like to express their appreciation to Dr. E. G. Funk of the University of Notre Dame for several discussions concerning the work of Schwerdtfeger *et al.*[9] on the decay of Pm^{148} and for kindly sending a copy of their manuscript prior to publication. They would also express appreciation to Dr. D. A. Shirley of the Lawrence Radiation Laboratory at Berkeley for several informative communications concerning nuclear alignment data on 5.4-day Pm^{148} and for kindly allowing them to quote some of the results prior to publication.

PHYSICAL REVIEW VOLUME 127, NUMBER 1 JULY 1, 1962

(d,t) Reaction Studies on Iron and Nickel*

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(Received February 28, 1962)

Angular distributions and absolute cross sections have been measured for the (d,t) reactions on Fe^{54}, Fe^{58}, Ni^{58}, Ni^{60}, and Ni^{61} at a deuteron energy of 21.6 MeV. Sum rules are used to interpret the resulting reduced widths in terms of the average occupation numbers of the $1f_{7/2}$, $2p$, and $1f_{5/2}$ orbits in the various target ground states. The average occupation numbers so obtained are compared with the predictions of the pairing model of Kisslinger and Sorensen. We examine whether our conclusions are consistent with the results of (d,p) experiments on Ni^{58} and Ni^{60}.

1. INTRODUCTION

THIS paper is the third of a series[1,2] concerning (d,t) reaction studies of various nuclei in the $(1f,2p)$ shell. Angular distributions and absolute cross sections have been measured for the (d,t) reactions on Fe^{54}, Fe^{58}, Ni^{58}, Ni^{60}, and Ni^{61} at a deuteron energy of 21.6 MeV. The resulting reduced widths are interpreted in terms of the structure of the nuclear states involved, and the conclusions for the Ni isotopes are compared with the predictions of the pairing model of Kisslinger and Sorensen. The paper concludes with a general survey of the current experimental and theoretical situation concerning the neutron configurations of fp-shell nuclei, particularly the ground states. It reviews what has been learned in this connection from (d,t) reactions on nuclei between V^{51} and Zn^{68} and examines whether our conclusions are consistent with other experimental informaton. In particular, the implications of (d,p)-reaction studies of fp-shell nuclei are considered.

2. EXPERIMENTAL METHOD AND RESULTS

The experimental equipment and techniques have been described previously.[1] The experiment was performed in the 60-in. scattering chamber. The detector consisted of a 0.012-in. NaI(Tl) dE/dx crystal mounted on a 0.160-in. NaI(Tl) E crystal. The light from the E crystal was coupled directly to a photomultiplier, while the light from the dE/dx crystal was coupled to its photomultiplier via an air light pipe. The particles were identified by using a multiplier circuit whose inputs consisted of the E and dE/dx singals. After adjustment of the circuit, the output spectrum of the multiplier consisted of peaks corresponding to deuterons, protons, and tritons. The position of these peaks was independent of the energy of the incident particles. A single-channel analyzer set to accept only the triton peak gated a multichannel analyzer which recorded the E pulse. The targets were metallic foils, enriched in the desired isotopes and rolled to a thickness of about 0.0003 in. The energy calibration was obtained both from range-energy relations and reactions of known Q value. The calibration is accurate to roughly 100 keV. Relative cross sections are reliable to within the statistical error involved. Absolute cross sections have a possible error of less than 10%, resulting mainly from uncertainties in the target thickness.

Energy spectra were obtained at 3° intervals between 12° and 30° for all targets. In previous studies[1] of (d,t) reactions in the fp shell, it appeared that the desired information concerning l values and reduced widths could be obtained from the behavior of the differential cross section within this limited angular range.[3] This

* Work performed under the auspices of the U. S. Atomic Energy Commission, with partial support from the National Science Foundation.

[1] B. Zeidman, J. L. Yntema, and B. J. Raz, Phys. Rev. 120, 1723 (1960).

[2] B. J. Raz, B. Zeidman, and J. L. Yntema, Phys. Rev. 120, 1730 (1960).

[3] See, however, the discussion of the extraction of reduced widths, in Sec. 3.

stems from the fact that the angular distributions corresponding to the various l values are clearly differentiated at the bombarding energy and Q values of the present study.

Angular distributions have been obtained for the unresolved groups in the various spectra except when the level spacing permitted unambiguous separation of the contributions from individual states. The fact that several of the observed triton groups involve more than one final state is clearly established by changes in peak shape in the energy spectra as the angle varies. All the angular distributions reported here correspond to pickup with $l=1$, $l=3$, or mixtures thereof. The two basic shapes are illustrated in Fig. 1, which shows angular distributions for the $Ni^{60}(d,t)Ni^{59}$ transitions to the ground state of Ni^{59} ($l=1$) and to a group of levels at an excitation energy of about 2.5 MeV ($l=3$).

The spectra at 21° (lab) for the iron isotopes are shown in Fig. 2, those for the nickel isotopes are shown in Fig. 3. The information extracted from these energy spectra and the corresponding angular distributions—the excitation energies, l values, and strengths (reduced widths) of the various triton groups—is summarized in Table I. Triton groups that are known to correspond to more than one level of the residual nucleus are indicated by square brackets following their excitation energy.

The experimental results will be discussed and analyzed in Sec. 4 and 5. There are, however, a few comments which should be made at this point.

The Fe^{58} target contained 53% Fe^{58}, 46% Fe^{56}, and small amounts of the other iron isotopes. In order to obtain spectra for the reaction $Fe^{58}(d,t)Fe^{57}$, it was necessary to subtract the contribution from Fe^{56} from the data obtained with the "Fe^{58} target." A typical spectrum obtained in this manner is shown in Fig. 2. Because of this subtraction, the possible errors in intensity and peak position are greater than those for the other targets. One of the peaks in the $Fe^{58}(d,t)Fe^{57}$ spectrum corresponds to a broad group around 4.7-MeV excitation in Fe^{57}. It is difficult to assign an l value to

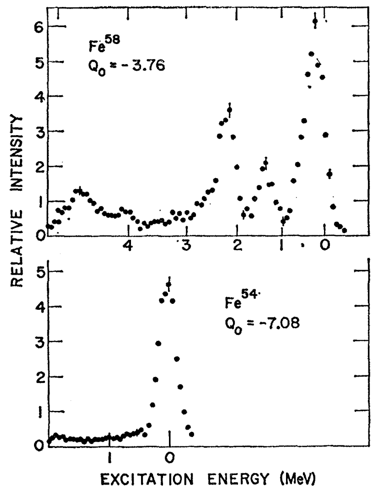

FIG. 2. Triton spectra at 21° (lab) for Fe isotopes. The ground-state Q values are indicated.

this group. Careful examination suggests a large $l=3$ component with perhaps an admixture of $l=1$, but it should be emphasized that the assignment is very uncertain.

Finally, it should be noted that weak $l=1$ components are readily observable in the presence of $l=3$ contributions, but that sizeable $l=3$ admixtures can easily be obscured by predominant $l=1$ contributions.

3. METHOD OF INTERPRETING THE EXPERIMENTAL RESULTS

The results of stripping and pickup reactions will be interpreted in terms of the structure of nuclear levels in the manner described in earlier studies.[2,4] The method uses the Butler formula for the differential cross section in the form

$$(d\sigma/d\omega) \propto (c)^2\Lambda\theta^2, \tag{3.1}$$

where θ^2 is the reduced width, Λ is a factor that measures the overlap of the internal wave functions of the deuteron and triton, and $(c)^2$ is an isotopic-spin coupling factor. The remaining terms in the Butler cross section,

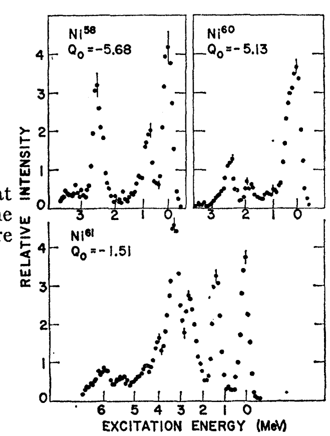

FIG. 3. Triton spectra at 21° (lab) for Ni isotopes. The ground-state Q values are indicated.

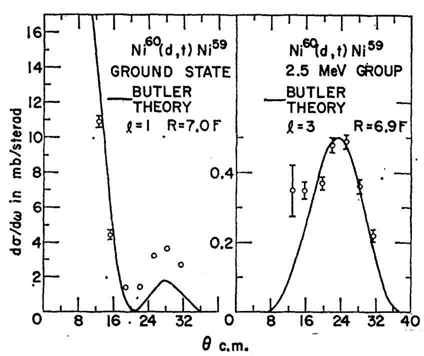

FIG. 1. The $l=1$ angular distribution for $Ni^{60}(d,t)Ni^{59}$ (ground state), and the $l=3$ angular distribution for $Ni^{60}(d,t)Ni^{59}$ (2.5 MeV).

[4] M. H. Macfarlane and J. B. French, Revs. Modern Phys. **32**, 567 (1960).

containing the angular dependence, are simple explicit functions of the masses, energies, and scattering angle and are given, for example, in Eq. (II. 37) of reference 4. It should be emphasized, however, that this analysis is in no sense dependent on the validity of the Butler theory as a detailed description of the stripping process. Rather, the Butler formula is used as a convenient basis for a semiphenomenological analysis, as explained in detail in reference 4.

Standard procedure is to determine the l value and reduced width by fitting the Butler cross section to the position and absolute magnitude of the first maximum in the observed differential cross section. However, for most of the $l=1$ transitions in the present study, the first maximum occurs below the angular range covered in the experiment. In order to obtain a first-peak cross section for use in extracting reduced widths, it is therefore necessary to use an extrapolation procedure which is discussed elsewhere.[1] The essential assumption is that the ratio of the cross section at the primary maximum to that at the secondary one is the same for all nuclides, the actual value of the ratio being taken from the $Fe^{56}(d,t)Fe^{55}$ ground-state transition where both maxima have been observed.[1] It is difficult to assess the errors

TABLE I. Reduced widths ($\Lambda\theta^2$) of prominent triton groups. A square bracket following the "final-state excitation" indicates a group which is known to involve more than one level of the residual nucleus.

Reaction	Final-state excitation (MeV)	Q(MeV)	$\frac{d\sigma}{d\omega}(28°)$ (mb/sr)	l	$\Lambda\theta^2$
$V^{51}(d,t)V^{50}$[a]	0.3]	−5.1	1.6	3	4.3
	1.1]	−5.9	1.5	3	3.7
	3.1]	−7.9	0.7	3	3.3
$Cr^{52}(d,t)Cr^{51}$[b]	0	−5.8	1.2	3	5.4
	0.75	−6.5	0.2	3	0.8
$Fe^{54}(d,t)Fe^{53}$	0	−7.1	1.0	3	3.0
$Mn^{55}(d,t)Mn^{54}$	0	−4.0	2.5	1	5.7
	1.1]	−5.1	0.95	{1	1.0
				+3	1.9
	2.7]	−6.7	0.40	1	1.0
	4.0]	−8.0	0.15	3	0.4
$Fe^{56}(d,t)Fe^{55}$	0	−4.9	2.0	1	4.0
	0.42	−5.4	0.65	1	1.2
	1.4]	−6.3	1.0	{1	0.4
				+3	2.7
	2.0]	−6.9	0.15	1	0.3
	2.5]	−7.4	0.10	3	0.2
$Fe^{57}(d,t)Fe^{56}$[c]	0	−1.4	0.40	1	1.1
	0.85	−2.2	1.40	{1	2.2
				+3	1.4
	2.9]	−4.3	2.0	1	3.3
	4.0]	−5.4	0.70	3	1.8
$Fe^{58}(d,t)Fe^{57}$[d,e]	0]	−3.8	4.9	{1	9.0
				(+3)	(<2)
	1.3]	−5.1	1.0	{1	3.5
				(+3)	(<0.5)
	2.2]	−6.0	1.1	3	4.0
	≈4.7]	−8.5	≈1.0	3(?)	≈3
$Ni^{58}(d,t)Ni^{57}$	0	−5.7	2.05	1	4.2
	0.85]	−6.5	0.20	{1	0.3
				+3	0.5
	1.15]	−6.8	0.20	1	0.4
	2.5]	−8.2	0.40	{1	0.1
				+3	1.1
$Co^{59}(d,t)Co^{58}$	0.3]	−4.5	5.3	1	12
				(+3)	(<4)
$Ni^{60}(d,t)Ni^{59}$	0	−5.1	3.6	1	7.6
	0.34]	−5.5	0.42	3	1.2
	0.47]	−5.6		1	0.7
$Ni^{60}(d,t)Ni^{59}$	0.87	−6.0	0.35	1	0.6
	1.31	−6.5	0.2	1	0.4
	1.8]	−6.9	0.2	{1	0.2
				+3	0.4
	2.5]	−7.6	0.36	3	1.1
	2.9]	−8.0	0.15	1	0.2
$Ni^{61}(d,t)Ni^{60}$[c]	0	−1.5	1.35	1	3.2
	1.33	−2.8	1.70	1	3.7
	2.2]	−3.7	0.30	1	0.6
	2.5]	−4.0	0.50	3	1.7
	3.3]	−4.8	4.3	1	6.0
	3.9]	−5.4	1.0	{1	1.3
				+3	0.2
	4.5]	−6.0	0.50	{1	0.6
				+3	0.1
	5.9]	−7.4	0.30	3	0.9
$Cu^{63}(d,t)Cu^{62}$	0.4]	−4.9	4.7	{1	9.8
				(+3)	(<4)
	1.4]	−5.9	0.7	{1	1.9
				+3	(<0.8)
$Zn^{64}(d,t)Zn^{63}$	0	−5.6	3.3	1	8.0
	0.19	−5.8		(3)	(<3)
	0.64	−6.2	0.80	1	1.7
	1.1]	−6.7	0.40	{1	0.8
				(+3)	(<0.4)
$Cu^{65}(d,t)Cu^{64}$	0.4]	−4.0	7.8	{1	16
				(+3)	(<6)
$Zn^{66}(d,t)Zn^{65}$	0]	−4.8	6.0	{1	13
				(+3)	(<5)
	0.86	−5.6	1.1	{1	3.8
				(+3)	(<1)
$Zn^{67}(d,t)Zn^{66}$[c,f]	0	−0.8	0.25	3	0.5
	1.0]	−1.8	0.35	{1	0.8
				+3	0.4
	2.7]	−3.5	1.85	{1	2.0
				+3	1.8
	3.7]	−4.5	3.85	{1	3.0
				+3	1.5
$Zn^{68}(d,t)Zn^{67}$	0.3]	−4.2	7.5	{1	14
				+3	7.2

[a] The upper limit of excitation in all experiments corresponds to $Q\approx-8.5$ MeV.
[b] The $Cr^{52}(d,t)$ energy spectrum (Fig. 2 of reference 1) suggests the existence of sizable $l=3$ groups to levels between 1.5 and 3 MeV in Cr^{51}.
[c] The Q value of this transition is sufficiently far from that of the standard $Fe^{56}(d,t)Fe^{55}$ ground-state reaction to lessen the reliability of our extrapolation procedure for $l=1$ reduced widths.
[d] The errors on the reduced widths for this target are larger than for the other targets.
[e] The symbol (<) indicates an upper limit for possible $l=3$ contributions to a predominantly $l=1$ transition.
[f] The $Zn^{67}(d,t)$ spectrum (Fig. 5 of reference 1) indicates the presence of fairly strong unresolved groups to levels at about 4 MeV in Zn^{66}.

introduced by this extrapolation procedure, but it is hoped that they do not exceed 20% since the Q values of the various reactions involved are all much the same. For $l=3$ transitions the cross section at the first maximum is measured directly and should be reliable to within 10%.

Values of l and $\Lambda\theta^2$ [strictly $(c)^2\Lambda\theta^2$] for the various observed triton groups, together with the corresponding excitation energies and Q values, are given in Table I. For convenience of reference, the reduced widths for other fp-shell nuclei measured in the earlier experimental study in the present series[5] are also listed.

In addition to the nuclear overlap factor $\mathcal{S}$ which contains the information sought, the measured reduced width θ^2 contains a dynamical factor θ_0^2, the single-particle reduced width, whose dependence on energy and l simulates the various distortion effects neglected by the simple Butler theory. These quantities are defined and discussed in Sec. 2 of reference 4. Laying aside for the moment the question of evaluating the empirical constants Λ and θ_0^2, we turn to a discussion of the spectroscopic factor $\mathcal{S}$.

In any reaction in which a single nucleon is transferred, the strength of a level is characterized by its "relative reduced width" $\mathcal{S}=\theta^2/\theta_0^2$, the reduced width in units of the single-particle width.[4] If, as is often convenient, one wishes to consider the combined strength of a group of levels in the residual nucleus of a stripping or pickup reaction, the appropriate "strengths" are

$$G=\sum(c)^2\mathcal{S} \tag{3.2}$$

for pickup reactions, and

$$\bar{G}=\sum(2J+1/2J_i+1)(c)^2\mathcal{S} \tag{3.3}$$

for stripping. The summations include all levels of the group to be considered and may embrace more than one value of j but only one value of n and l.

The strengths defined here include spin and isotopic-spin statistical factors. As a result, if spin and isotopic spin are ignored completely in extracting reduced widths, the measured strength of a group of levels is simply $\Lambda\theta_0^2G$ for (d,t) pickup and $\theta_0^2\bar{G}$ in the case of stripping. It is then unnecessary to know the spin and isotopic spin of each level of a group in order to evaluate its strength.[6]

Simple and general expressions may be given for the strengths G and $\bar{G}$ in terms of the shell-model wave function of the target ground state.[4,7] Treating neutrons and protons on a separate footing leads, for a single value of j (and of course of n and l), to

$$G_n=\langle\text{neutrons}\rangle_j, \tag{3.4}$$

$$G_p=\langle\text{protons}\rangle_j, \tag{3.5}$$

where, for example, $\langle\text{neutrons}\rangle_j$ signifies "the average number of j neutrons in the target ground state." Specifically, if the target wave function is given in the form $\sum_k A_k\phi_k$ and if n_k is the number of j neutrons in ϕ_k, then

$$\langle\text{neutrons}\rangle_j=\sum_k A_k{}^2 n_k.$$

Other such averages are defined in the same way.[8] The corresponding expressions for $\bar{G}_n$ and $\bar{G}_p$ are obtained by replacing particles by holes in Eqs. (3.4) and (3.5). Since the present study deals exclusively with (d,t) and (d,p) reactions in which a neutron is transferred, the subscript n on the strengths will henceforth be omitted.

Generally, it is easy to see that the additional sum over the two spin-orbit components j corresponding to given l can be performed immediately. The strength now embraces all observed transitions with given l (and n) and measures $\langle\ \ \rangle_l$ rather than $\langle\ \ \ \ \rangle_j$.

Evaluation of the Empirical Constants Λ and θ_0^2

From a purely phenomenological viewpoint, $\Lambda\theta_0^2$ may be regarded as a single-particle reduced width for (d,t) reactions. The overlap factor Λ can be isolated by comparing (d,t) and (p,d) transitions between the same states, the (p,d) reaction providing a value of θ_0^2. Since, however, distortion effects in (p,d) and (d,t) reactions are not the same, the dependence of $\Lambda\theta_0^2$ on its parameters may differ markedly from that of θ_0^2. Consequently Λ, when evaluated empirically, may depend on energy and l. Extreme caution must therefore be exercised both in carrying over an empirical value of Λ from one dynamical situation to another and in interpreting Λ in terms of the triton wave function.

The status of empirical determinations of Λ and θ_0^2 may be summarized as follows. Almost all the pertinent data concern nuclei with $A<45$. Both Λ and θ_0^2 appear to depend on the projectile energies, increasing steadily up to and above the Coulomb barrier.[4,9,10] Above this θ_0^2 seems to level out to the roughly constant values discussed in reference 4, but it is not clear that the same is true of Λ. A sizeable body of data indicates that

$$\theta_0^2(2p)\approx 2\theta_0^2(1f). \tag{3.6}$$

The only available information about Λ for $2p$ and $1f$ transitions is obtained by a direct comparison of the Fe^{54}(d,t) and Fe^{56}(d,t) cross sections measured in the present study with the corresponding (p,d) cross sections

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[5] A few misprints in reference 1 have been corrected in this table.

[6] It is also unnecessary that isotopic spin be a good quantum number for each of the individual levels.

[7] J. B. French and M. H. Macfarlane, Nuclear Phys. **26**, 168 (1961).

[8] For example, a possible (neutron) wave function for Ni^{60} is

$$\alpha(p_{\frac{3}{2}}{}^4)_0+\beta(p_{\frac{3}{2}}{}^2)_0(f_{\frac{5}{2}}{}^2)_0+\gamma[(p_{\frac{3}{2}}{}^2)_2(f_{\frac{5}{2}}{}^2)_2]_0+\delta(f_{\frac{5}{2}}{}^4)_0.$$

For this wave function,

$$\langle\text{neutrons}\rangle_{p_{\frac{3}{2}}}=4\alpha^2+2\beta^2+2\gamma^2,\quad \langle\text{neutrons}\rangle_{f_{\frac{5}{2}}}=2\beta^2+2\gamma^2+4\delta^2.$$

Usually we refrain from writing explicit target wave functions and talk directly in terms of average occupation numbers.

[9] D. A. Bromley, in *Proceedings of the International Conference on Nuclear Structure, Kingston* (University of Toronto Press, Toronto, 1960), pp. 272–305. See in particular p. 303.

[10] F. de S. Barros, P. D. Forsyth, A. A. Jaffe, and I. J. Taylor, Proc. Phys. Soc. (London) **77**, 853 (1961).

which have been measured by Goodman[11] at a proton energy of 22 MeV. It is found that

$$\Lambda(2p) \approx 180\ \mathrm{F}^{-1}, \tag{3.7}$$

$$\Lambda(1f) \approx 110\ \mathrm{F}^{-1}, \tag{3.8}$$

encouragingly close to the value of 190 ± 40 obtained[4] from analysis of $l=0$ transitions and $l=2$ transitions in light nuclei ($A \leqslant 26$).

In view of the uncertainties in both factors, it is probably unwise to separate Λ and $\theta_0{}^2$ in discussing the (d,t) data. Accordingly, the analysis in Sec. 4 deals directly with the "(d,t) single-particle widths" $\Lambda\theta_0{}^2$.

Role of Isotopic Spin

Isotopic spin should be a rather good quantum number for the ground states of fp-shell nuclei.[12] Nevertheless the present study employs a formalism in which neutrons and protons are treated separately, and in such a formalism the wave functions do not necessarily have definite isotopic spin T. Linear combinations of n-p states can always, of course, be chosen such that they will conserve isotopic spin. Such questions[13] are of no concern here, firstly, because the strength equations [(3.4) and its analog for stripping] are valid quite generally, whether or not the target wave function has good T; and secondly, because no attempt is made to write explicit wave functions, the discussion being in terms of average occupation numbers instead. No information could be obtained in the present study about a possible T splitting of the "single-hole giant resonances,"[7] mainly because the higher-T components should be very weak (~10% of the total strength). We shall make no further mention of isotopic spin.

4. (d,t) REACTIONS ON Fe^{54}, Fe^{58}, AND THE Ni ISOTOPES: DISCUSSION

This section discusses only the data on (d,t) reactions; other experimental information will be considered in Sec. 5. No attempt is made to separate $2p_{3/2}$ from $2p_{1/2}$ in considering $2p$ contsibutions.

$Fe^{54}(d,t)Fe^{53}$

The only strong triton group observed corresponds to an $l=3$ transition to the Fe^{53} ground state. The total $l=3$ strength of $\sum \Lambda\theta^2(1f) \approx 3$ observed in Fe^{54} is much smaller than the strengths of 11.6 found in V^{51} and 6.2 in Cr^{52}. This problem is discussed in Sec. 5f where it is concluded that the most probable explanation lies in the existence of strong $l=3$ groups which have as yet escaped detection and which probably extend above the region of excitation covered in the present study.

[11] C. D. Goodman (private communication).

[12] W. P. Alford and J. B. French, Phys. Rev. Letters **6**, 119 (1961).

[13] J. B. French, Nuclear Phys. **26**, 161 (1961).

The fact that no $l=1$ transitions are observed[14] confirms the expectation that the $f_{7/2}$ neutron shell should be hard to excite.

$Fe^{58}(d,t)Fe^{57}$

Two strong triton groups are observed at excitation energies around 2.2 and 4.7 MeV. The lower group has $l=3$ and so also may the upper. The fact that no $l=3$ transitions of significant strength are seen[15] to levels at these excitations in $Fe^{56}(d,p)Fe^{57}$, shows that these groups result from $f_{7/2}$ pickup. The total strength of these transitions seems to be considerably less than for the $f_{7/2}$ found in V^{51}. In view of the uncertainty concerning the 4.7-MeV group, it is not clear whether or not there is a serious discrepancy.

Since only an upper limit has been set on possible $f_{5/2}$ contributions in $Fe^{58}(d,t)$ (see Table I), an absolute value of $\Lambda\theta_0{}^2(2p)$ is needed in order to interpret the observed $l=1$ strength in terms of occupation numbers. If $\Lambda\theta_0{}^2(2p)=3.1$ as was deduced from the (d,t) results on Ni^{58} and Ni^{60}, the observed total strength $\sum \Lambda\theta^2(l=1) \approx 12.5$ implies

$$\langle \text{neutrons} \rangle_{2p} \approx 4 \quad (\text{in } Fe^{58}). \tag{4.1}$$

It will be remembered (Sec. 2) that the Fe^{58} reduced widths are subject to larger errors than those for the other targets in the present study. Accordingly, Eq. (4.1) should not be taken too literally. The most that can be said is that $2p$ is strongly favored over $1f_{5/2}$ in the ground-state (neutron) wave function of Fe^{58}.

$Ni^{58}(d,t)Ni^{57}$, $Ni^{60}(d,t)Ni^{59}$, and $Ni^{61}(d,t)Ni^{60}$

Strong $l=3$ groups of similar strength appear at around 2.5 MeV ($Q \approx -8$ MeV) in Ni^{57} and Ni^{59}. Since the relevant levels in Ni^{59} are known[16] to show no $l=3$ stripping from Ni^{58}, we conclude that these groups result from $f_{7/2}$ pickup. The same is probably true of a weak $l=3$ group at 1.8 MeV in Ni^{59}; but the inclusion of the contribution from this state in the total $f_{5/2}$ strength would have little effect on the conclusions. We therefore proceed on the assumption that the only observed transitions involving $1f_{5/2}$ pickup from Ni^{58} or Ni^{60} are those to the 0.85-MeV group in Ni^{57} and to the 0.34-MeV state in Ni^{59}.

The relation[17]

$$\langle \text{neutrons} \rangle_{2p} + \langle \text{neutrons} \rangle_{1f_{5/2}} \approx 2 \quad (\text{in } Ni^{58}) \tag{4.2}$$

[14] There may be a very weak peak in the $Fe^{54}(d,t)$ energy spectrum at an excitation of about 600 keV. The group in question, however, is too weak to permit positive identification, let alone determination of its l value. If an $l=1$ group is, in fact, present, its strength relative to the ground-state group is less than 3%. This would imply very minute $2p$ admixture in the ground state of Fe^{54}.

[15] G. Parry (private communication).

[16] A. W. Dalton, G. Parry, H. D. Scott, and S. Swiersczewski, Proc. Phys. Soc. (London) **77**, 682 (1961).

[17] Excitation of the $f_{7/2}$ core and the contributions of more highly excited orbits such as $g_{9/2}$ are neglected here.

and a knowledge of the absolute value of either $\Lambda\theta_0^2(1f)$ or $\Lambda\theta_0^2(2p)$ will enable one to use Eq. (3.4) and the observed $2p$ and $1f_{5/2}$ strengths (Table I) to solve for the average occupation numbers in Ni^{58} and for the remaining single-particle width $\Lambda\theta_0^2$. To this end, it is assumed that the total $l=3$ strength observed in $V^{51}(d,t)V^{50}$ is approximately that corresponding to a filled $f_{7/2}$ shell of neutrons. Then Eq. (5.8) yields

$$\Lambda\theta_0^2(1f)\approx 1.4. \tag{4.3}$$

This value of $\Lambda\theta_0^2(1f)$ leads at once to the occupation numbers

$$\begin{aligned}\langle\text{neutrons}\rangle_{2p}&\approx 1.7,\\ \langle\text{neutrons}\rangle_{1f_{5/2}}&\approx 0.3,\end{aligned} \tag{4.4}$$

in Ni^{58}. Furthermore

$$\Lambda\theta_0^2(2p)\approx 3.1. \tag{4.5}$$

We shall use the values (4.3) and (4.5) of the single-particle reduced widths in all subsequent analysis (Sec. 5) of (d,t)-reaction data on fp-shell nuclei. In particular, from the observed strengths in Ni^{60}, it follows that

$$\left.\begin{aligned}\langle\text{neutrons}\rangle_{2p}&\approx 3.1,\\ \langle\text{neutrons}\rangle_{1f}&\approx 0.9\end{aligned}\right. \quad (\text{in } Ni^{60}). \tag{4.6}$$

The fact that these occupation numbers satisfy

$$\langle\text{neutrons}\rangle_{2p}+\langle\text{neutrons}\rangle_{1f}\approx 4,$$

as expected, constitutes an important check on the reliability of our procedure.

In Ni^{61} the occupation number is

$$\langle\text{neutrons}\rangle_{2p}\approx 5. \tag{4.7}$$

It is not known whether or not the $l=3$ transition to a group of levels near 2.5 MeV in Ni^{60} involves $f_{7/2}$ or $f_{5/2}$ pickup. It is, however, clear that $2p$ is strongly favored over $1f_{5/2}$ in the ground state of Ni^{61}.

5. STUDIES OF fp-SHELL NUCLEI BY MEANS OF (d,t) AND (d,p) REACTIONS

We now review what has been learned about the structure of fp-shell nuclei in the present series of (d,t)-reaction studies. We also examine the consistency of our conclusions with the results of (d,p)-reaction studies in this mass region and compare them with the predictions of the pairing model of Kisslinger and Sorensen.[18]

a. Occupation Numbers and Single-Particle Reduced Widths

Values of $\Lambda\theta_0^2(1f)$ and $\Lambda\theta_0^2(2p)$ were obtained [Eqs. (4.3) and (4.5)] in Sec. 4 from the analysis of (d,t) reactions on the Ni isotopes. These single-particle widths were then used to obtain average occupation numbers from the $f_{5/2}$ and $2p$ strengths observed in all the (d,t) experiments of the present series. The results, covering fp-shell nuclei from V to Zn, are given in Table II. The very reasonable magnitudes of these occupation numbers and of their variation with neutron excess lend support to the reliability of our analysis. The only serious difficulty concerns the small occupation numbers found for the $2p$ and $1f_{5/2}$ neutron orbits in Zn^{67}—values only half as large as would be expected for a nucleus with 9 neutrons outside closed shells. It would clearly be desirable to re-examine the reaction $Zn^{67}(d,t)Zn^{66}$, especially by extending the measurements to higher energies and improving the resolution.

TABLE II. Occupation numbers of $2p$ and $1f_{5/2}$ orbits, deduced from data on (d,t) reactions. In this table, ($<$) indicates an upper limit on possible $l=3$ contributions to a predominantly $l=1$ group; and ($\leqslant$) indicates uncertainty as to whether the $l=3$ group in question is $1f_{7/2}$ or $1f_{5/2}$. In deriving the occupation numbers from the reduced widths, we used $\Lambda\theta_0^2(1f)=1.4$, $\Lambda\theta_0^2(2p)=3.1$, as discussed in Sec. 4 of the text.

Target	$\Sigma\,\Lambda\theta^2(2p)$	$\Sigma\,\Lambda\theta^2(1f_{5/2})$	$\langle\text{neutrons}\rangle_{2p}$	$\langle\text{neutrons}\rangle_{1f_{5/2}}$
V^{51}, Cr^{52}, Fe^{54}	0	0	0	0
$_{24}Mn_{30}^{55}$	7.7	0	2.5	0
$_{26}Fe_{30}^{56}$	5.9	≈ 0	1.9	≈ 0
$_{28}Ni_{30}^{58}$	5.0	0.5	1.7	0.3
$_{26}Fe_{31}^{57}$	6.6	1.4	2.1	1.0
$_{26}Fe_{32}^{58}$	12.5	(<2.5)	4.0	(<1.6)
$_{27}Co_{32}^{59}$	12	(<4)	4	(<3)
$_{28}Ni_{32}^{60}$	9.7	1.2	3.1	0.9
$_{28}Ni_{33}^{61}$	15	$(\leqslant 1.7)$	5	$(\leqslant 1)$
$_{29}Cu_{34}^{63}$	12	(<5)	4	(<3.5)
$_{30}Zn_{34}^{64}$	11	(<4)	3.5	(<2.5)
$_{29}Cu_{36}^{65}$	16	(<6)	5	(<4)
$_{30}Zn_{36}^{66}$	17	(<6)	5.5	(<4)
$_{30}Zn_{37}^{67}$	6.3	4.2	2	3
$_{30}Zn_{38}^{68}$	14	7.2	4.5	5.1

Values of Λ for $l=1$ and $l=3$ transitions were obtained in Sec. 3 by direct comparison of (d,t) and (p,d) cross sections of Fe^{54} and Fe^{56}. Combining these estimates of Λ [Eqs. (3.7) and (3.8)] with the values of $\Lambda\theta_0^2$ given in Eqs. (4.3) and (4.7) leads to

$$\theta_0^2(1f)\approx 0.013, \tag{5.1}$$

$$\theta_0^2(2p)\approx 0.017. \tag{5.2}$$

These are sufficiently close to the values found[4] in studies of (d,p) reactions on Ca and on lighter nuclei that they provide a further check on the consistency of our interpretation.

b. The Pairing Model and the Isotopes of Ni

For "single-closed-shell" nuclei (in which either neutrons or protons are in closed-shell configurations), the strengths (3.2) and (3.3) can be evaluated directly in terms of the parameters of the pairing model of Kisslinger and Sorensen.[18] In fact, if V_j is the probability amplitude for occupation of the pair state $[jm, j-m]$

[18] L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 9 (1960).

in the ground state of the target nucleus, if follows that

$$G_j=\langle\text{neutrons}\rangle_j=(2j+1)V_j^2 \quad (5.3)$$

for pickup of a j neutron, and

$$\bar{G}_j=\langle\text{neutron holes}\rangle_j=(2j+1)(1-V_j^2) \quad (5.4)$$

for stripping. Substitution of the appropriate values of V_j from the paper of Kisslinger and Sorensen yields

$$\langle\text{neutrons}\rangle_{2p}=1.4,\quad \langle\text{neutrons}\rangle_{1f_{\frac{5}{2}}}=0.6 \quad (5.5)$$

in Ni^{58}, and

$$\langle\text{neutrons}\rangle_{2p}=2.5,\quad \langle\text{neutrons}\rangle_{1f_{\frac{5}{2}}}=1.4 \quad (5.6)$$

in Ni^{60}. These occupation numbers are in fair agreement with the estimates (4.4) and (4.6) made on the basis of our interpretation of the mickup data. On the other hand, the pairing-model prediction that the occupation numbers in the $2p$ and $1f$ orbits are in the ratio of 3:2 in Ni^{61} is in clear disagreement with the pickup data [Eq. (4.7)].

Thus while the pairing model seems to give a reasonable general picture of the occupation numbers in Ni, it cannot follow the sizable ffuctuations in detail from nucleus to nucleus. That the pairing model can be expected to give only an over-all picture and not a detailed description of the properties of low-lying levels of the Ni isotopes is already clear from the presence of a pronounced $l=1$ transition in the proton pickup reactions[19,20] $Ni^{58}(n,d)$ and $Ni^{58}(d,He^3)$. The implication is that core excitation may be important even for the lowest states of the Ni isotopes. The treatment of such excitations demands consideration of neutron-proton pairing forces.

Studies of neutron occupation numbers in the isotopes of Sn by means of (d,p) and (d,t) reactions have been carried out by Cohen and Price.[21] Their treatment of the experimental data and their technique of theoretical analysis are very similar to ours in spite of the very different notations. We have preferred not to use the language of the pairing model (as Cohen and Price do) since it is clear that the use of stripping and pickup reactions to measure average occupation numbers is of wider applicability than the simple pairing model.

c. Comparison of (d,p) and (d,t) Data on the Ni Isotopes

The (d,t) reactions on a given target measure the average number of neutrons in each orbit, while the (d,p) reactions measure the number of neutron holes. The information obtained in the two cases is clearly the same. The next question is whether the (d,t) occupation numbers are consistent with the results of (d,p) experiments[16] on Ni^{58} and Ni^{60}. The values of $\bar{G}\theta_0^2(2p)$ [Eq. (3.3)] are found to be 0.073 for $Ni^{58}(d,p)$ and 0.051 for $Ni^{60}(d,p)$. The ratio of these strengths measures the relative number of p-neutron holes in the ground states of the corresponding target nuclei. In fact, the ratio is $\bar{G}(Ni^{58})/\bar{G}(Ni^{60})=[6-\langle\text{neutrons}\rangle_{2p}]Ni^{58}/$

$$[6-\langle\text{neutrons}\rangle_{2p}]Ni^{60} \\ =0.073/0.051\approx 1.4, \quad (5.7)$$

in excellent agreement with the value 1.5 obtained for this ratio by use of the neutron occupation numbers [Eqs. (4.4) and (4.6)].

d. Detection of $l=3$ and $l=4$ Contributions

One of the main difficulties in the present series of experiments has been the fact that sizable $l=3$ contributions are masked by dominant $l=1$ groups. The same is true of $l=4$ transitions, which may be present in pickup from the isotopes of Zn. A possible way of detecting such high-l components (without performing experiments with very high energy resolution) is suggested by recent studies of (He^3,α) and (α,He^3) reactions.[22,23] The point here is that stripping and pickup involving α particles seem to be more favorable to higher values of l than are processes such as (d,p) and (d,t) involving deuterons. This possibility should receive further experimental investigation.

e. Spin-Orbit Splitting of the $2p$ Contributions

We have made no attempt to separate $l=1$ strengths into $2p_{1/2}$ and $2p_{3/2}$ components. For pickup reactions, there is only one item of pertinent information: the spins involved demand that the $Fe^{57}(d,t)Fe^{56}$ ground-state $l=1$ transition should proceed by $p_{1/2}$ transfer. For stripping, the appearance of two $l=1$ "resonances"[24] in (d,p) reactions on many fp-shell nuclei invites an interpretation in terms of a spin-orbit splitting of the $2p$ resonance; the upper "resonance" would correspond to $2p_{1/2}$. There is, however, very little direct experimental evidence on this point and, in fact, serious difficulties have been encountered with the idea of a simple spin-orbit splitting. In Ti^{49}, recent measurements[25,26] of $Ti^{48}(n,\gamma\gamma)$ have been taken to indicate a spin of $\frac{1}{2}^-$ for a state around 1.7 MeV, which is known[27] to contribute about one-third of the total strength of the lower $l=1$

[19] R. N. Glover and K. H. Purser, Nuclear Phys. **24**, 431 (1961).

[20] J. L. Yntema, T. H. Braid, B. Zeidman, and H. W. Broek, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company, Ltd., London, 1961), p. 521.

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resonance in Ti^{48} (d,p). A similar situation has been encountered in Cr^{53}, where a spin of $\frac{1}{2}^-$ has been tentatively assigned[28] to the first-excited state. However, more recent information[29] on Ti^{49} suggests that the 1.7-MeV "level" is at least a doublet, while the spin assignment in Cr^{53} is far from certain. The experimental situation is thus unclear.

Since (d,p) reactions with deuterons having energies of 10 MeV or less on fp-shell nuclei very often produce angular distributions that peak at positions half way between those characteristic (in Butler theory) of adjacent l values, it has been suggested (p. 676 of reference 4) that the proposed "$2p_{1/2}$ resonance" actually involves $l=2$ capture. It now seems that this interpretation is incorrect. In the first place, distorted-wave calculations[30] for (d,p) reaction on Ni^{58} and Ni^{60} at a deuteron energy of 9 MeV decide the ambiguities in l assignment quite clearly in favor of the lower l value (in our case $l=1$); secondly, the (negative) parity of many of the component levels of the proposed $2p_{1/2}$ resonances is established by the fact that they are excited by thermal-neutron capture γ rays.[31] All this, of course, has an indirect bearing on the question of the separation of the $2p$ resonance into spin-orbit components. By ruling out the only plausible alternative interpretation of the two well-separated resonances in question, the above arguments establish the fact that the $2p$ resonance splits into two parts. It is then hard to imagine any other interpretation of this splitting than that which assigns the upper "resonance" as $2p_{1/2}$, the lower as $2p_{3/2}$. Clear and direct experimental evidence is, however, sadly lacking. And until such evidence has been obtained the possibility remains that the usual interpretation of giant resonances for particles, wherein each resonance is taken to correspond to a single (jj) shell-model orbit, is seriously in error.

f. The $1f_{7/2}$ Single-Hole Resonance

For any nucleus with a complete $f_{7/2}$ neutron shell, Eq. (3.4) yields

$$G(f_{7/2})=(2j+1)_{j=7/2}=8. \tag{5.8}$$

Thus $\sum \Lambda\theta^2(l=3)$ should have about equal values in the (d,t) reactions on V^{51}, Cr^{52}, and Fe^{54}. The observed strengths (Table I) are in fact 11, 6, and 3, respectively. The most likely explanation of this discrepancy is that large $f_{7/2}$ contributions have escaped detection in the (d,t) reactions on Cr^{52} and Fe^{54}. This could happen in two ways—through a distribution of small $f_{7/2}$ contributions over many levels of the residual nucleus, or through the existence of sizable $f_{7/2}$ strengths above the upper limit of excitation ($Q\approx-9$ MeV) covered in our experiments. Such an explanation of the small $f_{7/2}$ strength would imply a spreading of the $f^{7/2}$ single-hole resonance over an energy range of 2 MeV or more in Fe^{54}. A spread of about 3 MeV is observed in $V^{51}(d,t)$ and single-hole widths of this order of magnitude are found in ($p,2p$) studies in light nuclei.[32] Further experimental studies of the reaction $Fe^{54}(d,t)Fe^{53}$ are planned to clarify this point. {*Note added in proof*. Recent studies of (d,t) reactions on Fe^{54} and Cr^{52} confirm the presence of strong $l=3$ contributions over an energy range of 4 MeV [B. Zeidman and T. H. Braid, Bull. Am. Phys. Soc. **7**, 315 (1962)].}

g. Alternative Ways of Analyzing the (d,t) Data

Our treatment of the data rests on two main assumptions: (1) The value of $\sum \Lambda\theta^2(l=3)$ observed in $V^{51}(d,t)$ is the full $f_{7/2}$ single-hole strength. (2) Variations of the single-particle reduced widths with Q value may be neglected for the transitions under consideration. Both assumptions are, of course, open to question. However, the various consistency checks, described at the end of Sec. 4 and in Sec. 5a, give us considerable assurance that the errors involved are not serious.

With these assumptions, our procedure has been to determine $\Lambda\theta_0^2(1f)$ from $V^{51}(d,t)$ and to use the single-particle width so obtained with the (d,t) data on Ni^{58} to determine the empirical constants $\Lambda\theta_0^2(1f)$ and $\Lambda\theta_0^2(2p)$ completely. Instead, we might have supplemented the Ni^{58} data by determining the ratio $\Lambda\theta_0^2(2p)/\Lambda\theta_0^2(1f)$ from independent (d,p), (p,d) and (d,t) data on neighboring nuclei. This procedure, which has the advantage of being independent of assumptions about the $f_{7/2}$ single-hole strength in V^{51}, was in fact used in an earlier analysis[33] of the data on Fe and Ni. Since, of course, the various relevant items of information do not fix $\Lambda\theta_0^2(2p)/\Lambda\theta_0^2(1f)$ precisely, ranges of "allowed" values are obtained for the occupation numbers and single-particle widths. The fact that the results of the present study lie within these "allowed" ranges may be regarded as independent verification that the observed value of $\sum \Lambda\theta^2(l=3)$ in $V^{51}(d,t)$ is indeed close to the full $f_{7/2}$ single-hole strength.

6. CONCLUSIONS

With the help of simple and general sum rules, (d,p) and (d,t) reactions have been used to measure the average occupation numbers of various (neutron) orbits in the ground states of fp-shell nuclei. The main uncertainties in our analysis stem from incomplete knowledge of the empirical constants involved in extracting

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[33] M. H. Macfarlane, B. J. Raz, J. L. Yntema, and B. Zeidman, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company, Ltd., London, 1961), p. 511.

absolute reduced widths, and from the masking of $l=3$ transitions by dominant $l=1$ groups. The first difficulty could be overcome either by experimental study of the essential empirical constants or, perhaps better, by extracting reduced widths with the help of distorted-wave calculations. The detection of higher l values could be improved by using higher energy resolution or, perhaps, by performing stripping and pickup reactions involving α particles. One problem of interest for future study concerns the shape of energy spectra in pickup reactions up to high excitations (at least 15 MeV) in the residual nucleus.

ACKNOWLEDGMENTS

We wish to acknowledge the cooperation of W. Ramler and the cyclotron group and the assistance of W. J. O'Neill and S. Piascek in obtaining and analyzing data. Target foils were rolled by F. Karasek of the Argonne Metallurgy Division. We also wish to extend our thanks to A. G. Blair, C. D. Goodman, G. Parry, and D. M. van Patter for sending us their experimental results before publication, and to G. A. Bartholomew, J. B. French, R. D. Lawson, L. L. Lee, P. A. Moldauer, D. M. van Patter, and J. P. Schiffer for helpful discussions.

PHYSICAL REVIEW VOLUME 127, NUMBER 1 JULY 1, 1962

Kinetic Energy Distributions of Fragments from the Fission of Au, Tl, Pb, and Bi*†

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(Received February 19, 1962)

Single fragment and total kinetic energy distributions have been measured for fission induced by helium ions and deuterons in Au, Tl, Pb, and Bi targets. The average total kinetic energy release has the same proportionality with $Z^2/A^{\frac{1}{3}}$ found for fission of heavier elements. The widths for the total kinetic energy distributions are comparable to those observed for fission of heavy elements. A preliminary mass distribution curve has been obtained by measuring the energies of coincident fragments.

I. INTRODUCTION

A FEW years ago Fairhall[1] showed that fission of bismuth induced by 20-MeV deuterons results in a mass distribution which is symmetric and rather narrow. The width of this mass distribution was comparable to the widths of the individual light and heavy mass groups resulting from low-energy fission of heavier elements. This predominance of symmetric fission has been found to be characteristic of fission induced in low atomic number targets by protons, deuterons or helium ions.[2,3] Later investigations[4–6] of the excitation functions and angular distributions have provided information about fission thresholds and saddle-point shapes for these systems. The present work is concerned with the kinetic energy distributions, which yield information about the nucleus at scission.

* Based on work performed under the auspices of the U. S. Atomic Energy Commission.
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II. SINGLE FRAGMENT KINETIC ENERGY MEASUREMENTS

The energy distributions of the fragments were observed using surface barrier solid-state detectors having a sensitive area of about 0.3 cm^2. Two detectors were used simultaneously, allowing two independent determinations at the same time. Unfortunately one of the two detectors proved to be unstable in many of the runs, so that final results are often reported for only one of the two detectors. The detectors were located approximately 7 cm from the target and at $\pm 150°$ with respect to the beam direction. The choice of a backward angle of observation was made for purposes of reducing pileup pulses from projectile particles scattered by the target into the detectors. The targets were prepared by vacuum volatilization of 300–600 μg/cm^2 of the metal (natural isotopic composition) onto 0.001-cm Al backing foils. The energy calibration of the detectors was performed with a Cf^{252} spontaneous fission source. As there was an appreciable noise level from the scattered projectile particles even in the backward direction, the calibration was performed with a "beam on" technique. The Cf^{252} source was located on the back of the target holder in such a way that it faced in the opposite direction to that of the target material and was not in the path of the beam. By rotating the target 180° from the normal position, the Cf^{252} source was in a position facing the detectors. The cyclotron beam could still pass through