Electromagnetic Radiation from a Nuclear Explosion in Space

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The γ rays from a nuclear explosion in space Compton-scatter electrons near the surface of the device or in a surrounding material shield. The scattered electrons leave the surface and are accelerated back toward it by the positively charged matter. Provided they are asymmetrically distributed, the accelerating electrons radiate an electromagnetic signal. The electron motions are analyzed, the electromagnetic signal is estimated, and its detectability is discussed. For a typical nuclear explosion, the electromagnetic signal is independent of the yield and contains frequencies up to 10 to 100 Mc/sec and thus will penetrate the ionosphere. Taking into account dispersion by the ambient interplanetary plasma ($\approx 10^2$ electrons/cc), the peak electric field strength at a distance R kilometers from the explosion is $\approx 10^4 R^{-3/4}$ v/m. The pulse length is $\approx 10^{-10}R$ sec. If only background cosmic noise limits detectability of the signal, the maximum detectable range is about 10^6 km.

I. INTRODUCTION

DURING the technical discussions at the Geneva conference on the nuclear test ban, it was suggested that a nuclear explosion in space might radiate a high-frequency radio signal. A number of different mechanisms were considered, but the magnitude and spectrum of the signal were only conjectured at the time. More recently a particular mechanism, due to the asymmetric emission of x rays from the explosion causing an asymmetric acceleration of the ambient charges in the low-density medium surrounding the explosion, has been quantitatively considered by Johnson and Lippmann.[1] The maximum detection range of the signal which they estimate is about a few times 10^4 km from an explosion producing one megaton of x-ray energy; the frequency of the signal is less than 10 Mc/sec.

In the present discussion a quite different mechanism is considered. This mechanism,[2] based on the emission of γ rays from the explosion rather than x rays, produces a signal of greater magnitude and with higher frequencies. An estimate of this signal will be given here and the possibility of observing it will be discussed.

II. GENERAL CONSIDERATIONS

During the course of a nuclear explosion, γ rays are produced, both directly in the fission process and indirectly by inelastic scattering of neutrons in the materials of the device. These γ rays, while moving through the device or any ambient matter, can Compton-scatter electrons. If the scattered electrons are produced sufficiently close to the surface of the matter, they can leave it and start to move into the surrounding space. As the electrons leave, the matter becomes positively charged and accelerates the electrons back towards it. Provided they are asymmetrically distributed, the accelerating electrons radiate. There are at least two possible ways by which such an asymmetrical distribution of electrons might be generated: (1) Because of the design of the device, the γ-ray flux at the surface may be asymmetric, or (2) the γ rays may be emitted isotropically, but there may be an asymmetric material shield.[3]

It is the electromagnetic radiation generated by the Compton-recoil electrons from the device or from an external shield which we wish to estimate. Some of the essential quantities determining this radiation, however, depend upon the design of the specific device; we shall use an idealized and simplified model of the explosion device and typical values for the relevant parameters—thereby obtaining only an order of magnitude estimate of the intensity and detectability of the radiation generated by the Compton-recoil mechanism.

III. SOURCE OF THE SIGNAL

During the course of the nuclear explosion the γ rays will be assumed to be produced at approximately an exponentially increasing rate. If $t=0$ is the time at which the explosion starts, then the number of γ rays produced up to time t is given by $e^{\alpha t}$, where, typically, α is about 10^8 sec^{-1}. As the explosion proceeds, the generation of γ rays continues to increase exponentially until a substantial fraction of the full yield has been produced, at which time the rate of generation of γ rays reaches a maximum. The generation from then on proceeds at a considerably slower rate than the buildup. For simplicity, we shall assume the generation increases exponentially at the rate $\alpha e^{\alpha t}$ until time T and then decreases to zero exponentially at the rate $\sigma\beta e^{-\beta(t-T)+\alpha T}$ ($\alpha \gg \beta$, $\sigma \lesssim 1$). About 0.03% of the total yield is assumed to be produced as γ rays. These γ rays have an average energy of about 1 Mev, and there are 7.5×10^{21} γ rays produced per kiloton (4×10^{19} ergs) of yield; thus,

$$(1+\sigma)e^{\alpha T}=7.5\times10^{21}Y, \tag{1}$$

[1] M. H. Johnson and B. A. Lippmann, Phys. Rev. **119**, 827 (1960).

[2] This mechanism was discussed by O. I. Leipunski at the 1958 Geneva Conference of Experts (unpublished).

[3] Such shields have been proposed to reduce the x-ray flux radiated in a given direction from the explosion and thus make more difficult long-range detection of x rays. The shield might be a flat plate or a hemisphere a few meters from the device so oriented as to shadow the explosion from possible x-ray detectors.

where Y is the total yield of the explosion expressed in kilotons.

The γ rays Compton-scatter electrons within the device or in a surrounding shield, if one is present. The resultant recoil electrons will be distributed both in energy and direction, but will move predominantly in a forward direction with a mean energy of about $\frac{1}{4}$ Mev, corresponding to those electrons produced at a depth of one-half the electron range R_e ($R_e \approx 0.1$ g/cm^2) from the surface of the device or the shield.

The total number of electrons emitted per unit area up to time t is

$$n(t)=\frac{R_e}{2\lambda_\gamma}\frac{1}{4\pi a^2}e^{\alpha t},\quad \text{for}\quad t\leq T,$$
$$=\frac{R_e}{2\lambda_\gamma}\frac{1}{4\pi a^2}e^{\alpha T}[1+\sigma-\sigma e^{-\beta(t-T)}],\quad \text{for}\quad t>T,\quad (2)$$

where a is the radius of the nuclear device or the distance from the source of γ rays to the surface of the shield, and λ_γ is the mean free path for Compton scattering ($\lambda_\gamma \approx 15$ g/cm^2). If the thickness of the matter in which the Compton-scattered electrons are produced is less than R_e the electron range, then R_e in Eq. (2) should be replaced by the thickness of the matter and the mean energy of the electrons is increased. The limiting mean energy for small thicknesses is about $\frac{1}{2}$ Mev.

For simplicity we shall henceforth consider only the problem of a thick hemispherical shield of radius a centered about the device. This problem also provides an estimate of the radiation from a flat-plate shield or from a spherical device of radius a whose γ-ray asymmetry is roughly equivalent to a distribution wherein γ rays escape predominantly into a hemisphere.[4]

As electrons leave the surface of the hemispherical shield it becomes positively charged and the electrons are accelerated back towards the shield. Assuming the electrons are all emitted in a direction normal to the surface (and that they do not move far from the surface[5]), a dipole moment normal to the surface is generated. From symmetry the dipole moment is uniform over the hemisphere and has a strength per unit area given by

$$Z(t)=-e\sum_i x_i(t)\eta(x_i(t)),\qquad (3)$$

where $\eta(x)=0$ if $x<0$, $\eta(x)=1$ if $x>0$, and $x_i(t)$ is the distance of the ith electron from the surface. The sum includes all electrons which are emitted from the surface in the unit area considered.

The acceleration of the dipole moment is found by summing the accelerations of the individual electrons outside the surface and including a contribution due to the flux of electrons in and out of the surface, namely,

$$\ddot{Z}(t)=-e\sum_i[\ddot{x}_i\eta(x_i)+v_0^2\delta(x_i)+v_r^2\delta(x_i)],\qquad (4)$$

where v_0 and v_r are the speeds of electrons leaving and returning to the surface and $\delta(x)$ is the delta function. Strictly, Eq. (4) should be averaged over the spectrum of electron energies. However, for the present order of magnitude estimates, it is sufficient to assume that all electrons have the same energy, namely, their mean energy. Quantitatively, this assumption is unimportant. However, it introduces certain artificialities into the behavior of the dipole moment which will be discussed and treated below.

If we neglect magnetic and retardation effects, as well as relativistic effects,[6] the acceleration of the ith electron is given by

$$\ddot{x}_i=-(e/m)E(x_i,t).\qquad (5)$$

Provided x_i is much smaller than the dimensions of the shield,[5] the electric field, $E(x_i,t)$, is determined by the local surface charge density only. Thus

$$E(x_i,t)=2\pi e\sum_j[1-\eta(x_i-x_j)]+2\pi e\sum_j\eta(x_j-x_i)$$
$$=4\pi e\sum_j\eta(x_j-x_i).\qquad (6)$$

For a constant electron emission rate it is possible to obtain an exact solution of Eqs. (5) and (6) for the average motion of the electrons and of Eq. (3) for the (time-independent) dipole moment. This steady-state solution is presented in Appendix 1.

To determine the average motion of the electrons for a time-dependent electron emission rate requires treatment of the complicated self-consistent-field problem expressed by Eqs. (5) and (6). Rather than attempt an exact solution, we will estimate the solution for times $t\leq T$ by two independent approximations, one accurate at early times and the other accurate at late times. In the first case, the screening of the electrons by one another will be disregarded. That is, each electron will be assumed to move in a constant accelerating field equal to the field at the emission surface *at the time of the electron's emission*. At the very early times during which the emitted electrons are moving only outwardly, the no-screening approximation is exact. As electrons emitted at an earlier time begin to return to the emitting surface, screening becomes important and the no-screening approximation provides only a semiquantitative description of the electron behavior. In order to obtain a more accurate description for the late-time behavior of the electrons, a second and independent approximation will be made. Namely, the electron motions will be assumed to be quasi-static. That is, at each instant the electrons will be assumed to be in that collective steady-state motion corresponding to the instantaneous value of the emission rate. The validity of this approximation will be discussed below.

[4] More generally, it is equivalent, in first order, to any asymmetry in which the electrons are distributed in angle approximately as $Y_0(\theta,\phi)+Y_1^0(\theta,\phi)$.

[5] From the solution of Eq. (5), it is easy to show that this approximation is valid.

[6] All these effects are less than a few percent.

Finally, with the aid of the results derived for $t \leq T$, the solution for $t \geq T$ will be obtained.

1. No-screening Approximation—$t \leq T$

When screening is neglected, Eqs. (5) and (6) reduce to

$$\ddot{x}_i(t) = -\frac{4\pi e^2}{m} \sum_i \eta(x_i(t_i)) = -\frac{4\pi e^2}{m} N(t_i), \tag{7}$$

where $N(t_i)$ is the number of electrons per unit area outside the surface at time t_i, which is the time that the ith electron is emitted. The motion is parabolic, and the ith electron returns with the initial speed v_0 at the time

$$t_r(t_i) = t_i + mv_0/2\pi e^2 N(t_i). \tag{8}$$

For times before the first electron returns,

$$N(t) = n(t), \quad t \leq t_0 + 1/\alpha, \tag{9}$$

and for times after the first electron returns,

$$N(t) = n(t) - \int_{t_r(\tau)<t} d\tau \frac{dn(\tau)}{d\tau}, \quad t \geq t_0 + \frac{1}{\alpha}, \tag{10}$$

where $n(t)$ is given by Eq. (2). Here t_0 is the time at which the first returning electron is emitted, and is found by differentiating Eq. (8) with $N(t) = n(t)$:

$$e^{\alpha t_0} = 4\pi a^2 \frac{2\lambda_\gamma}{R_e} \frac{mv_0\alpha}{2\pi e^2}. \tag{11}$$

Equations (8)–(10) cannot, in general, be solved analytically for $N(t)$, but can be solved numerically. In Fig. 1 we show $N(t)$ versus $\alpha(t-t_0)$ obtained numerically for early times.

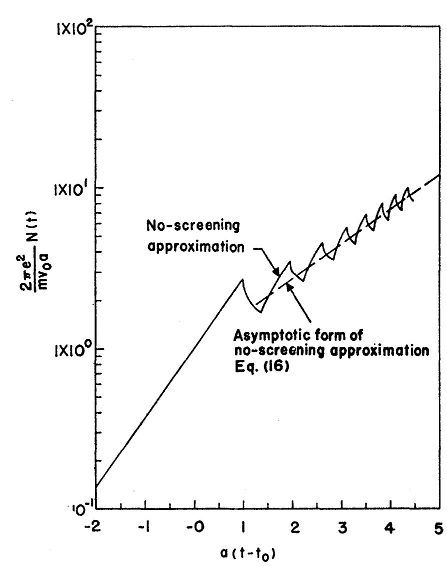

FIG. 1. Number of electrons per unit area outside surface as a function of time.

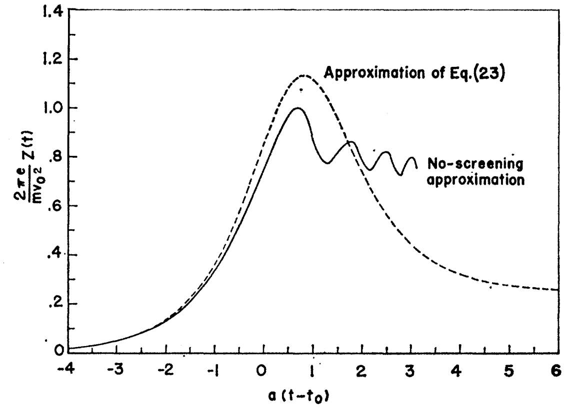

FIG. 2. Dipole moment per unit area as a function of time.

The qualitative behavior of $N(t)$ for early times is simply explained. Until time t_0+1/α no electrons have returned, so $N(t)$ just increases exponentially as $n(t)$. At t_0+1/α, there is a sudden flux of returning electrons which momentarily exceeds the emission rate, causing $N(t)$ to decrease. Electrons emitted during this period will have a decreased acceleration and will return at a slower rate, which shows up as an increased $N(t)$ at a later time. This oscillating behavior persists, but the time between discontinuities decreases as $N(t)$ increases. That $N(t)$ increases is expected since the emission rate is increasing.

The magnitude and frequency of the oscillations about the mean growth rate of $N(t)$ depend on t_0 and hence on the initial velocity v_0. Since there is actually a distribution of electron velocities instead of a single one, it is clear that in the actual case the oscillations will be substantially averaged out and only the smooth growth of $N(t)$ will remain.

From the solution for $N(t)$, Eq. (7) can be integrated for the electron motion. Equation (3) then determines $Z(t)$, the dipole moment per unit area. Namely,

$$Z(t) = -e \int_{t_r(\tau)>t>\tau} d\tau\, \dot{n}(\tau) \times \left[v_0(t-\tau) - \frac{2\pi e^2}{m} N(\tau)(t-\tau)^2 \right]. \tag{12}$$

For times before $t = t_0 + 1/\alpha$, $Z(t)$ is given simply by:

$$Z(t) = -(mv_0^2/2\pi e) e^{\alpha(t-t_0)} [1 - \tfrac{1}{4} e^{\alpha(t-t_0)}], \quad \alpha(t-t_0) \leq 1. \tag{13}$$

Thus, as is easily shown, $Z(t)$ reaches a maximum and starts to decrease before any electron has returned. Its maximum occurs at $\alpha(t-t_0) = \ln 2$. For later times $Z(t)$ is shown in Fig. 2 where one expects again that in the real case the oscillations will not be present.

For large t, we can obtain an asymptotic expression for $N(t)$ by using the fact that the time between

emission, t_1, and return, t_r, is very short compared to $1/\alpha$ and t. Thus

$$N(t)\simeq n(t)-n(t_1)$$
$$=(mv_0\alpha/2\pi e^2)e^{\alpha(t-t_0)}[1-e^{-\alpha(t-t_1)}]$$
$$\simeq(mv_0\alpha/2\pi e^2)e^{\alpha(t-t_0)}\alpha(t-t_1) \qquad (14)$$

and

$$t-t_1=mv_0/2\pi e^2N(t_1)\simeq e^{-\alpha(t-t_0)}/\alpha^2(t-t_1).$$

Finally, then,

$$\alpha(t-t_1)\simeq e^{-\alpha(t-t_0)/2} \qquad (15)$$

and

$$N(t)\simeq(mv_0\alpha/2\pi e^2)e^{\alpha(t-t_0)/2}. \qquad (16)$$

This asymptotic limit for $N(t)$ is also shown in Fig. 1.

In this limit we find for the dipole moment per unit area

$$Z(t)\simeq-(mv_0^2/12\pi e)[1+\tfrac{1}{4}e^{-\frac{1}{2}\alpha(t-t_0)}], \qquad (17)$$

which has the very small acceleration

$$\ddot{Z}(t)\simeq-(mv_0^2/2\pi e)(\alpha^2/96)e^{-\alpha(t-t_0)/2}. \qquad (18)$$

The results, Eqs. (16)–(18), for $N(t)$ and $Z(t)$ are easily seen to hold for times t such that $e^{\alpha(t-t_0)/2}\gg1$; and we note that the undesired oscillations have implicitly been omitted.

2. Quasi-static Approximation—$t\leq T$

The no-screening approximation, Eqs. (16)–(18), is not expected to be accurate at late times when screening becomes important. However, at late times an alternative approximation becomes valid. Namely, if the electron emission rate changes slowly compared to the time between an electron's emission and return, that is, if

$$(\dot{n}/n)(t_r-t_1)=\alpha(t_r-t_1)\ll1,$$

then we may assume that the electron motions at a given time are in that state of steady motion corresponding to the instantaneous emission rate.

From Appendix 1 we find the exact solution for a steady state to be

$$N_{ss}=\dot{n}(t_r-t_1)=(\dot{n}mv_0/\pi e^2)^{\frac{1}{2}}. \qquad (19)$$

Assuming that $\alpha(t_r-t_1)\ll1$, we obtain a good estimate of $N(t)$ and $Z(t)$ by substituting the slowly changing rate $\dot{n}(t)$ from Eq. (2) into Eq. (19). Thus

$$N_{ss}(t)\simeq(mv_0\alpha/\sqrt{2}\pi e^2)e^{\alpha(t-t_0)/2}, \qquad (20)$$

$$\alpha(t_r-t_1)\simeq\sqrt{2}e^{-\alpha(t-t_0)/2}, \qquad (21)$$

$$Z(t)\simeq-mv_0^2/8\pi e. \qquad (22)$$

From Eq. (21) it is deduced that the quasi-static solution, Eq. (22), is accurate for late times t such that $e^{\alpha(t-t_0)/2}\gg1$.

A comparison of Eqs. (17) and (22) shows that the no-screening and the quasi-static approximations agree to within a factor of $\frac{2}{3}$ and both predict essentially no radiation at late times. This close quantitative agreement suggests that in fact the no-screening approximation is actually quite good and can be expected to provide a rather accurate estimate of the over-all behavior of the electron motions.

3. Solution for $t\geq T$

In view of the behavior of $Z(t)$ for $t\leq T$, it is clear that for $t\geq T$, $\ddot{Z}(t)$ will be small until $t\gg T$. Therefore, in order to estimate $\ddot{Z}(t)$ for $t\geq T$, we can for convenience arbitrarily assume that at $t=T$ the electrons are in a state of steady motion (using the results of the quasi-static approximation). In view of the accuracy of the no-screening approximation, this approximation can be used to determine both the steady motion at $t=T$ and the subsequent time-dependent motion for $t>T$. Using Eq. (12) and the results of Appendix 2, it readily follows that $\ddot{Z}(t)\sim\beta^2$ for $t>T$. However, from Eqs. (13) and (17), $\ddot{Z}(t)\sim\alpha^2$ for $t<T$. Since $(\beta/\alpha)^2\ll1$, the radiation at late times $t>T$ is small compared to that at early times $t<T$. In addition, it consists of lower frequencies which, as can be shown from subsequent results, are more difficult to detect at great distances. Accordingly, the radiation at times $t>T$ may be neglected.

Combining the results of Secs. 1–3, we conclude that the dipole moment will increase almost exponentially to a maximum value, decay somewhat more slowly to a steady-state value, then increase again slightly and finally decay slowly to zero. The important times t, during which radiation is emitted, are in the neighborhood of $\alpha(t-t_0)$ equal to the order of one or a few.

For convenience in subsequent order of magnitude calculations an approximate expression for the dipole moment will be used. This expression was chosen to give the qualitative behavior of the dipole moment as well as correct values for its maximum and quasi-steady state. Namely, we shall use

$$Z(t)=-(mv_0^2/16\pi e)\{1+\tanh[\alpha(t-t_0')]+8\,\mathrm{sech}[\alpha(t-t_0')]\}, \qquad (23)$$

where $\alpha t_0'=\alpha t_0+\ln2$. For comparison this function is shown in Fig. 2 along with the more exact results.

IV. THE NEAR RADIATED SIGNAL

The electromagnetic field radiated by the Compton electrons is composed of a coherent and an incoherent part. The coherent part arises from the mean-time behavior of $Z(t)$ given by Eq. (23). The incoherent part arises from the fluctuations in the mean variation of $Z(t)$. In Appendix 3 an upper limit on the magnitude of the incoherent radiation is determined. Compared to the coherent part, even this upper limit on the incoherent part is quite small and, therefore, the incoherent radiation may be neglected.

The coherent part of the electric field radiated to a distance R from the explosion is[7]

$$\mathbf{E}(R,t) = -\frac{1}{c^2R}\int d\omega\, \hat{Z}(\omega)\omega^2 \int\int d^2\mathbf{r}\, e^{i\omega t_{\text{ret}}}\mathbf{e}_R \times(\mathbf{e}_n\times\mathbf{e}_R), \quad (24)$$

where

$$Z(t) = \int d\omega\, e^{i\omega t}\hat{Z}(\omega) \quad (25)$$

defines $\hat{Z}(\omega)$. The second and third integrations in Eq. (24) are carried out over the radiating surface; t_{ret} is the retarded time from the observation point to the element of area at the point $\mathbf{r}$. $\mathbf{e}_n$ is a unit vector in the direction of the dipole moment at the point $\mathbf{r}$, and $\mathbf{e}_R$ is a unit vector parallel to $\mathbf{R}$. For a hemisphere of radius a with a radial dipole moment, we find

$$E(R,t) = \frac{2\pi a}{cR}\int d\omega\, e^{i\omega(t-R/c)}\hat{Z}(\omega)\omega^2 J_1\left(\frac{\omega a}{c}\sin\theta\right)\Big/\omega, \quad (26)$$

where θ is the angle between the axis of symmetry of the hemisphere and the direction to the observer. The electric field is polarized in the plane containing these two directions.

For $Z(t)$ given by Eq. (23) the Fourier transform of $Z(t)$ is

$$\hat{Z}(\omega) = -(mv_0^2/32\pi e a)e^{-i\omega t_0'} \times[8\,\text{sech}(\pi\omega/2\alpha) - i\,\text{csch}(\pi\omega/2\alpha)]. \quad (27)$$

From this result we conclude that frequencies much larger than α are sharply reduced. Moreover, as can be seen from Eq. (26), the finite size of the hemisphere, expressed by the factor $J_1(\omega a\sin\theta/c)/\omega$, tends to cut off frequencies above about $1/\tau_0 = c/a\sin\theta$. For $\alpha = 10^8$ sec^{-1}, and $a = 10$ m, the radiated signal will therefore contain frequencies only up to about 10 to 100 Mc/sec. From Eqs. (23), (26), and (27), the maximum signal strength is about

$$E_{\max} \approx (mv_0^2/8eR)(\alpha a/c)^2 \approx 10^3(R_{\text{km}})^{-1}\text{ v/m}, \quad (28)$$

for $\frac{1}{2}mv_0^2 = \frac{1}{4}$ Mev; the signal duration is $\approx 10^{-7}$ sec.

V. THE DISTANT RADIATED SIGNAL

To determine the signal radiated to great distances it is necessary to take into account the fact that interplanetary space is not empty, but filled with a low-density ionized plasma, containing from 10^2 to 10^3 electrons per cm^3, which has a characteristic frequency of 10^5 to 3×10^5 cps. For these low densities, collisions can be neglected, and the effect of the plasma is to attenuate signals below the plasma frequency and disperse those above. Since we are concerned with very great distances (10^5 km or greater) for detection, the attenuation of the lower frequencies will be essentially complete, while the higher frequencies will be greatly dispersed and the character of the signal considerably altered.

The refractive index of the interplanetary medium is given by

$$\begin{aligned} n(\omega) &= [1-(\omega_p/\omega)^2]^{\frac{1}{2}}, \quad \omega > \omega_p,\\ &= -i[(\omega_p/\omega)^2-1]^{\frac{1}{2}}, \quad \omega<\omega_p, \end{aligned} \quad (29)$$

where

$$\omega_p^2 = 4\pi n_e e^2/m \quad (30)$$

and n_e is the electron density in space. The electric field at great distances from the explosion is

$$E(R,t) = \frac{2\pi a}{cR}\int d\omega\, e^{i\omega t}e^{-i\omega n(\omega)R/c}\hat{Z}(\omega)\omega^2\frac{J_1(\omega\tau_0)}{\omega}, \quad (31)$$

where the contour for the ω integration runs below the real axis and $\tau_0 = a\sin\theta/c$.

The integral in Eq. (31) can be evaluated by the method of steepest descents for times such that

$$\omega_p(t-t_0'-R/c) \gtrsim 1. \quad (32)$$

The points of steepest descent occur at

$$\omega = \pm\frac{\omega_p|t-t_0'|}{[(t-t_0')^2-R^2/c^2]^{\frac{1}{2}}} = \pm\omega_p x_0. \quad (33)$$

Noting that $\hat{Z}(-\omega) = [\hat{Z}(\omega)]^*$, we find

$$E(R,t) = \frac{2(2\pi)^{\frac{3}{2}}a}{cR}\omega_p^2\left(\frac{c}{\omega_p R}\right)^{\frac{1}{2}} x_0(x_0^2-1)^{\frac{3}{4}}J_1(\omega_p\tau_0 x_0) \times\text{Re}\{e^{i\pi/4}e^{i\omega_p(t-t_0')/x_0}\hat{Z}(\omega_p x_0)\}. \quad (34)$$

This represents a rapidly oscillating electric field whose amplitude rises from nearly zero at $t = t_0'+R/c$ to a maximum and then decays as $t^{-\frac{3}{2}}$ for late times. The maximum amplitude, using the approximation for $\hat{Z}(\omega)$ given by Eq. (27), occurs at $x_0\approx\alpha/\omega_p$, that is, at

$$t-t_0'-R/c \approx (\omega_p/\alpha^2)R/c \quad (35)$$

and is approximately

$$E_{\max}(R) \approx (2\pi)^{\frac{3}{2}}\frac{a}{c^{\frac{1}{2}}R^{\frac{3}{2}}}\frac{\alpha^{\frac{3}{2}}}{\omega_p}J_1\left(\frac{\alpha a\sin\theta}{c}\right)\frac{mv_0^2}{5\pi e}. \quad (36)$$

Assuming $a\approx10^3$ cm, $\omega_p\approx 6\times10^5$ sec^{-1} (corresponding to an electron density of $10^2/cc$), $\alpha\approx10^8$ sec^{-1}, $\frac{1}{2}mv_0^2 = \frac{1}{4}$ Mev, it is found that

$$E_{\max}(R) \approx 10^4(R_{\text{km}})^{-\frac{3}{2}}\text{ v/m}, \quad (37)$$

and the signal duration is approximately

$$\Delta t = t-t_0'-R/c \approx 10^{-10}R_{\text{km}}\text{ sec}. \quad (38)$$

The bandwidth is 10^7 to 10^8 cps.

[7] The effect of dispersion by the ambient medium is treated in the next section.

VI. DETECTABILITY OF THE SIGNAL

Since the frequencies in the signal extend up to about 10 to 100 Mc/sec, the signal can penetrate the ionosphere, and can be observed from the surface of the earth, provided background noise is not too great. We shall estimate the maximum detection range of the signal assuming that the only interfering noise arises from background cosmic noise. Thus, it will be assumed that the receiver can be effectively shielded against man-made disturbances.

In the frequency region from 10 to 100 Mc/sec the background cosmic noise is characterized by an effective temperature of about 10^4 °K. That is, the mean cosmic noise flux per cycle over the frequency range of the signal is

$$S_N = 4\pi kT/\lambda^2 = 1.7\times10^{-22}(T/\lambda^2) \text{ w/m}^2 \text{ cps} \\ = (1.7\times10^{-18}/\lambda^2) \text{ w/m}^2 \text{ cps}. \qquad (39)$$

If the detection receiver has a bandwidth σ cps, the average cosmic noise power received will be

$$P_N = \sigma S_N = (1.7\times10^{-18}/\lambda^2)\sigma \text{ w/m}^2. \qquad (40)$$

For a band width $\sigma \approx 10^8$ cps and $\lambda \approx 10$ m (corresponding to a frequency of 3×10^7 cps)

$$P_N \approx 10^{-12} \text{ w/m}^2. \qquad (41)$$

The signal radiated by the explosion has a peak intensity of

$$\frac{cE_{\max}^2}{4\pi} \approx \frac{3\times10^5}{(R_{\text{km}})^3} \text{ w/m}^2. \qquad (42)$$

For distances greater than 10^5 to 10^6 km, the peak signal power is less than the mean noise flux. For detection, however, the peak signal power need not exceed the mean noise power. By suitable circuit design of detectors the mean noise power can be biased out leaving only the fluctuations of the noise. It is sufficient for detection that the total signal energy exceed a threshold level determined by the requirement that the probability for a noise fluctuation to exceed the threshold is less than some acceptable value. A reasonable threshold for detection is ten times the dispersion of the total noise energy during a time equal to the signal duration time.

In Appendix 4 it is shown that for a receiver with negligible internal noise, having an effective antenna area A m^2, a frequency bandwidth σ cps, and which integrates the noise power for a time period τ sec, the dispersion in the received energy is

$$[\langle(\epsilon_N - \bar{\epsilon}_N)^2\rangle_{\text{av}}]^{\frac{1}{2}} = \left(\frac{\sigma\tau A\lambda^2}{2\pi}\right)^{\frac{1}{2}} \frac{4\pi kT}{\lambda^2} \\ = 0.7\times10^{-22}\left(\frac{\sigma\tau A}{\lambda^2}\right)^{\frac{1}{2}} T \text{ joules}. \qquad (43)$$

Clearly, optimum detection results when σ equals the signal bandwidth and τ equals the signal duration time. Due to the dispersive effects of the interplanetary medium the signal duration depends upon distance from the explosion. An estimate of the signal duration is given by Eq. (35).

From Eq. (31) the total received signal energy is

$$\epsilon = \frac{c}{4\pi} A \int_{-\infty}^{\infty} dt [E(R,t)]^2 \\ = \frac{4\pi^2 A a^2}{cR^2} \int_{\omega_p}^{\infty} d\omega\, \omega^2 \left[J_1\left(\frac{\omega a \sin\theta}{c}\right)\right]^2 |\hat{Z}(\omega)|^2. \qquad (44)$$

For the approximation to $\hat{Z}(\omega)$ given by Eq. (27),

$$\epsilon = \frac{A}{32\pi^3}\left(\frac{a}{R}\right)^2 \frac{m^2 v_0^4 \alpha}{e^2 c} I\left(\frac{2\alpha a}{\pi c}\sin\theta\right), \qquad (45)$$

where

$$I[(2\alpha a/\pi c)\sin\theta] = 7.9 \quad \text{for} \quad (2\alpha a/\pi c)\sin\theta = 2.12 \\ = 12.0 \quad \text{for} \quad (2\alpha a/\pi c)\sin\theta = 1.06 \\ = 46.5[(2\alpha a/\pi c)\sin\theta]^2 \\ \text{for} \quad (2\alpha a/\pi c)\sin\theta \ll 1.$$

Clearly, the signal energy ϵ is a rather insensitive function of the angle θ, except when θ is very small. Moreover, detailed evaluation of the integral of Eq. (44) indicates that nearly all the signal energy is contained in a bandwidth $\Delta\omega \approx 8\alpha/\pi$, or $\sigma \approx 4\alpha/\pi^2 = 4\times10^7$ cps for $\alpha = 10^8$. Using this band width $\sigma = 4\alpha/\pi^2$ and an integration time $\tau = (\omega_p/\alpha)^2 R/c$, the maximum detection range (corresponding to a signal equal to N times the dispersion of the noise) is given by

$$\frac{A}{32\pi^3}\left(\frac{a}{R}\right)^2 \frac{m^2 v_0^4 \alpha}{e^2 c} I = N\left(\frac{\sigma\tau A\lambda^2}{2\pi}\right)^{\frac{1}{2}} \frac{4\pi kT}{\lambda^2} \\ = N\left(\frac{4\alpha}{\pi^2}\frac{\omega_p^2}{\alpha^2}\frac{R}{c}\frac{A}{2\pi}\right)^{\frac{1}{2}} \frac{4\pi kT}{\lambda} \qquad (46)$$

or

$$R_{\max} = \frac{1}{8\pi}\left[\frac{a^4 m^4 v_0^8 \alpha^3 \lambda^2 I^2 A}{N^2 \omega_p^2 (kT)^2 c e^4}\right]^{1/5}. \qquad (47)$$

For $a = \lambda = 10$ m, $\omega_p = 6\times10^5$ sec^{-1}, $A = 10^2$ m^2, $T = 10^4$ °K, $N = 10$, and $I = 10$, we find

$$R_{\max} = 1.3\times10^6 \text{ km}. \qquad (48)$$

We note that the detection range is independent of the explosion yield. This result is valid so long as $e^{\alpha T} \gg e^{\alpha t_0}$.

VII. DISCUSSION

In treating the propagation of the explosion signal, effects of the earth's ionosphere were ignored. To justify neglecting ionosphere effects, we note that the dispersion of the signal depends approximately only on the total number of electrons per cm^2 along the

propagation path. For distances greater than about 10^5 to 10^6 km, the total number of ionospheric electrons is small compared to the total number of interplanetary electrons. Second, we note that the cutoff frequency of the ionosphere (~6 Mc/sec) is small compared to the signal bandwidth (~40 Mc/sec). Consequently, only a few percent of the signal energy *cannot* penetrate the ionosphere.

The key uncertainty in this method of detecting nuclear explosions is the feasibility of eliminating all sources of background noise except cosmic noise. Removing this uncertainty will require experimental detector studies.

Note added in proof. We were recently informed by Dr. Richard Garwin that in an unpublished Los Alamos report which appeared about 10 years ago, he had considered the effect of Compton electrons from the device case in producing electromagnetic signals from nuclear explosions in the atmosphere.

APPENDIX 1. STEADY-STATE MOTION

For a constant emission rate, the electron motions approach a steady state. In this steady state the electric field distribution is independent of time and is given by

$$dE(x)/dx = -4\pi e\rho(x), \tag{1.1}$$

where $\rho(x)$ is the electron density at a distance x from the emission surface. $\rho(x)$ is related to the emission rate $\dot{n}_0$ by

$$\rho(x) = 2\dot{n}_0/v(x), \tag{1.2}$$

where $v(x)$ is the velocity of the outward moving electrons at x. (The inward moving electrons have an equal and opposite velocity.) From the electron equation of motion, Eq. (5),

$$\ddot{x} = v(x)dv(x)/dx = -(e/m)E(x). \tag{1.3}$$

Differentiating Eq. (1.3) with respect to x and using Eqs. (1.1) and (1.2), we find that

$$\frac{d^2v^2(x)}{dx^2} = \frac{16\pi e^2}{m}\frac{\dot{n}_0}{v(x)}. \tag{1.4}$$

A straightforward solution of this equation gives for the velocity of the outward moving electrons

$$v(x) = [v_0^{\frac{3}{2}} - 3A^{\frac{1}{2}}x]^{\frac{2}{3}}, \tag{1.5}$$

where

$$A = 4\pi\dot{n}_0e^2/m.$$

The path of the outward moving electrons is obtained by integrating Eq. (1.5), namely,

$$x = 3A^{\frac{1}{2}}[v_0^{\frac{3}{2}} - (v_0^{\frac{1}{2}} - A^{\frac{1}{2}}t)^3]. \tag{1.6}$$

We observe that the time between emission and maximum displacement is

$$t = (v_0/A)^{\frac{1}{2}}, \tag{1.7}$$

and the time between emission and return is

$$t_r - t_1 = 2t = 2(v_0/A)^{\frac{1}{2}}. \tag{1.8}$$

The total number of electrons outside the emitting surface is then

$$N_{ss} = \dot{n}_0(t_r - t_1) = 2\dot{n}_0(v_0/A)^{\frac{1}{2}},$$

or

$$N_{ss} = (\dot{n}_0 m v_0/\pi e^2)^{\frac{1}{2}}. \tag{1.9}$$

The dipole moment is

$$Z(t) = -e\int dx\, \rho(x)x.$$

Substituting from Eqs. (1.2) and (1.5) and observing that the integration extends from 0 to the maximum displacement $v_0^{\frac{3}{2}}/3A^{\frac{1}{2}}$, we find

$$Z(t) = -emv_0^2/8\pi e^2. \tag{1.10}$$

APPENDIX 2. SOLUTION FOR $t \geq T$

For a constant emission rate $\dot{n}_0$ and using the no-screening approximation, Eq. (7) gives for the motion of an individual electron

$$\ddot{x}(t) = -(4\pi e^2/m)N_0, \tag{2.1}$$

where $N_0 = \dot{n}_0\tau$ and τ is the time interval between emission and return of the electron. Thus, if the electron is emitted at $t = t_0$,

$$x(t) = -(2\pi e^2/m)N_0(t-t_0)^2 + v_0(t-t_0). \tag{2.2}$$

From this result,

$$\tau = mv_0/2\pi e^2 N_0 = N_0/\dot{n}_0. \tag{2.3}$$

Thus

$$N_0 = [\dot{n}_0 m v_0/2\pi e^2]^{\frac{1}{2}}. \tag{2.4}$$

Assume now that Eq. (2.4) is valid for $t \leq T$ and that for $t \geq T$, $\dot{n}(t) = \dot{n}_0 e^{-\beta(t-T)}$. Then from Eqs. (2) and (10),

$$\begin{aligned} N(t) &= (\dot{n}_0/\beta)(1 - e^{-\beta(t-T)}) - \dot{n}_0[t_1(t) - T] \\ &\qquad \text{for} \quad T - t \leq t_1(t) \leq T \\ &= (\dot{n}_0/\beta)(e^{-\beta[t_1(t)-T]} - e^{-\beta(t-T)}) \\ &\qquad \text{for} \quad t_1(t) \geq T, \end{aligned} \tag{2.5}$$

where $t_1(t)$, the time of emission of an electron which returns at time t, is given by

$$t = t_1(t) + \frac{mv_0}{2\pi e^2}\frac{1}{N(t_1(t))}. \tag{2.6}$$

Eqs. (2.5) and (2.6) can be solved approximately in two limits—$\beta[t - t_1(t)] \ll 1$ and $\beta[t - t_1(t)] \gg 1$, where t and $t_1(t) \gg T$. In the case that $\beta[t - t_1(t)] \ll 1$, $t \gg T$ and $t_1(t) \gg T$, Eq. (2.5) gives

$$N(t) \approx \dot{n}_0 e^{-\beta(t-T)}[t - t_1(t)]. \tag{2.7}$$

Substitution of Eq. (2.7) into Eq. (2.6) leads to

$$N(t) \approx (mv_0\dot{n}_0/2\pi e^2)^{\frac{1}{2}} e^{-\beta(t-T)/2}. \tag{2.8}$$

In the case that $\beta[t-t_1(t)]\gg 1$, $t\gg T$ and $t_1(t)\gg T$, Eq. (2.5) gives

$$N(t)\approx(\dot{n}_0/\beta)\exp\{-\beta[t_1(t)-T]\}. \tag{2.9}$$

Substitution of Eq. (2.9) into Eq. (2.6) gives

$$N(t)\approx\frac{mv_0\beta}{2\pi e^2}\frac{1}{\beta(t-T)+\ln(mv_0\beta^2/2\pi e^2\dot{n}_0)}. \tag{2.10}$$

APPENDIX 3. INCOHERENT RADIATION

We will show here only that the incoherent part of the radiated field is small compared to the coherent part. For this purpose it is sufficient to overestimate the intensity of the incoherent field. For an overestimate, it will be assumed that each electron radiates independently at all frequencies. The no-screening approximation will be used and it will be assumed that the electrons are in a steady state corresponding to a constant γ-ray flux over a time $1/\alpha\approx 10^{-8}$ sec. In this case, the total power per unit area radiated to the distance R is

$$P\approx\frac{8\pi}{3}a^2N_0\frac{e^2}{c^3}\ddot{x}^2\frac{1}{4\pi R^2}, \tag{3.1}$$

where $\ddot{x}$ is the acceleration of each electron as given by Eq. (2.1) and N_0 is the number of electrons outside the emitting surface as given by Eq. (2.4). Then

$$P\approx\frac{128\pi^3}{3}\frac{e^6a^2}{m^2c^3}\left(\frac{mv_0}{2\pi e^2}\dot{n}_0\right)^{\frac{1}{2}}\frac{1}{4\pi R^2}, \tag{3.2}$$

where, using Eq. (1),

$$\dot{n}_0\approx\frac{R_e\alpha}{2\lambda_\gamma}\frac{7.5\times 10^{21}}{4\pi a^2}Y.$$

Using $a=10^3$ cm, we obtain

$$P\approx\frac{7\times 10^{-3}}{R_{\mathrm{km}}{}^2}Y^{\frac{1}{2}}\text{ ergs/cm}^2\text{ sec.} \tag{3.3}$$

Thus

$$(\langle E^2\rangle)^{\frac{1}{2}}\approx\frac{5\times 10^{-2}Y^{\frac{1}{4}}}{R_{\mathrm{km}}}\text{ v/m.} \tag{3.4}$$

Comparison of Eq. (3.4) with Eq. (28) establishes that the incoherent radiation is negligible.

APPENDIX 4. NOISE FLUCTUATIONS

Cosmic background noise is generally described by specifying an equivalent "noise temperature" for the frequency band of interest. This equivalent temperature is related to the average cosmic noise power per unit area per cycle per second at wavelength λ by

$$S_N=4\pi\frac{kT}{\lambda^2}=1.7\times 10^{-22}\frac{T}{\lambda^2}\text{ w/m}^2\text{ cps,} \tag{4.1}$$

where T is in degrees Kelvin and λ in meters.

If a receiver with an antenna area A m^2 and band width σ cps integrates the received noise power for a period of τ seconds, the total received energy will be, on the average,

$$\bar{\epsilon}=S\sigma\tau A=1.7\times 10^{-22}T(A/\lambda^2)\sigma\tau\text{ joules.} \tag{4.2}$$

The fluctuation of the mean noise energy is given by the fluctuation of the radiant energy contained in a hohlraum of volume $Ac\tau$ and temperature T. For the band width σ, this fluctuation in received energy is

$$\langle(\epsilon-\bar{\epsilon})^2\rangle_{\mathrm{av}}=Ac\tau\frac{8\pi h^2\nu^4e^{h\nu/kT}\sigma}{c^3(e^{h\nu/kT}-1)^2}. \tag{4.3}$$

For the case of radio waves, $h\nu\ll kT$, and

$$\begin{aligned}\langle(\epsilon-\bar{\epsilon})^2\rangle_{\mathrm{av}}&=A\tau(8\pi\nu^2(kT)^2/c^2)\sigma\\&=A\tau\sigma(\lambda^2/2\pi)S_N{}^2.\end{aligned} \tag{4.4}$$

Thus, the dispersion of the mean received noise energy is

$$\begin{aligned}[\langle(\epsilon-\bar{\epsilon})^2\rangle_{\mathrm{av}}]^{\frac{1}{2}}&=(\sigma\tau A\lambda^2/2\pi)^{\frac{1}{2}}S_N\\&=0.7\times 10^{-22}T(\sigma\tau A/\lambda^2)^{\frac{1}{2}}\text{ joules.}\end{aligned} \tag{4.5}$$