

# Ultrasonic Absorption in Liquid Helium at Temperatures below 0.6°K

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An explanation is given for the sound absorption observed in liquid He II in the phonon region below 0.6°K, arising from an interaction between acoustic phonons and thermal phonons. This interaction occurs between the acoustic wave and only those thermal phonons which propagate parallel to the wave. There is a "bunching" of thermal phonons within certain regions of the acoustic wave, analogous to the bunching of electrons in a traveling wave tube. The sound absorption calculated varies with temperature, frequency, and pressure in agreement with all available experimental data and is of the right order of magnitude. When the mean free path of the thermal phonons is reduced, for instance, by adding He<sup>3</sup>, the absorption is expected to decrease as found experimentally by Harding and Wilks.

## INTRODUCTION

THE absorption of sound in liquid He<sup>4</sup> has been measured below 1°K,<sup>1-3</sup> and shows a broad maximum at 0.9°K. The high-temperature side of this maximum, including the maximum itself was explained by Khalatnikov<sup>4</sup> to arise from relaxation processes between the thermal excitations of the liquid, phonons, and rotons. However, at temperatures below 0.8°K this relaxation process could not even approximately account for the observed absorption. As can be seen from Fig. 1, the measured absorption decreases with about the fourth power of the temperature, whereas the relaxation process alone (solid line) would lead to a much more rapid decrease. At 0.6°K the relaxation process already gives values about 3 orders of magnitude too small, and the discrepancy becomes even more serious at lower temperatures. Thus, it is evident that some other mechanism must be responsible for the absorption at these low temperatures.

In He II below 0.6°K the thermal excitations are predominantly phonons with a long mean free path which according to the heat conductivity data<sup>5</sup> is at least 1 mm, and thus considerably longer than the acoustical wavelength used ( $10^{-1}$ – $10^{-2}$  mm). During one acoustical cycle and in one wavelength the thermal phonons cannot, therefore, establish statistical equilibrium and, thus, thermodynamical concepts are not very useful here.

On the other hand, from the fact that the mean free path of the thermal phonons is longer than the acoustical wavelength, it follows that the uncertainty in their energy is less than  $\hbar\omega$ ,<sup>6</sup>  $\omega$  being the acoustical frequency. In He II below 0.6°K, therefore, it is appropriate to consider processes in which a thermal

phonon may absorb one acoustical phonon, thereby increasing its energy by  $\hbar\omega$ .

It is the purpose of this paper to show that an acoustic wave influences the propagation of thermal phonons in a manner which leads to a net energy transfer from the wave to the thermal phonons, as pointed out earlier in a short note.<sup>7</sup> This process, which can also be looked at as a three-phonon process, can explain the ultrasonic absorption in liquid He<sup>4</sup> below 0.6°K and also seems to account for the experimental results of Harding and Wilks<sup>8</sup> which showed a decrease of the absorption if small amounts of He<sup>3</sup> are added to He<sup>4</sup>. A more recent theoretical treatment by Woodruff,<sup>9</sup> based on the Boltzmann equation, leads to results which are similar in some respect to the ones presented here.

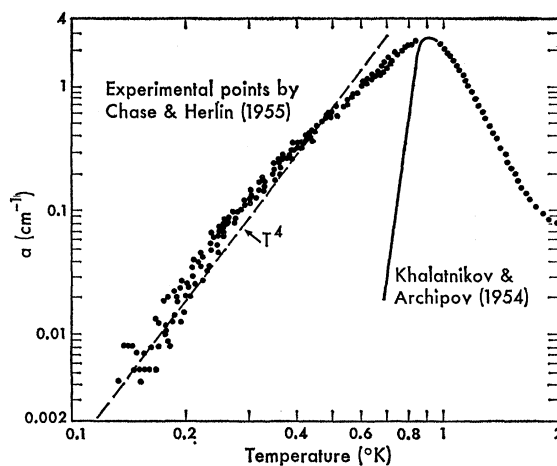


FIG. 1. Absorption of 12 Mc/sec ultrasonic waves in liquid helium. Experimental points by Chase and Herlin,<sup>1</sup> theoretical curve according to calculations by R. B. Archipov [Doklady Akad. Nauk. U.S.S.R. 98, 747 (1954)], based on a relaxation process.

<sup>1</sup> C. E. Chase, Proc. Roy. Soc. (London) A220, 116 (1953); C. E. Chase and M. A. Herlin, Phys. Rev. 97, 1447 (1955).

<sup>2</sup> K. Dransfeld, J. A. Newell, and J. Wilks, Proc. Roy. Soc. (London) A243, 500 (1957).

<sup>3</sup> W. M. Whitney, Phys. Rev. 105, 38 (1957).

<sup>4</sup> I. N. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 243 (1950) [translation: Soviet Phys.—JETP 23, 8, 21 (1952)].

<sup>5</sup> H. A. Fairbank and J. Wilks, Proc. Roy. Soc. (London) A231, 545 (1955).

<sup>6</sup> L. D. Landau and G. Rumer, Physik. Z. Sowjetunion, 11, 18 (1937).

<sup>7</sup> K. Dransfeld, Bull. Am. Phys. Soc. 3, 221 (1958).

<sup>8</sup> G. O. Harding and J. Wilks, *Proceedings of the Seventh International Conference on Low-Temperature Physics, Toronto, 1960* (University of Toronto Press, Toronto, 1960), p. 647, detailed paper to be published.

<sup>9</sup> T. O. Woodruff (private communication).

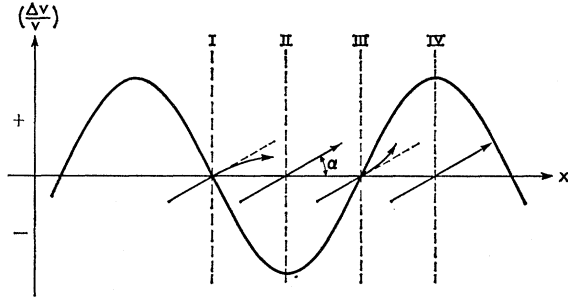


FIG. 2. Variation of the velocity of thermal phonons caused by an ultrasonic wave travelling to the right. The arrows represent the paths of thermal phonons going in almost the same direction as the sound wave.

### ACOUSTIC ABSORPTION BY THREE-PHONON PROCESSES IN He II

The interaction considered here takes place between the acoustic phonons, representing the impressed ultrasonic wave and the thermal phonons of energy  $kT$  whose mean free path we will assume to be limited only by the walls of the apparatus. Although the wavelength of the thermal phonons is much shorter than the acoustic wavelength, it is still at least several hundred Å units below 0.6°K, and we will therefore neglect any dispersion due to the structure of the liquid, i.e. we will ascribe the same velocity to acoustic and thermal phonons.

The interaction between acoustical and thermal phonons takes place, of course, by virtue of the elastic nonlinearity of the medium. We will only mention here the simplest case in which the elastic energy of the liquid contains a term in the third power of the strain. It is well known<sup>10</sup> that a third-order term leads to three-phonon processes, in which, for example, two phonons combine to give a third. If the wavelength of all three phonons is large compared to the interatomic distance, as in our case, the "momentum" as well as the energy is conserved<sup>10</sup>

$$\mathbf{k}_1 + \mathbf{k} = \mathbf{k}_2, \quad (1)$$

$$\epsilon_1 + \epsilon = \epsilon_2. \quad (2)$$

$\mathbf{k}_1$  and  $\mathbf{k}_2$  are the wave vectors of the initial and final thermal phonon and  $\epsilon_1$ ,  $\epsilon_2$  are their respective energies.  $\mathbf{k}$  and  $\epsilon = \hbar\omega$  refer to the low-frequency acoustic phonon which is absorbed in the process.

Since all three phonons, being longitudinal, have the same velocity, Eqs. (1) and (2) can be written

$$\omega_1 \frac{\mathbf{k}_1}{|\mathbf{k}_1|} + \omega \frac{\mathbf{k}}{|\mathbf{k}|} = \omega_2 \frac{\mathbf{k}_2}{|\mathbf{k}_2|}, \quad (1a)$$

$$\omega_1 + \omega = \omega_2. \quad (2a)$$

<sup>10</sup> See, for example, J. M. Ziman, *Electrons and Phonons* (Clarendon Press, Oxford, 1960).

It is evident that (1a) and (2a) can be satisfied simultaneously only if

$$\frac{\mathbf{k}_1}{|\mathbf{k}_1|} = \frac{\mathbf{k}}{|\mathbf{k}|} = \frac{\mathbf{k}_2}{|\mathbf{k}_2|},$$

i.e., if all three phonons are traveling parallel.<sup>6</sup> Thus, acoustic phonons can be absorbed only by those thermal phonons which travel parallel with the sound wave.

The elastic nonlinearity of the liquid, as defined by a third-order term in the elastic energy can be expressed also by "Grüneisen's constant"  $\gamma$  which relates a density change  $\Delta\rho/\rho$  to the corresponding change of the velocity of sound,  $\Delta v/v$ .

$$\Delta v/v = \gamma(\Delta\rho/\rho). \quad (3)$$

The low-frequency acoustic wave has naturally a wavelength several orders of magnitude larger than the wavelength of the high-frequency ( $kT/h$ ) thermal waves. The density variations of the acoustic waves cause therefore, according to (3), a long-wavelength modulation of the velocity of sound in the liquid. As the sound wave moves to the right with velocity  $v$  the sinusoidal spatial variation of the velocity moves to the right.

At a fixed time, therefore, the velocity of thermal phonons shows a variation in space as indicated in Fig. 2. Let us look at a thermal phonon traveling for a distance  $x_0$  within a small angle  $\alpha$  with respect to the sound wave. If the angle  $\alpha$  is small, the thermal phonon remains within the same phase of the sound wave for an appreciable time. However, for angles  $\alpha$  larger than corresponding to

$$\alpha_L^2 = (\Lambda/x_0), \quad (4)$$

one can see by geometrical arguments (Fig. 3) that the thermal phonon will lag behind the sound wave by more than half an acoustic wavelength  $\Lambda$  after having traversed the distance  $x_0$  ( $x_0 \gg \Lambda$ ). Thus, assuming that the phonons propagate in straight lines and equally as fast as the sound wave, we conclude that only those phonons whose angle with the sound wave is smaller than  $(\Lambda/x_0)^{1/2}$  can keep in step with the sound wave for the whole distance  $x_0$ .

This conclusion is, however, not yet quite correct because, in general, the thermal phonons do *not* propagate in straight paths. The propagation of thermal phonons through a liquid with a velocity gradient is similar to the case of light passing through a medium with gradually changing refractive index: Both the phonon path and the beam of light are bent towards the direction of decreasing velocity.

A thermal phonon moving at an angle  $\alpha$  with respect to the sound wave (Fig. 2) and the velocity gradient  $(\partial v/\partial x)$  will experience a rate of change of its direction which is given by geometrical optics

$$d\alpha/dt = (\partial v/\partial x) \sin\alpha \approx (\partial v/\partial x)\alpha$$

for a small  $\alpha$ . Thus, the angle can increase or decrease

in time, depending on the sign of the gradient  $(\partial v/\partial x)$ . A phonon traveling in region "I" of Fig. 2 will, for example, become more and more parallel to the  $x$  axis, while another one in "III" would increase its angle in time. While traveling for a distance  $x_0$ , a phonon changes its initial angle  $\alpha$  by a small increment

$$\Delta\alpha = (\partial v/\partial x)\alpha(x_0/v_0), \quad \text{for } \Delta\alpha < \alpha, \quad (5)$$

which is negative in region I and positive in III.

Consequently, a phonon in region I will stay in step with the sound wave for the whole distance  $x_0$ , if its initial angle is smaller than

$$\alpha_I = \alpha_L + \Delta\alpha_L, \quad (6)$$

therefore, allowing larger angles than (4). Vice versa, a phonon in region III can only stay in phase with the wave if its initial angle is smaller than

$$\alpha_{III} = \alpha_L - \Delta\alpha_L. \quad (7)$$

The density  $N_I$  of those phonons in the region I, which can keep in step with the wave for a distance  $x_0$ , because their initial angle is smaller than  $\alpha_I$ , is

$$N_I \approx N/2(\alpha_I^2\pi/4\pi),$$

where  $N$  is the total density of thermal phonons. In regions III, since  $\alpha_{III}$  is smaller, there are fewer phonons  $N_{III}$  able to stay in phase

$$N_{III} \approx N/2(\alpha_{III}^2\pi/4\pi).$$

Thus, in regions I there are more phonons in step with the wave than in the regions III, after the wave has traveled for a distance  $x_0$ . For the excess number  $\Delta N = N_I - N_{III}$ , one finds from (5), (6), (7)

$$\begin{aligned} \Delta N/N &= \frac{1}{8}(\alpha_I^2 - \alpha_{III}^2) = \frac{1}{2}\alpha_L\Delta\alpha \\ &= (\alpha_L^2/2)(\partial v/\partial x)_0(x_0/v_0), \end{aligned} \quad (8)$$

where  $(\partial v/\partial x)_0$  is the gradient at the center of region I.  $(\partial v/\partial x)_0$  is related to the amplitude of the velocity modulation  $\Delta v$  as  $(\partial v/\partial x)_0 = (2\pi/\Lambda)\Delta v$ . Using (4) equation (8) reduces to

$$\Delta N/N = \pi(\Delta v/v). \quad (9)$$

We conclude: *The sound wave causes a bunching of the number of phonons staying in regions I.* This excess number does not vary with the distance  $x_0$  traveled by the wave, but it increases linearly with the amplitude of the wave.

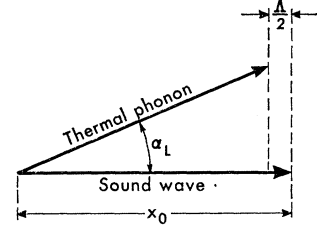
While the phonon bunching was a consequence of the *spatial* variation of the phonon-velocity only, we will now consider also the fact that the acoustic wave causes, at any fixed point, a change of the velocity *in time* at a rate

$$\partial v/\partial t = (\partial v/\partial x)v_0.$$

Using again  $(\partial v/\partial x) = \pm(2\pi/\Lambda)\Delta v$  for regions I and III, respectively, we find

$$(1/v)(\partial v/\partial t) = \pm(2\pi/\Lambda)\Delta v = \pm\omega(\Delta v/v),$$

FIG. 3. A thermal phonon travelling a distance  $x_0$  under the small angle  $\alpha_L = (\Lambda/x_0)^{1/2}$  with respect to the sound wave lags behind the wave by half an acoustic wavelength  $\Lambda$ .



which is the relative rate of change of the velocity at points in space where region I or III is passing by, with  $\omega$  being the acoustic frequency.

If a wave is propagating through a medium, while the *local* velocity of propagation changes in time by  $(\partial v/\partial t)$ , one can show in general that the frequency  $\nu$  of the wave for an observer at rest is also changed by

$$(1/\nu)(\partial \nu/\partial t) = 1/v(\partial v/\partial t).$$

A phonon of initial energy  $h\nu$  traveling in region I of the sound wave is therefore gaining energy at an average rate

$$h(\partial \nu/\partial t) = h\nu(1/v)(\partial v/\partial t) = h\nu\omega(\Delta v/v), \quad (10)$$

while phonons in region III are losing the same amount (for  $\partial v/\partial t < 0$ ).

However, in virtue of the bunching of thermal phonons there are more phonons in regions I than in III and thus there is a net energy transferred to the thermal phonons, at the expense of the acoustic wave. The energy loss of the ultrasonic wave per unit volume and unit time is, by using (9) and (10)

$$\begin{aligned} dW/dt &= \Delta N h(\partial \nu/\partial t) = N(\Delta N/N)h\nu\omega(\Delta v/v) \\ &= N h\nu\omega\pi(\Delta v/v)^2. \end{aligned}$$

Introducing the thermal energy density of the liquid  $U = N h\nu$  and remembering (3), we have

$$dW/dt = U\omega\pi\gamma^2(\Delta\rho/\rho)^2,$$

with  $\Delta\rho/\rho$  being the acoustic amplitude. The absorption coefficient  $a$  is defined by

$$2a = \left( \frac{\text{energy absorbed per sec per cc}}{\text{energy transported through cm}^2 \text{ per sec}} \right) = \frac{dW/dt}{W}.$$

Since the energy transport  $W$  of an acoustic wave with amplitude  $(\Delta\rho/\rho)$  is

$$W = \rho v^3(\Delta\rho/\rho)^2,$$

we arrive at the following absorption coefficient for sound in liquid helium

$$a = (\omega\pi U\gamma^2/2\rho v^3)[\text{cm}^{-1}]. \quad (11)$$

## DISCUSSION

(1) Comparison with Experiments in Pure He<sup>4</sup>

Although the derivation of the absorption coefficient (11) is only semiquantitative in view of various simplifying assumptions made, the agreement with the available experimental data is satisfactory. Values for  $U$  and  $\gamma$  can be derived from specific-heat work by Kramers *et al.*<sup>11</sup> and from acoustic experiments by Atkins *et al.*,<sup>12</sup> respectively. For a frequency of 12 Mc/sec and at a temperature of 0.5°K, the absorption according to (11) is  $a=0.4\text{ cm}^{-1}$ , while the experimental value (see Fig. 1) is  $a=0.6\text{ cm}^{-1}$ . Thus, the absorption coefficient (11) is of the right order of magnitude. Besides it agrees well with the observed dependence on temperature,<sup>1,2</sup> frequency<sup>2</sup> and pressure,<sup>2,3</sup> as previously pointed out.<sup>7</sup>

## (2) Mean-Free-Path Effects

A proper three-phonon process as described above can only occur if the mean free path of the thermal phonons is long enough so that they remain bunched in the sound wave until they have, on the average, absorbed one acoustic quantum  $\hbar\omega$ . In order to absorb just one acoustic quantum, the bunched thermal phonons need on the average a certain "interaction time"  $\tau$  which is, according to Eq. (10),

$$\hbar(\partial\nu/\partial t)\tau = \hbar\omega = \hbar\nu\omega(\Delta\nu/\nu)\tau$$

or

$$\tau = (1/2\pi\nu)/(\Delta\nu/\nu).$$

This corresponds to an "interaction length"

$$l = \tau v = (\lambda/2\pi)/(\Delta\nu/\nu)$$

( $\lambda$ =wavelength of thermal phonons) which can be larger than the acoustic wavelength, for small amplitudes of the sound wave.

If He<sup>3</sup> impurities restrict the mean free path of the thermal phonons to less than this interaction length a

simple three-phonon process as described above will probably no longer occur. The acoustic absorption due to 3-phonon processes is, therefore, expected to fall with increasing He<sup>3</sup> content, in agreement with the measurements of the absorption in He<sup>4</sup>—He<sup>3</sup> mixtures by Harding and Wilks.<sup>8</sup>

If the mean free path of the thermal phonons becomes even smaller than the acoustic wavelength the phonon energy is uncertain by more than  $\hbar\omega$  and one can no longer speak of the absorption of an acoustic quantum  $\hbar\omega$  by a single thermal phonon. In this case the sound wave can no longer interact with individual phonons but only with the whole ensemble of thermal excitations, and the absorption of sound is caused by relaxation processes within the ensemble, as described in detail by Khalatnikov<sup>4</sup> for liquid helium at higher temperatures.

## (3) Effects at Large Amplitudes

All previous discussions refer to small acoustic amplitudes, i.e.,  $\Delta\rho/\rho \sim \Delta v/v \ll \Lambda/x_0$ . For amplitudes larger or equal to  $\Lambda/x_0$ , a sound wave, after traveling a distance  $x_0$ , does not preserve its sinusoidal shape, but transforms into a shock wave having a steep gradient in region I and almost no gradient  $\partial v/\partial x$  in region III. All thermal phonons which are bunched in region I, would be further compressed by being sharply focused in the immediate vicinity of the steep gradient. All these modifications would tend to greatly increase the absorption above the low-amplitude value expressed in Eq. (11). At a frequency of 20 Mc/sec and over a path length of 1 cm, these effects would become noticeable as soon as the amplitude  $\Delta\rho/\rho \sim \Delta v/v$  approaches  $10^{-3}$ , which is almost certainly above the peak amplitudes used in all previous experiments so that these effects can be neglected here.

## ACKNOWLEDGMENTS

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<sup>12</sup> K. R. Atkins and R. A. Stasior, *Can. J. Phys.* **31**, 1156 (1953).