where K is given by (89) and

$$a_{\pm}=\tfrac{1}{2}(E+m)^{-2}[g_V P\mp g_A(m+E)]^2,$$
$$b_{\pm}=\tfrac{1}{2}(E+m)^{-1}(E\pm P)\{[g_V-f_V(m+E)]^2+[g_A-h_A(E-m)]^2\}, \quad (98)$$
$$c=(E+m)^{-1}m\{[g_V-f_V(m+E)]^2-[g_A-h_A(E-m)]^2\}.$$

If we assume further that the conserved current hypothesis holds, then g_V and f_V are given explicitly by (43). The functions g_A and h_A can then be determined by using (97) and (98). In particular, the difference,

$$(d\sigma_\nu-d\sigma_{\bar{\nu}})\equiv\sum_{s=L,R}[d\sigma_\nu(l_s^-)-d\sigma_{\bar{\nu}}(l_s^+)],$$

gives a sensitive determination of g_A. We have, similar to (44) but without the $v_l=1$ approximation,

$$(d\sigma_\nu-d\sigma_{\bar{\nu}})=(4\pi m^2k_\nu^2)^{-1}q^2[4mk_\nu-q^2-m_l^2]\times g_Vg_Ad(q^2). \quad (99)$$

(vi) If $v_l=1$, then

$$K=(8\pi k_\nu^2)^{-1}[(k_\nu+k_l)^2-P^2],$$
$$y=(E+P)^{-1}m,$$

and

$$y^{-1}=(E-P)^{-1}m.$$

The $a_\pm$, b, d functions of Sec. III are related to the present ones by

$$a_+=a_+(R),$$
$$a_-=a_-(L),$$
$$b=\sum_{s=L,R}[yb_+(s)+y^{-1}b_-(s)+c(s)],$$

and

$$d=[yb_+(R)+y^{-1}b_-(R)+c(R)]-[yb_+(L)+y^{-1}b_-(L)+c(L)]. \quad (100)$$

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Self-Consistent Model for Nonleptonic Decays of Σ and Λ Hyperons*

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A self-consistent calculation of pionic Σ and Λ decays has been carried out in the pole approximation of an S matrix approach in order to get information on (a) the angular momentum in which the decay $\Sigma^+\to n\pi^+$ takes place, (b) the relative ($\Sigma\Lambda$) parity, (c) the possible existence of other than global symmetric solutions. On the basis of existing experimental data, the model predicts that $\Sigma^+\to n\pi^+$ decay must occur in the s wave, and, somewhat less definitely, that ($\Sigma\Lambda$) parity is even.

INTRODUCTION

RECENTLY, Beall *et al.* have established that the helicities of the protons in the nonleptonic decays of Σ^+ and Λ^0 are opposite.[1] This result, while confirming an important prediction of the global-symmetry hypothesis, contradicts the predictions of several other models of hyperon decay. In particular, it disagrees with the bound-pion model of Barshay and Schwartz,[2] in which the Λ decay is taken as the primary decay, and thus invalidates one of the arguments used by Nambu and Sakurai in favor of odd Σ-Λ parity.[3] We have, therefore, considered a simple self-consistent model in which both these decays are treated as equally fundamental, with parameters to be determined by requirements of consistency. We have, then, tried to seek answers to the following questions: (1) Are there solutions other than the global-symmetric one that fit the experimental data? (2) Does odd Σ-Λ parity fit the data better, or vice versa? (3) Can one predict which of the two decays—$\Sigma^+\to n\pi^+$ or $\Sigma^-\to n\pi^-$—goes into s wave and which into p wave? With regard to the last question, it has been well known for some time, from the experimental data on the Σ triangle of Gell-Mann and Rosenfeld,[4,5] that one of these decays must go into s-wave and the other into p wave, but it has not been possible so far to say which goes to which.

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‡ On leave from the Atomic Energy Establishment Trombay, Bombay, India.

[1] E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters, **7**, 285 (1961).

[2] S. Barshay and M. Schwartz, Phys. Rev. Letters **4**, 618 (1960).

[3] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **6**, 377 (1961).

[4] M. Gell-Mann and A. H. Rosenfeld, in *Ann. Rev. Nuclear Sci.* **7**, 407 (1957).

[5] B. Cork, L. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. **120**, 100 (1960).

THE MODEL

Ours is essentially an S matrix approach carried out in the pole approximation which has given reasonable results in the theory of strong interactions and has also been successful in the treatment of the π decay. The diagrams considered are shown in Fig. 1. The contributions of the black boxes to the matrix elements are shown in the figure.[6–9] Here, g_N, g_Λ, g_Σ are coupling constants; a_Λ, a_Σ, b_Λ, b_Σ (as also g_Λ, g_Σ) are to be fitted from experiment[10]; and Γ takes the value γ_5 or 1 according as the relative Σ-Λ parity is even or odd. Time-reversal invariance implies that b_Λ and b_Σ are real. Then the matrix elements for $\Sigma^+\to p\pi^0$, $\Sigma^+\to n\pi^+$, and $\Sigma^-\to n\pi^-$, respectively, are given by

$$M_0=\sqrt{2}[B_\Sigma(g_\Sigma+g_N)+i\gamma_5 A_\Sigma(g_\Sigma-g_N)],$$

$$M_+=\{B_\Lambda g_\Lambda-B_\Sigma(g_\Sigma+2g_N)+i\gamma_5[A_\Lambda g_\Lambda-A_\Sigma(g_\Sigma-2g_N)]\},$$

and

$$M_-=[B_\Lambda g_\Lambda+B_\Sigma g_\Sigma+i\gamma_5(A_\Lambda g_\Lambda+A_\Sigma g_\Sigma)].$$

Here we define

$$B_\Sigma=(a_\Sigma b_\Sigma)/(m_\Sigma+m_N)\quad\text{and}\quad A_\Sigma=a_\Sigma/(m_\Sigma-m_N),$$
$$B_\Lambda=a_\Lambda b_\Lambda/(m_\Lambda+m_N)\quad\text{and}\quad A_\Lambda=a_\Lambda/(m_\Lambda-m_N),$$

for $\Gamma=\gamma_5$, and

$$B_\Lambda=ia_\Lambda/(m_\Lambda-m_N)\quad\text{and}\quad A_\Lambda=ia_\Lambda b_\Lambda/(m_\Lambda+m_N)$$

for $\Gamma=1$. Also, we have

$$M_\Lambda=\sqrt{2}[(B_\Lambda g_N-B_\Sigma g_\Lambda)-i\gamma_5(A_\Lambda g_N+A_\Sigma g_\Lambda)]$$

for $\Gamma=\gamma_5$, and

$$M_\Lambda=-\sqrt{2}i[(A_\Lambda g_N+A_\Sigma g_\Lambda)-i\gamma_5(B_\Lambda g_N-B_\Sigma g_\Lambda)],$$

for $\Gamma=1$. Introducing the conditions that the asymmetries in $\Sigma^+\to n\pi^+$ and $\Sigma^-\to n\pi^-$ are zero, and that the s/p ratios in $\Sigma^+\to p\pi^0$ and $\Lambda\to p\pi^-$ decays have values x_0 and x_Λ, respectively, we can eliminate the A_Λ, A_Σ, B_Λ, and B_Σ and get a relation between the various strong-coupling constants. Another relation between the coupling constants is given by the ratio $|M_\Lambda|/|M_0|$, which is known from the measured lifetimes of Σ and Λ.[11] We now have four cases to consider: $\Gamma=1$ or γ_5; and pure s wave or pure p wave in $\Sigma^+\to n\pi^+$. The corresponding relations between the coupling constants are given below.

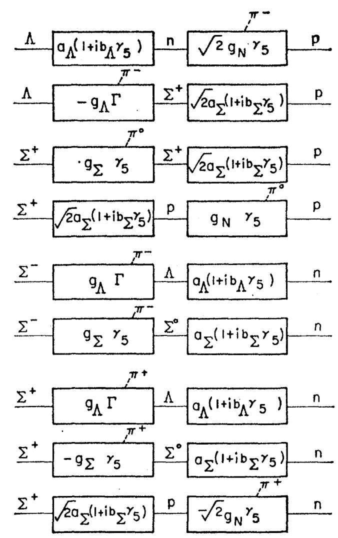

FIG. 1. Diagrams for Σ and Λ decays via baryon poles.

Case I. $\Gamma=\gamma_5$; $\Sigma^+\to n\pi^+$, *s wave.*

$$\frac{(g_\Sigma-g_N)(g_\Lambda^2+g_\Sigma g_N)}{(g_\Sigma+g_N)(g_\Lambda^2+g_\Sigma g_N-2g_N^2)}=\frac{x_\Lambda\mu_\Lambda}{x_0\mu_\Sigma},$$

$$\frac{g_\Lambda^2+g_\Sigma g_N}{g_\Lambda(g_\Sigma+g_N)}=\pm\frac{|M_\Lambda|}{|M_0|}\left(\frac{1+x_0^2}{1+x_\Lambda^2}\frac{x_\Lambda^2}{x_0^2}\right)^{\frac{1}{2}}.$$

Case II. $\Gamma=\gamma_5$; $\Sigma^+\to n\pi^+$, *p wave.*

$$\frac{(g_\Sigma-g_N)(g_\Lambda^2-g_\Sigma g_N-2g_N^2)}{(g_\Sigma+g_N)(g_\Lambda^2-g_\Sigma g_N)}=\frac{x_\Lambda\mu_\Lambda}{x_0\mu_\Sigma},$$

$$\frac{g_\Lambda^2-g_\Sigma g_N}{g_\Lambda(g_\Sigma-g_N)}=\pm\frac{|M_\Lambda|}{|M_0|}\left(\frac{1+x_0^2}{1+x_\Lambda^2}\frac{\mu_\Sigma^2}{\mu_\Lambda^2}\right)^{\frac{1}{2}}.$$

Case III. $\Gamma=1$; $\Sigma^+\to n\pi^+$, *s wave.*

$$\frac{(g_\Sigma-g_N)(g_\Lambda^2+g_\Sigma g_N)}{(g_\Sigma+g_N)(g_\Lambda^2+g_\Sigma g_N-2g_N^2)}=\frac{1}{x_0x_\Lambda\mu_\Sigma\mu_\Lambda},$$

$$\frac{g_\Lambda^2+g_\Sigma g_N}{g_\Lambda(g_\Sigma+g_N)}=\pm\frac{|M_\Lambda|}{|M_0|}\left(\frac{1+x_0^2}{1+x_\Lambda^2}\right)^{\frac{1}{2}}\frac{1}{x_0\mu_\Lambda}.$$

[6] We have used the $|\Delta T|=\frac{1}{2}$ rule in writing these contributions. Until the recent experimental results of Beall *et al.* (reference 1), Fowler *et al.* (reference 7), and Leitner *et al.* (reference 8), there has always been the possibility, emphasized by Okubo *et al.* (reference 9) that the $|\Delta T|=\frac{1}{2}$ rule for Λ decays could be accidental, since the same branching ratio in Λ decay is also predicted by the current-current form of the universal Fermi interaction which violates the $|\Delta T|=\frac{1}{2}$ rule. However, the prediction of the latter theory that the proton helicity in Λ decay must be negative is contradicted by these experiments, and one is now left with the $|\Delta T|=\frac{1}{2}$ rule as the only explanation of the observed branching ratio in Λ decay.

[7] W. B. Fowler, R. W. Birge, P. Eberhard, R. Ely, M. L. Good, W. M. Powell, and H. K. Ticho, Phys. Rev. Letters **6**, 134 (1961).

[8] J. Leitner, L. Gray, E. Harth, S. Lichtman, M. Block, B. Brucker, A. Engler, R. Gessaroli, A. Kovacs, T. Kikuchi, C. Meltzer, H. O. Cohn, W. Bugg, A. Pevsner, P. Schein, M. Meer, N. T. Grinellini, L. Lendinara, L. Monari, and G. Puppi, Phys. Rev. Letters **7**, 264 (1961.)

[9] S. Okubo, R. E. Marshak, and E. C. G. Sudarshan, Phys. Rev. **113**, 944 (1959).

[10] These six parameters have to fit six experimental numbers—four decay asymmetries and two lifetimes. However, it is not obvious that a fit will be obtainable with reasonable values of the coupling constants.

[11] For recent measurements of the lifetimes, see William E. Humphrey, Lawrence Radiation Laboratory Report UCRL-9752, 1961 (unpublished).

Case IV. $\Gamma=1$, $\Sigma^+\to n\pi^+$, *p wave.*

$$\frac{(g_\Sigma-g_N)(g_\Lambda^2-g_\Sigma g_N-2g_N^2)}{(g_\Sigma+g_N)(g_\Lambda^2-g_\Sigma g_N)}=\frac{1}{x_0x_\Lambda\mu_\Sigma\mu_\Lambda},$$

$$\frac{g_\Lambda^2-g_\Sigma g_N}{g_\Lambda(g_\Sigma-g_N)}=\pm\frac{|M_\Lambda|}{|M_0|}x_\Lambda\mu_\Sigma\left(\frac{1+x_0^2}{1+x_\Lambda^2}\right)^{\frac{1}{2}}.$$

Here μ_Λ and μ_Σ—kinematical factors for Λ and Σ decays, respectively—are given by

$$\mu_\Lambda=q_\Lambda/(E_\Lambda+m_N)\simeq0.053$$

and

$$\mu_\Sigma=q_\Sigma/(E_\Sigma+m_N)\simeq0.10,$$

where, q_Λ and q_Σ are the momenta, and E_Λ and E_Σ the energies of the proton in the decays at rest of Λ and Σ, respectively. When the $|\Delta T|=\frac{1}{2}$ rule is assumed in the analysis of experimental data, x_0 is known to be[5] very nearly -1, while x_Λ has a greater uncertainty attached to it. For further discussion, we shall take $x_0=-x_\Lambda=-1$, and $|M_\Lambda|/|M_0|=1$, which values are consistent with the experimental data. To simplify the calculations, we will also take $\mu_\Lambda=0.05$ (instead of the more exact value of ~0.053). We then get the following solutions for the coupling constants:

Case I. $\Gamma=\gamma_5$, $\Sigma^+\to n\pi^+$, *s wave.*[12]

$$g_\Sigma=-\tfrac{2}{3}g_N,$$
$$g_\Lambda^2=g_\Sigma^2=(4/9)g_N^2.$$

Case II. $\Gamma=\gamma_5$, $\Sigma^+\to n\pi^+$, *p wave.*[13]

$$g_\Sigma=-(5/3)g_N,$$
$$g_\Lambda^2=g_\Sigma^2/25=g_N^2/9.$$

Case III. $\Gamma=1$, $\Sigma^+\to n\pi^+$, *s wave.*

$$g_\Sigma\simeq-g_N,$$
$$g_\Lambda^2\simeq3g_N^2.$$

Case IV. $\Gamma=1$, $\Sigma^+\to n\pi^+$, *p wave.*

$$g_\Sigma\simeq0.02g_N,$$
$$g_\Lambda^2\simeq25g_\Sigma^2\simeq0.01g_N^2.$$

[12] In this case, $g_\Sigma=\pm g_\Lambda=1$ might appear to be an acceptable solution. A look at our expression for M_0 shows, however, that these values of the coupling constants would give zero asymmetry in $\Sigma^+\to p\pi^0$.

[13] This is the only solution where all coupling constants are nonzero. One could write down two other solutions in this case: (i) $g_\Sigma=\pm g_\Lambda=1$ which is not acceptable for the same reason as in case I; (ii) $g_\Sigma=0$, $g_\Lambda=\pm2$. Though the possibility of a vanishing $\Sigma\Sigma\pi$ coupling has been considered by some authors in the discussion of the branching ratios of Y^* etc., we have, as discussed below, thought it more interesting to look for solutions where all the strong coupling constants are of the same order. For the same reason, we are neglecting an alternative solution to case III, viz. $g_\Sigma\simeq2$, $g_\Lambda=\pm1/30$.

DISCUSSION

In the absence of definite knowledge of any of the strong strange-particle coupling constants, it is impossible to make a clear choice between the four cases considered. We have, therefore, taken the following as an additional criterion in the choice of a favored solution: that all the strong couplings be comparable with each other. In that case our results above may be taken as an indication that the decay $\Sigma^+\to n\pi^+$ takes place in the *s* wave. If it took place in the *p* wave, one would have $g_\Sigma^2\simeq25g_\Lambda^2$ for the case of even Σ-Λ parity, and $g_\Lambda^2\simeq25g_\Sigma^2\simeq0.01g_N^2$ for the case of odd Σ-Λ parity. The question of relative Σ-Λ parity is somewhat more difficult to decide.[14] But if neither of g_Λ^2 and g_Σ^2 is to be greater than g_N^2, we are left with $\Gamma=\gamma_5$, i.e., even Σ-Λ parity. The case which thus remains (Case I) is of the global symmetry type[15,16] in that $g_\Lambda^2=g_\Sigma^2$.

Once we have thus chosen the *g*'s the parameters a_Λ, a_Σ, b_Λ, and b_Σ are completely determined in this self-consistent model. We will not, however, give expressions for them, since we have no way of deciding what should be the reasonable values for them until we have analyzed the weak boxes further. When that is done, we hope we can make more definite statements about all these questions and about the relative Σ-Λ parity in particular. It may also be remarked that in the above calculation, only the relative sign of x_0 and x_Λ has been used, and not the absolute sign of either. The latter affects only the signs of *a*'s and *b*'s.

We would like to remark upon the relation of our model to the similar models of Feldman *et al.*,[17] and of Wolfenstein.[18] Feldman *et al.* take *K* poles also into account, in the spirit of a completely dispersion theoretical approach, where no particles are to be regarded as more fundamental than others. In doing so, however, they introduce two additional parameters—$(g_{K\Lambda}f_K)$, $(g_{K\Sigma}f_K)$, where f_K is the strength of the $K\pi$ vertex—into a problem in which there are already a large number of parameters. It is then impossible to make a definite statement on any of the questions to which we have sought answers. In fact, it is impossible even to

[14] We could perhaps make a somewhat stronger statement in discarding case III, if we note that in this case where Σ and Λ have opposite parities one of the coupling constants g_Σ, g_Λ is a scalar coupling constant while the other is a pseudoscalar coupling constant. With our criterion, the scalar coupling constant would be expected to be $\sim0.1\ g^2{}_N$.

[15] Note that our result $g_\Sigma^2=g_\Lambda^2$ refers to the renormalized coupling constants of conventional field theory, and not to bare coupling constants as in the usual symmetry schemes based on Lagrangians.

[16] There is, however, an important difference. Our solution has a negative relative sign for g_Σ and g_N, whereas global symmetry has a positive relative sign. In fact, the calculations of J. J. deSwart and C. Dullemond [Ann. Phys. 16, 263 (1961)] are sensitive to this relative sign, and if our preferred solution is the correct one, it would be an evidence against global symmetry. We are indebted to Dr. J. J. Sakurai for pointing this out to us.

[17] G. Feldman, P. T. Mathews, and A. Salam, Phys. Rev. **121**, 302 (1961).

[18] L. Wolfenstein, Phys. Rev. **121**, 1245 (1961).

predict the relative helicity of the protons in the Σ^+ and Λ decays, which depends on the sign $(g_{K\Sigma}/g_{K\Lambda})$ in the case considered by them in detail. In our model, on the other hand, the same helicity for the proton is almost definitely ruled out if the relative Σ-Λ parity is even, since a fit requires $g_\Sigma=2g_N$, $g_\Lambda=\pm 2g_N$ for $\Sigma^+\rightarrow n\pi^+$ going in s wave, and $g_\Sigma^2=(8.5\pm 2.9)g_N^2$, $g_\Lambda^2=(g_\Sigma+2g_N)^2$, for $\Sigma^+\rightarrow n\pi^+$ going in p wave. The choice is more difficult in the case of odd Σ-Λ parity, since the values of the coupling constants turn out to be practically the same as those which give rise to opposite proton helicities in the two decays.

Wolfenstein's model assumes that K decay is the more fundamental decay and that Σ and Λ decays are secondary. He, therefore, neglects baryon poles completely, but has to include two-particle intermediate states. His model, like that of Feldman *et al.*, also has (KYN) vertices, and his prediction regarding the angular-momentum states involved in $\Sigma^\pm$ decays into a neutron depends on the (KYN) and $(\Sigma\Lambda)$ parities assumed. Further, while in our model the fact that $\Sigma^+\rightarrow n\pi^+$ goes into s wave and $\Sigma^-\rightarrow n\pi^-$ into p wave is due to a dynamical cancellation between various diagrams, in the model of Wolfenstein, the Σ^+ goes into s wave only because a certain parity is assumed for the K meson and for $(\Sigma\Lambda)$, so that only a single diagram (K-pole diagram) contributes to it.[19]

We have already remarked about our omission of the K-pole diagrams. Apart from the fact that their inclusion would have increased the number of parameters in the problem, we were encouraged to neglect them by the frequently expressed conjecture that the K couplings are weak compared to the π couplings. It is, therefore, interesting that we are able to fit the experimental data without the inclusion of these diagrams. We have also omitted diagrams involving more than one-particle intermediate states, which would have to be included in a complete S-matrix approach. The lowest mass two-particle diagram has a pion and a nucleon in the intermediate state. Because the $J=\frac{1}{2}$ πN interaction is known to be small at the relevant energies, the contribution of this diagram may be expected to be small. We expect the πY intermediate-state diagrams to make an even smaller contribution since there is no strong $J=\frac{1}{2}$ interaction of the πY system either.[20]

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We wish to thank Dr. O. W. Greenberg for an interesting discussion. One of us (BMU) wishes to thank Dr. David L. Judd for his hospitality at the Lawrence Radiation Laboratory.

[19] Wolfenstein's expressions could be used to evaluate g_Σ/g_Λ from his model in the way we have done. In the case considered by him, if one assumes $x_0=-x_\Lambda$ and $|M_\Lambda|=|M_0|$, one is led to $g_\Lambda=g_\Sigma$ in his model too. It is curious that with both, our model which includes only baryon poles and excludes K poles, and Wolfenstein's model which excludes baryon poles, one is led to a global-symmetric solution.

[20] Note that the only charged Y^* known has a spin $>\frac{3}{2}$; see Robert P. Ely, Sun-Yiu Fung, George Gidal, Yu-Li Pan, Wilson M. Powell, and Howard S. White, Phys. Rev. Letters **7**, 461 (1961).

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N/D Method*

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A rigorous mathematical discussion of the N/D method is presented. It is shown in particular that we can always make sure that the N/D solution is the correct one in the sense that the D function has no redundant zeros, by examining the high-energy behavior of the phase shift in the solution obtained. It is also shown how the N/D method exhausts all the possible solutions of the original equation systematically according to the high-energy behavior of the phase shift. Virtually, the same arguments are shown to be applicable to the inverse method. It is, however, pointed out that the N/D method is preferable to the inverse method for both technical and physical reasons.

1. INTRODUCTION AND THE STATEMENT OF THE PROBLEM

THE N/D method[1] has been widely used in the dispersion theoretic approach to the scattering problem; the partial-wave scattering amplitude $F(z)$ (z stands for the complex c.m. energy) is represented as

$$F(z)=N(z)/D(z), \tag{1}$$

where $N(z)$ and $D(z)$ are individually analytic everywhere except for certain poles and cuts, and the coupled equations for $N(z)$ and $D(z)$ are solved. The inverse method,[2] which deals with the equation for the inverse amplitude has also been used.

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[1] See, for example, J. S. Ball and D. Y. Wong, Phys. Rev. Letters **7**, 390 (1961), which quotes virtually all other references.

[2] B. H. Bransden and J. W. Moffat, Phys. Rev. Letters **6**, 708 (1961) **8**, 145 (1962).