conditions upon the metric field, which would exhibit generally this unique dependence upon the mass distribution, have not yet been formulated. According to general relativity this is the sole local influence of distant matter, and, in accordance with the equivalence principle, in a freely falling, nonrotating laboratory there are no observable gravitational effects having their origin in distant matter (aside from, generally weak, tidal effects).

If one were to have some small faith in the proposition that nature is not capricious, that the physical world is

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ultimately simple, then the simplest of the three fields, the scalar field, might be expected to exist and to play an important role in the universe. However, it is generally believed to be missing. The basis for this belief is the lack of any indication in laboratory experiments of such a scalar interaction.

While, the scalar interaction might be so weak as to be undetectable, this is generally considered unlikely, for gravitation is the only extremely weak interaction known. Apparently strong interactions (and moderately weak interactions) are much more common than the extremely weak. Also, it is argued that the interaction would need to be excessively weak, even when compared with gravitation, or it would appear as a gravitational anomaly in the Eötvös experiment and as an anomaly in the motions of the planets.

It is the purpose of this note to point out that both of these assumptions are incorrect, that the scalar interaction, if it exists, is expected to be very weak, and, with assumption discussed in paragraph (3) below, that the scalar interaction masquerades like gravitation, being almost indistinguishable from true gravitation.

The long-range scalar interaction (neutral, massless, scalar field) has been discussed by Bergmann,[1] in curved space as part of the gravitational field by Jordan[2] and others,[3–5] and as a matter field in curved space by Dicke.[6] The most important properties of a scalar interaction are summarized here:

(1) If a scalar field φ interacts with a particle, the particle's mass is a function of the scalar.

(2) The equations of motion of a particle in a gravitational and a scalar field are

$$(d/d\tau)(mu_i)-\tfrac{1}{2}mg_{jk,i}u^ju^k+(dm/d\varphi)\varphi_{,i}=0. \quad (1)$$

(3) To avoid difficulties with the Eötvös experiment, one must assume a common functional dependence upon φ for all elementary particles, $m=m_0f(\varphi)$, with f the same for all particles.

(4) Equation (1) is obtained from the variational principle,

$$0=\delta\int m(-g_{ij}u^iu^j)^{\frac{1}{2}}d\tau. \quad (2)$$

However the factor f can be taken under the integral sign and combined with g_{ij} to define a new metric tensor $\bar{g}_{ij}=f^2g_{ij}$. Thus, the scalar interaction can be combined with the tensor interaction to give a new effective gravitational interaction, associated with a new metric tensor. The scalar interaction no longer appears explicitly.

(5) The source strength of a body for the generation of a scalar field is given by an integral over the body of the contracted energy-momentum tensor of matter. Because of the virial theorem, and for a stationary body, this is equal, when time averaged, to the negative of the mass of the body. Thus, for those cases where experimental tests exist, both the scalar and gravitational fields are proportional to the same parameter, the mass of the source.

As a result of propositions (3) and (4) above, it is clear that the effect of a given scalar field, externally produced, is indistinguishable from the gravitational field. (This, however, does not mean that there are no observable effects of a scalar field. For example, a light ray is not deflected by a scalar interaction, and the expected perihelion rotation of Mercury may be noticably different[5] from the general relativity value. The radial dependence of $\bar{g}_{ij}$ for the Schwarzshild solution is different from the usual result of general relativity.)

One question remains: Why should the scalar interaction be so weak? We shall show that, if it exists, the scalar interaction would be expected to have a strength of the order of magnitude of the normal gravitational interaction.

It will be assumed that $f(\varphi)$ can be approximated by a power dependence $f\cong a\varphi^{-n}$ when φ falls in a sufficiently narrow range. The wave equation satisfied by φ is then[5,6]

$$\Box\varphi=\frac{1}{(-g)^{\frac{1}{2}}}[(-g)^{\frac{1}{2}}\varphi^{,i}]_{,i}=\frac{n}{\varphi}T, \quad (3)$$

where n is a constant and T is the contracted energy momentum tensor of matter. If a function of φ were to appear inside the d'Alembertian operator, the scalar φ could be redefined to eliminate it.

For a static configuration of astronomical bodies of mass m_i, Eq. (3) has an approximate solution:

$$\varphi\cong\frac{n}{4\pi}\sum\frac{m_ic^2}{\varphi_ir_i}. \quad (4)$$

In Eq. (4) it is assumed that the curvature of the space is sufficiently small that a Minkowskian coordinate system can be used, at least over a limited region; r_i is radial distance in this coordinate system.

It is evident that as the sum in Eq. (4) is extended over larger and larger regions, the neglect of the radial expansion of the universe and the non-Euclidean character of space becomes increasingly intolerable. It would be physically reasonable to expect that, for purposes of an order of magnitude calculation, the sum in Eq. (4) could be cut off at the Hubble radius, the farthest extent of the visible universe. This conclusion has been justified quantitatively, for a particular cosmological model. Assuming outgoing-wave boundary conditions on φ, essentially this result in Eq. (4) has been

[1] P. G. Bergmann, Am. J. Phys. **24**, 38 (1956).
[2] P. Jordan, *Schwerkraft und Weltall* (Friedrick Vieweg und Sohn, Braunschweig, 1955).
[3] G. Ludwig, *Fortschritte der projektiven Relativitätstheorie* (Friedrick Vieweg und Sohn, Braunschweig, 1951).
[4] K. Just, Z. Physik **140**, 485 (1955).
[5] C. Brans and R. H. Dicke, Phys. Rev. **123**, 925 (1961).
[6] R. H. Dicke, Phys. Rev. **125**, 2163 (1962).

obtained by expressing the solution in terms of a Green's function.[5]

Thus, for the spatial dependences of φ about a local body, such as the sun, one can write

$$\varphi \cong \frac{n}{4\pi\varphi_0} \frac{m_s c^2}{r} + \varphi_0, \tag{5}$$

with

$$\varphi_0 \sim \frac{n}{4\pi\varphi_0} \int_0^R \frac{\rho c^2}{r} 4\pi r^2 dr = \frac{n}{\varphi_0} \frac{R^2}{2} \rho c^2. \tag{6}$$

Equation (6) is a crude approximation only. ρ is the mean matter density of the universe and R is the Hubble radius. With the observed mass density,

$$G\rho R^2/c^2 \sim 1, \tag{7}$$

and Eq. (6) can be written

$$\varphi_0 \sim (n/G)^{\frac{1}{2}} c^2. \tag{8}$$

From Eq. (1) the scalar interaction force produced by the sun on a nearby body of mass m is

$$F = c^2 \frac{dm}{d\varphi} \frac{d\varphi}{dr} = -c^2 nm \frac{1}{\varphi} \frac{d\varphi}{dr}$$

$$= n^2 \frac{m m_s c^4}{4\pi \varphi_0^2 r^2} = (n/4\pi)(Gmm_s/r^2). \tag{9}$$

Because of the crude order of magnitude nature of Eqs. (6) and (7), the factor n in Eq. (9) is not significant. All that can be asserted is that the scalar interaction strength is of the same rough order of magnitude as gravitation. It could differ by at least a factor of 10.

To summarize, it is concluded that the absence of overt effects, observed in the laboratory, due to a long-range scalar field does not imply the nonexistence of the field. The effect of nearby matter in generating this scalar is minuscule compared with the dominant effect of the enormous amounts of matter in distant parts of the universe. As a result the scalar interaction is very weak. A further complication is introduced by the fact that the scalar field masquerades as gravitation and can be distinguished from gravitation only with difficulty. The difficulties in observing the effects of such a weak scalar field have been discussed.[7,8] These difficulties are sufficiently great that a conclusive test for the scalar field has never been applied. To assert that the scalar field does not exist because it is unobserved is analogous to the claim of a blind man that light does not exist. While the effects of the scalar field could be observed only with difficulty, the question of the existence of the field is of considerable interest. In particular, if this field exists, the great weakness of the gravitational interaction becomes understandable. The extremely small value of the gravitational coupling constant ($Gm_e^2/\hbar c \sim 10^{-40}$) is then recognized as the effect of the enormous amount of matter in the universe generating a scalar field which acts to depress the value of m_e, the mass of an elementary particle.

[7] R. H. Dicke, Varenna summer school notes, 1961 (to be published).
[8] R. H. Dicke, Revs. Modern Phys. **34**, 110 (1962).