

TABLE II. Expected resonance absorptions, for absorbers of one-half value thickness for electronic attenuation, for all the transitions studied so far with the centrifuge method.

Isotope	Abundance (%)	$E_\gamma$ (keV)	$\Gamma_0$ (eV)	Resonance absorption (%)
As <sup>75</sup>	100	265	$4.0 \times 10^{-6}$ <sup>a</sup>	17.1
Pr <sup>141</sup>	100	145	$2.3 \times 10^{-7}$ <sup>b</sup>	0.07
Gd <sup>155</sup>	15.1	105	$1.1 \times 10^{-6}$ <sup>c</sup>	0.05
Lu <sup>175</sup>	97.4	343	$1.4 \times 10^{-6}$ <sup>d</sup>	0.07
Hg <sup>198</sup>	10.0	411	$2.1 \times 10^{-6}$ <sup>e</sup>	0.45
Hg <sup>199</sup>	16.8	208	$3.9 \times 10^{-6}$ <sup>f</sup>	0.11
Tl <sup>203</sup>	29.5	279	$1.3 \times 10^{-6}$ <sup>g</sup>	0.05

<sup>a</sup> This paper.

<sup>b</sup> F. R. Metzger, J. Franklin Inst. **261**, 219 (1956).

<sup>c</sup> B. I. Deutch, F. R. Metzger, and F. J. Wilhelm, Nuclear Phys. **16**, 81 (1960).

<sup>d</sup> B. I. Deutch, Nuclear Phys. **30**, 191 (1962).

<sup>e</sup> W. G. Davey and P. B. Moon, Proc. Phys. Soc. (London) **A66**, 956, (1953).

<sup>f</sup> V. Knapp, Proc. Phys. Soc. (London) **A70**, 142 (1957).

<sup>g</sup> B. I. Deutch and F. R. Metzger, Phys. Rev. **122**, 848 (1961).

out the importance of carrying out self-absorption studies whenever they are feasible and an uncertainty concerning the shape of the incident line exists.

As far as the combination of transmission studies with the centrifuge technique is concerned, the As<sup>75</sup> experiment confirmed the expectation that, for large absorptions, the transmission study is a very efficient

way of measuring cross sections. All the data reported in this paper, for instance, was accumulated in less than 10 hours of centrifuge operation. Unfortunately, most absorption effects encountered in typical centrifuge experiments are rather small. This fact is illustrated in Table II, where all the isotopes studied so far with the centrifuge method are listed, and where the resonance absorption effects, expected for absorbers of one-half value thickness for electronic attenuation, are given. It becomes clear from this tabulation that As<sup>75</sup> is indeed a very favorable case, and that transmission studies with any of the other isotopes would be, to say the least, very tedious.

As improved rotors become available, studies with lower  $Z$  nuclei will become more feasible, and the opportunity for transmission studies will grow. Higher rotor speeds are often achieved at a sacrifice of space available for the source. Since the activities required for transmission experiments are approximately one order of magnitude smaller than those necessary for scattering studies, the trend towards higher rotor speeds will favor once more the transmission-type experiment.

A transmission study is also indicated whenever  $\Gamma_0/\Gamma$  is small, i.e., for large branching (internal conversion), since the absorption is proportional to  $\Gamma_0$  while the scattering is proportional to  $\Gamma_0^2/\Gamma$ .

## Study of the $\text{Al}^{27}(\alpha, \text{Be}^7)\text{Na}^{24}$ Reaction from Threshold to 41 MeV\*

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Excitation functions for the formation of  $\text{Na}^{24}$ ,  $\text{Be}^7$ , and  $\text{Na}^{22}$  in the interaction of  $\text{Al}^{27}$  with alpha particles have been measured from threshold to 41 MeV. The cross sections at 41.2 MeV are 0.33, 0.14, and 8.0 mb, respectively, and the excitation functions exhibit a sharp rise with bombarding energy. The recoil properties of the above nuclides have been investigated by means of thick-target integral range measurements at 40 MeV. The results for  $\text{Be}^7$  are consistent with approximately equal contributions from direct interaction and evaporation processes while the results for  $\text{Na}^{24}$  and  $\text{Na}^{22}$  indicate that these nuclides are primarily formed through processes involving compound-nucleus formation. It is concluded that at 40 MeV approximately 60% of the yield of  $\text{Na}^{24}$  may be attributed to the  $(\alpha, \alpha\text{He}^3)$  reaction.

### I. INTRODUCTION

THE formation of  $\text{Be}^7$  in medium-energy nuclear reactions was first studied by Bouchard and Fairhall.<sup>1</sup> These authors detected  $\text{Be}^7$  as a product of the reactions of oxygen, aluminum, and copper with 33–42 MeV alpha particles. The formation of  $\text{Be}^7$  in the bombardment of aluminum with alpha particles is of particular interest since  $\text{Na}^{24}$ , the complementary

product in the reaction, may also be detected by radiochemical techniques. As a result, a detailed study of this reaction is possible. Bouchard and Fairhall<sup>1</sup> thus reported approximately equal cross sections for the formation of  $\text{Be}^7$  and  $\text{Na}^{24}$  from Al with 40-MeV alpha particles. These workers also studied the emission of  $\text{Be}^7$  recoils in the forward and backward directions and on the basis of the absence of backward emission concluded that the  $\text{Al}^{27}(\alpha, \text{Be}^7)\text{Na}^{24}$  reaction proceeds by a pickup mechanism.

More recently, Lindsay and Carr<sup>2</sup> also investigated

\* Research performed under the auspices of the U. S. Atomic Energy Commission.

<sup>1</sup> G. H. Bouchard, Jr. and A. W. Fairhall, Phys. Rev. **116**, 160 (1959).

<sup>2</sup> R. H. Lindsay and R. J. Carr, Phys. Rev. **120**, 2168 (1960).

the same system. On the basis of their excitation-function measurements they conclude that above 37 MeV the yield of  $Na^{24}$  is significantly higher than that of  $Be^7$ . This difference is attributed to the  $Al^{27}(\alpha, \alpha He^3)Na^{24}$  reaction and it is concluded that this reaction is responsible for about 30% of the yield of  $Na^{24}$  at 40 MeV. These authors also studied the recoil properties of  $Be^7$  formed in the bombardment of thin magnesium targets with alpha particles. On the basis of the observed forward-backward emission ratio they conclude that  $Be^7$  is produced in a process involving compound-nucleus formation. Furthermore, these workers feel that the recoil studies of Bouchard and Fairhall<sup>1</sup> were carried out with too thick a target and they conclude that the previous recoil measurements on the  $Be^7$ -Al system<sup>1</sup> are consistent with compound-nucleus formation.

The present study, which has been previously reported on,<sup>3</sup> was carried out in order to obtain detailed information on the formation of  $Be^7$  and  $Na^{24}$  in the reaction of aluminum with medium-energy alpha particles. In the course of the investigation results were also obtained for  $Na^{22}$ , formed by the  $(\alpha, 2\alpha n)$  reaction on aluminum. The excitation functions for the formation of these products have been measured from threshold to 41 MeV. The results of these measurements are presented in Sec. II. The recoil properties of these three nuclides have been investigated at 40 MeV by means of integral forward-backward range measurements using thick targets. Contrary to the view expressed by Lindsay and Carr,<sup>2</sup> recoil experiments with thick targets can yield useful information about the reaction mechanism. It is necessary, however, to analyze the results with the aid of expressions that take the variation of cross section with bombarding energy into account. The results and analysis of the recoil experiments are presented in Sec. III. It is found that the results and conclusions of the present investigation differ in a number of respects from the earlier findings of both Bouchard and Fairhall<sup>1</sup> and Lindsay and Carr.<sup>2</sup>

## II. EXCITATION-FUNCTION MEASUREMENTS

### A. Experimental

The excitation functions were measured by the stacked foil technique with the deflected helium ion beam of the Brookhaven 60-in. cyclotron. The existing target assembly<sup>4</sup> was modified in order to permit irradiations to be carried out in vacuum. The assembly was attached to the cyclotron face plate without the intervening Dural window and the cyclotron vacuum was allowed to extend to the target foils. This modification was necessary because it was found that the presence of air in the beam path led to spuriously high  $Be^7$  cross sections. The beam intensity was determined by current integration as before<sup>4</sup> and the energy of the incident

helium ions was determined by the copper ratio method<sup>5</sup> and by range measurements. A range-energy relation based on the range-energy relation for protons of Bichsel *et al.*<sup>6</sup> was used to determine the bombarding energy for each foil in the stack. The target foils consisted of 0.001-in. thick high-purity (99.99%) aluminum. The target stack, consisting of 15 such foils, was bombarded for a period of several hours. Two irradiations at the same incident energy were performed.

In view of the steeply rising excitation functions reported<sup>1,2,7,8</sup> for the reactions in question the cross sections for all three nuclides were determined in the same bombardment. This precluded the occurrence of errors in the relative cross sections due to small shifts in the energy of the incident beam. The counting and chemical procedures followed were as follows. First, the activity of  $Na^{24}$  in the target foils was determined by measurement of the 2.75-MeV  $\gamma$  ray with a 3-inch NaI crystal connected to a 100-channel pulse-height analyzer. The efficiency of the detector was determined with a  $Na^{24}$  source whose disintegration rate was known on the basis of a  $\beta$ - $\gamma$  coincidence calibration. After  $Na^{24}$  had decayed, the activity of  $Na^{22}$  in the target foils was determined by 0.51–0.51 MeV  $\gamma$ -ray coincidence measurements as well as by determination of the 1.28-MeV  $\gamma$ -ray activity with the above mentioned scintillation spectrometer. The detectors were calibrated with a  $Na^{22}$  source whose disintegration rate was known on the basis of 511–511- $\gamma$  triple coincidence measurements. Finally, the target foils were dissolved and beryllium was separated radiochemically.<sup>2</sup> The  $Be^7$  activity was determined by measurement of the 0.477-MeV  $\gamma$  ray with a NaI well-counter connected to a 100-channel pulse-height analyzer. The detector was calibrated with the aid of a number of standard sources including  $Sr^{85}$  with its 0.513-MeV  $\gamma$  ray. The chemical yield was determined at the completion of the activity measurement by spectrophotometry.

The cross sections were obtained from the activity measurements on the basis of the known<sup>9</sup> intensities of the measured radiations. The measured excitation functions had to be corrected in all cases for the net transfer of activity from any one foil to the adjacent foil on the low-energy side resulting from the recoil behavior of the nuclides in question. The correction was applied on the basis of the measured recoil properties described in the following section. The cross sections for the formation of  $Na^{24}$  also had to be corrected for the contribution of the  $Al^{27}(n, \alpha)$  reaction. The contribution of this reaction was evident from the observed constancy of the cross section below 33 MeV. The  $Na^{24}$  activity observed at

<sup>5</sup> N. T. Porile and D. L. Morrison, *Phys. Rev.* **116**, 1193 (1959).

<sup>6</sup> H. Bichsel, R. F. Mozley, and W. A. Aron, *Phys. Rev.* **105**, 1788 (1957).

<sup>7</sup> R. H. Lindsay and R. J. Carr, *Phys. Rev.* **118**, 1293 (1960).

<sup>8</sup> M. Lindner and R. N. Osborne, *Phys. Rev.* **91**, 342 (1953).

<sup>9</sup> *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U.S. Government Printing Office, Washington, D. C., 1960).

<sup>3</sup> N. T. Porile, *Bull. Am. Phys. Soc.* **5**, 405 (1960).

<sup>4</sup> S. Amiel and N. T. Porile, *Rev. Sci. Instr.* **29**, 1112 (1958).

these energies was about a factor of 7 lower than the activity observed at 41 MeV. A correction was made by assuming a constant  $\text{Na}^{24}$  production by the  $(n, \alpha)$  reaction for all target foils.

### B. Results

The measured cross sections are listed in Table I. The uncertainties in the results, based on reproducibility and an estimate of systematic errors, are approximately 4% and 10% for  $\text{Na}^{22}$  and  $\text{Be}^7$ , respectively. The uncertainty in the  $\text{Na}^{24}$  cross section is approximately 10% at 41 MeV and increases to over a factor of 2 below 36 MeV. The listed average energies have not been corrected for the variation of cross section over the target thickness. This correction would increase the average bombarding energies by a fraction of a MeV. The excitation functions are shown in Fig. 1. It is seen that the cross sections for the formation of  $\text{Na}^{24}$  are larger than those for the formation of  $\text{Be}^7$ . As has been pointed out previously on the basis of threshold and barrier considerations,<sup>2</sup> the difference between these cross sections may be ascribed to the  $\text{Al}^{27}(\alpha, \alpha\text{He}^3)\text{Na}^{24}$  reaction. The cross section for this reaction, given by the dashed line in Fig. 1, is seen to be larger than the cross section for the  $\text{Al}^{27}(\alpha, \text{Be}^7)\text{Na}^{24}$  reaction above 36 MeV. At the highest bombarding energy this reaction, in fact, accounts for nearly 60% of the observed  $\text{Na}^{24}$  activity.

The cross section for the formation of  $\text{Na}^{22}$  is seen to be an order of magnitude larger than that for the formation of  $\text{Na}^{24}$ . This fact is probably related to the difference in thresholds for these reactions. The  $\text{Al}^{27}(\alpha, 2\alpha n)$  reaction has a threshold of 25.8 MeV, which is 1.4 MeV lower than the threshold for the  $\text{Al}^{27}(\alpha, \alpha\text{He}^3)$  reaction. A difference in threshold is not the only factor governing the magnitude of the cross sections as may be seen from the fact that the  $(\alpha, \alpha\text{He}^3)$  reaction has a higher cross section than the  $(\alpha, \text{Be}^7)$  reaction in spite of the fact that the threshold for the latter is 1.8 MeV lower. The detailed level structure of the residual nuclei undoubtedly is of importance in determining the magnitude of the cross sections in view of the light mass numbers under consideration. It is shown in the following

TABLE I. Cross sections for the formation of  $\text{Be}^7$ ,  $\text{Na}^{24}$ , and  $\text{Na}^{22}$ .

Average bombarding energy (MeV)	$\sigma(\text{Be}^7)$ (mb)	$\sigma(\text{Na}^{24})$ (mb)	$\sigma(\text{Na}^{22})$ (mb)
41.2	0.14	0.33	8.0
40.3	0.088	0.25	5.5
39.4	0.054	0.14	3.0
38.4	0.031	0.092	1.6
37.4	0.015	0.046	0.74
36.3	0.008	0.024	0.32
35.2	0.005	0.008	0.11
34.2	0.002	0.003	0.044
33.1	...	...	0.006

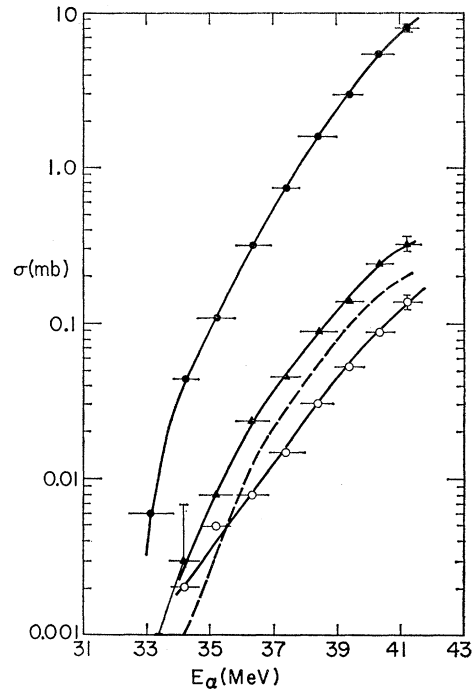


FIG. 1. Experimental excitation functions. ●— $\text{Na}^{22}$ ; ▲— $\text{Na}^{24}$ ; ○— $\text{Be}^7$ ; dashed line, difference between  $\text{Na}^{24}$  and  $\text{Be}^7$  cross sections, attributed to  $(\alpha, \alpha\text{He}^3)$  reaction.

section that at an incident energy of 40 MeV the emission of  $\text{Be}^7$  leaves the residual  $\text{Na}^{24}$  nucleus with an average excitation energy of only 3.4 MeV. Since the distribution of excited states is nonstatistical at low-excitation energies in light-mass nuclides, it is clear that a somewhat different average residual excitation energy following an  $(\alpha, \alpha\text{He}^3)$  reaction could have a substantial effect on the reaction cross section. In view of these considerations detailed evaporation calculations, which usually assume a statistical distribution of levels, have not been carried out.

The present results may be compared with previous measurements of these cross sections. The comparison for the highest energies is presented in Table II. The energies quoted by Lindsay and Carr<sup>2,7</sup> have been adjusted to take into account the fact that these authors used the older range-energy relation of Aron *et al.*<sup>10</sup> to obtain their beam energies. In the other studies no mention is made of how the beam energy was determined and the listed energies have been taken at face value. It is seen that the results for  $\text{Na}^{22}$  and  $\text{Na}^{24}$  are in fair agreement with the results of Bouchard and Fairhall.<sup>1</sup> The observed difference can be entirely accounted for on the assumption that the reported cyclotron energies differ from each other by 0.3 to 0.4 MeV. A difference of this magnitude is consistent with uncertainties in energy determination. The cross section

<sup>10</sup> W. A. Aron, B. G. Hoffman, and F. C. Williams, Atomic Energy Commission Report AECU-663, 1949 (unpublished).

for Be<sup>7</sup> reported by Bouchard and Fairhall<sup>1</sup> is, on the other hand, some 50% larger than the present value. This difference would require an energy shift of about 2 MeV and consequently the difference must probably be attributed to other causes. The results of the present work do not agree with the results of Lindsay and Carr.<sup>2,7</sup> The cross sections for all three nuclides are thus found to be lower by a factor of 2 to 3 than the values reported by these authors and the discrepancy persists at lower energies.

### III. RECOIL MEASUREMENTS

#### A. Experimental

The recoil properties of the product nuclei were investigated at 40 MeV. Measurements of the fraction of the total activity of a given nuclide found in the target foil, forward catcher, and backward catcher were performed. The target foils were in all cases thicker than the range of the recoils in the target material. In the case of Na<sup>22</sup> and Na<sup>24</sup> the catchers were nickel foils having a thickness of 2 mg/cm<sup>2</sup>. Silver foils having a thickness of 12 mg/cm<sup>2</sup> were used for the Be<sup>7</sup> experiments. Two additional nickel or silver foils were included in the target stack to serve as blanks. These foils were placed on either side of the recoil catchers and the activation correction was determined by interpolation. The correction was usually less than 15%. In the case of Na<sup>22</sup> and Na<sup>24</sup>, however, no backward recoil emission above the activation level was observed.

The recoil experiments were performed with the evacuated irradiation assembly described in the previous section. Bombardment times of 8 to 10 h were necessary to produce sufficient activity. Duplicate experiments were performed in all cases. In addition, the recoil properties of Na<sup>24</sup> produced by the Al<sup>27</sup>( $n,\alpha$ ) reaction were investigated with helium ions degraded to 28 MeV and the results at 40 MeV were corrected for the contribution of the ( $n,\alpha$ ) reaction. The previously described counting and chemical procedures were used in all cases. In addition, sodium was separated from nickel by a procedure consisting primarily of NaCl precipitations with alcoholic HCl and anion exchange purification.

TABLE II. Comparison of reported cross sections for formation of Be<sup>7</sup>, Na<sup>24</sup>, and Na<sup>22</sup> from Al at 39–42 MeV.

Nuclide and bombarding energy (MeV)	Bouchard and Fairhall <sup>a</sup> (mb)	Lindsay and Carr <sup>b</sup> (mb)	Lindner and Osborne <sup>c</sup> (mb)	This work (mb)
Be <sup>7</sup> 41.5	0.26			0.16
40.8		0.32		0.12
Na <sup>24</sup> 40	0.25		0.28	0.21
39.2		0.37		0.14
Na <sup>22</sup> 40	5.2			4.5
39.2		7		2.7

<sup>a</sup> See reference 1.

<sup>b</sup> See references 2 and 7.

<sup>c</sup> See reference 8.

TABLE III. Experimental data obtained from recoil studies at 40 MeV.

Nuclide	Target thickness (mg/cm <sup>2</sup> )	$F$	$B$	$F/B$
Be <sup>7</sup>	9.8	0.231±0.007	0.029±0.002	8.05±0.60
Na <sup>22</sup>	2.50	0.292±0.010	≤0.004	≥73
Na <sup>24</sup>	2.50	0.290±0.014	≤0.007	≥41

tion. The experimental results are summarized in Table III. This table lists the target thickness, the fraction of the total activity recoiling in the forward and backward directions,  $F$  and  $B$ , and the ratio of forward to backward emission,  $F/B$ . The listed value of  $F$  for Na<sup>24</sup> is about 10% higher than the uncorrected value.

#### B. Analysis and Discussion

The results will be analyzed in terms of the two-stage model that has been applied in the case of high-energy fission.<sup>11</sup> In the first stage of the reaction the struck nucleus acquires a velocity  $\mathbf{v}$  along the direction of the incident particle. Consider the emission of Be<sup>7</sup> from the struck nucleus. Let the velocity of the fragment in the system of the moving struck nucleus be  $\mathbf{V}$ . The analysis of the experimental results then gives the value of  $\eta$ , ( $\eta = v/V$ ), and of the range,  $R$ , which is related to the velocity of the fragment in the system of the moving nucleus by  $R = kV^N$ , where  $k$  and  $N$  are constants. In the case of nonunique values of  $\mathbf{V}$  or  $\mathbf{v}$ , average values of the above quantities will be obtained, as discussed by Winsberg.<sup>12</sup> In order for the analysis in these terms to be meaningful,  $\mathbf{v}$  must be smaller than  $\mathbf{V}$ , and the emission of the fragments must be symmetric about 90° in the moving frame of reference. The first condition is fulfilled as long as the energy of the Be<sup>7</sup> fragments is greater than 1.2 MeV. This is undoubtedly the case since the Coulomb barrier for Be<sup>7</sup> emission is 8.8 MeV. It is not known if the second condition is met although the fact that backward recoil emission is observed indicates that at least some of the fragments come off in the backward direction. Insofar as fragment emission in the forward direction can be associated with a direct interaction mechanism while symmetric emission can be associated with evaporation from a compound nucleus, the analysis of the experimental results in the above terms will actually test the validity of the assumption and thereby discriminate between these two mechanisms.

Expressions for the analysis of recoil studies in which the above conditions are fulfilled have been given by Porile and Sugarman<sup>11</sup> for  $N=1$  and by Winsberg<sup>12</sup> and Sugarman<sup>13</sup> for  $N=2$ . These expressions assume that

<sup>11</sup> N. Sugarman, M. Campos, and K. Wielgoz, Phys. Rev. **101**, 388 (1956); N. T. Porile and N. Sugarman, Phys. Rev. **107**, 1410 (1957).

<sup>12</sup> L. Winsberg, University of California Radiation Laboratory Report UCRL 8618, 1959 (unpublished).

<sup>13</sup> N. Sugarman (private communication).

the cross section for the formation of the fragments remains constant throughout the bombarding energy range corresponding to the target thickness. This condition clearly is not met in the present experiment; the cross section for the production of Be<sup>7</sup> in fact varies by a factor of 2 over the target thickness. The following more general expressions may be derived on the assumption that the variation of cross section over the target thickness is linear, i.e.,  $\sigma = p + qt$ .

For  $N=1$  and  $W(\theta)=1$  (isotropic emission),

$$F' = \frac{R}{4W(p + \frac{1}{2}qW)} [p(1+\eta)^2 + \frac{1}{3}qR(1+\eta)^3] \quad (1)$$

and

$$\frac{F}{B} = \frac{p(1+\eta)^2 + \frac{1}{3}qR(1+\eta)^3}{(p+qW)(1-\eta)^2 - \frac{1}{3}qR(1-\eta)^3}. \quad (2)$$

For  $N=1$  and  $W(\theta)=a+b\cos^2\theta$ ,

$$F = \frac{R}{4W(p + \frac{1}{2}qW)(1 + \frac{1}{3}b/a)} \left\{ p \left[ (1+\eta)^2 + (b/a) \left( \frac{\eta^4}{6} + \frac{2\eta}{3} + \frac{1}{2} \right) \right] + \frac{qR}{3} \left[ (1+\eta)^3 + \frac{b/a}{10} (\eta^5 + 10\eta^2 + 15\eta + 6) \right] \right\} \quad (3)$$

and

$$\frac{F}{B} = \frac{p[(1+\eta)^2 + (b/a)(\frac{1}{6}\eta^4 + \frac{2}{3}\eta + \frac{1}{2})] + \frac{1}{3}qR[(1+\eta)^3 + \frac{1}{10}(b/a)(\eta^5 + 10\eta^2 + 15\eta + 6)]}{(p+qW)[(1-\eta)^2 + (b/a)(\frac{1}{6}\eta^4 - \frac{2}{3}\eta + \frac{1}{2})] - \frac{1}{3}qR[(1-\eta)^3 + \frac{1}{10}(b/a)(-\eta^5 + 10\eta^2 - 15\eta + 6)]}. \quad (4)$$

In the above expressions  $W$  is the target thickness. Equations (1-4) reduce in the limiting case of constant cross section to the expressions given by Porile and

Sugarman<sup>11</sup> if terms in  $\eta$  containing higher powers than  $\eta^2$  are neglected.

For  $N=2$  and  $W(\theta)=1$  we obtain

$$F = \frac{R}{W(p + \frac{1}{2}qW)} \left\{ \frac{4}{15} p(1+\eta)^2 \left[ 1 + \frac{\eta}{2} - \frac{(1-\eta)^2[1 - (1-\eta^2)^{\frac{1}{2}}]}{8\eta^2} \right] + \frac{qR}{24} [2 + 9\eta + 16\eta^2 + 14\eta^3 + 6\eta^4 + \eta^5] \right\} \quad (5)$$

and

$$\frac{F}{B} = \frac{(4/15)p(1+\eta)^2\{1 + \frac{1}{2}\eta - (1-\eta)^2[1 - (1-\eta^2)^{\frac{1}{2}}]/8\eta^2\} + (qR/24)[2 + 9\eta + 16\eta^2 + 14\eta^3 + 6\eta^4 + \eta^5]}{(4/15)(p+qW)(1-\eta)^2\{1 - \frac{1}{2}\eta - (1+\eta)^2[1 - (1-\eta^2)^{\frac{1}{2}}]/8\eta^2\} - (qR/24)[2 - 9\eta + 16\eta^2 - 14\eta^3 + 6\eta^4 - \eta^5]}. \quad (6)$$

Equations (5) and (6) reduce to the expressions given by Winsberg<sup>12</sup> for the case of constant cross section. The ranges of heavy ions in aluminum have been measured by Northcliffe.<sup>14</sup> It appears that for Be<sup>7</sup> fragments of 5-15 MeV,  $N \sim 2$ .

The results for Na<sup>22</sup> and Na<sup>24</sup> can also be analyzed in terms of a two-stage process. The formalism for the analysis of the recoil properties of residual nuclei has been developed by Winsberg and Alexander.<sup>15</sup> The forward component of velocity imparted to the struck nucleus is again defined as  $\mathbf{v}$ . As a result of the emission of particles, again assumed to be symmetric about 90° in the moving frame of reference, the residual nucleus acquires a velocity  $\mathbf{V}$  which is smaller than  $\mathbf{v}$ . In order to separate the effects of  $\mathbf{v}$  and  $\mathbf{V}$  on the measured recoil properties we define a range  $R$  that is related solely to  $\mathbf{v}$ , i.e.  $R = kv^N$ . The following expression relating  $R$  and  $F$  may be derived for the case of isotropic particle emission on the assumption that the cross section again

varies linearly over the target thickness:

$$F = \frac{R}{4W(p + \frac{1}{2}qW)\mu} \left\{ p \left[ \frac{1-\mu^2}{N+1} [(1+\mu)^{N+1} - (1-\mu)^{N+1}] \right] + \frac{1}{N+3} [(1+\mu)^{N+3} - (1-\mu)^{N+3}] \right. \\ \left. + \frac{qR}{4} \left[ \frac{(1-2\mu^2+\mu^4)}{2N} [(1+\mu)^{2N} - (1-\mu)^{2N}] \right] + \frac{1-\mu^2}{N+1} [(1+\mu)^{2N+2} - (1-\mu)^{2N+2}] \right. \\ \left. + \frac{1}{2N+4} [(1+\mu)^{2N+4} - (1-\mu)^{2N+4}] \right\}. \quad (7)$$

In this expression,  $\mu = V/v$ . In the case of a constant production rate this equation reduces to one given by Winsberg and Alexander.<sup>15</sup> A range-energy relation for 1-3 MeV sodium atoms in aluminum has been obtained by Poskanzer<sup>16</sup> on the basis of measurements on Ne<sup>22</sup>.

<sup>14</sup> L. C. Northcliffe, Phys. Rev. **120**, 1744 (1960).

<sup>15</sup> L. Winsberg and J. M. Alexander, Phys. Rev. **121**, 518 (1961).

<sup>16</sup> A. M. Poskanzer (private communication).

Range-energy data for the same system at energies above 5 MeV may be obtained from measurements on Ne<sup>20</sup> in emulsion.<sup>17</sup> An interpolation between these two sets of data indicates that  $N \sim 1$  in the energy region of interest in the present study. In this case Eq. (7) reduces to

$$F = \frac{R[p + \frac{1}{2}qR(1 + \frac{1}{3}\mu^2)]}{W(p + \frac{1}{2}qW)}. \quad (8)$$

The recoil expressions for Be<sup>7</sup> were solved with the aid of an IBM 7090 computer. The variation of the target thickness was obtained from the shape of the excitation function. The results for  $N=2$  and isotropic emission are summarized in Table IV. The listed errors are based on the experimental uncertainties. An additional error due to scattering at the foil interface has not been included. It is seen that the target thickness is in fact larger than the value of  $R(1+\eta)^2$  which corresponds to the minimum thickness required for the analysis of the data by use of the above expressions. The values of  $R$  and  $\eta$  are about 5 and 20% larger, respectively, than the values obtained from the simpler expressions for constant cross section. The expressions for  $N=1$  predict  $R$  and  $\eta$  values that are about 10 and 30% larger than the values for  $N=2$ , respectively. The average kinetic energy of Be<sup>7</sup> in the system of the struck nucleus was obtained with the aid of Northcliffe's range-energy relation.<sup>14</sup> The resultant value is seen to be 0.5 MeV larger than the Coulomb barrier energy of 8.8 MeV. The maximum kinetic energy of Be<sup>7</sup> in the center-of-mass system at the bombarding energy in question is 12.7 MeV. It is thus seen that the residual Na<sup>24</sup> nucleus has an average excitation energy of 3.4 MeV following the emission of Be<sup>7</sup>. This value is about 4 MeV too small to permit additional particle emission.

The kinetic energy imparted to the struck nucleus may be obtained from the calculated values of  $\eta$  and  $E_F$  by the relation

$$E_{SN} = (M_{SN}/M_F)E_F\eta^2, \quad (9)$$

where  $SN$  and  $F$  refer to the struck nucleus and Be<sup>7</sup> fragment, respectively, and  $E$  and  $M$  are kinetic energy and mass. The value of  $E_{SN}$  obtained on the assumption that the moving struck nucleus has  $M_{SN}=31$  is listed in Table IV, and may be compared with the value expected for compound nucleus formation. It is seen that  $E_{SN}$  is some 45% larger than  $E_{CN}$ . One possible interpretation of this difference is that some of the Be<sup>7</sup> fragments are emitted only in the forward direction as a result of a direct-interaction process. The observed value of  $F/B$  will therefore be larger than that corresponding to compound-nucleus formation, leading in turn to a larger value of  $\eta$ . Furthermore, it is reasonable to assume that the value of  $E_F$  will be approximately independent of the reaction mechanism because of the

TABLE IV. Analysis of recoil results.

Nuclide	Range (mg/cm <sup>2</sup> )	Fragment energy (MeV)	$\eta$	Energy of struck nucleus (MeV)	$E_{CN}$ (MeV)
Be <sup>7</sup>	4.04±0.21	9.3±0.9	0.43±0.02	7.5±1.0	5.15
Na <sup>22</sup>	0.76±0.03	3.8±0.3		5.3±0.4	5.15
Na <sup>24</sup>	0.75±0.04	3.7±0.4		4.8±0.5	5.15

small difference in energy between the Coulomb barrier and the maximum possible kinetic energy. A value of  $E_{SN}$  larger than that corresponding to compound nucleus formation will then be obtained. The contribution of a direct interaction process to the formation of Be<sup>7</sup> may be estimated on the assumption that the fragments are emitted along the beam direction and that their energy is the same regardless of mechanism. On this basis we estimate that direct interactions account for approximately 40% of the Be<sup>7</sup> yield. This percentage will be increased if the Be<sup>7</sup> fragments actually are emitted at an angle to the beam, which seems likely. On the other hand, the kinetic energy of the fragments emitted in a direct process is probably somewhat larger than that of evaporated fragments and this effect will in turn lead to a lower estimate of the direct interaction contribution. These two effects will therefore largely cancel each other. An additional uncertainty in this interpretation is due to the uncertainty in the value of  $N$ . If  $N < 2$ ,  $R$  and  $\eta$  will both increase leading in turn to a larger contribution from a direct interaction process. The opposite situation holds if  $N > 2$ .

An alternative interpretation of these results is that Be<sup>7</sup> is formed exclusively by evaporation from a compound nucleus and has an angular distribution in the center-of-mass system that is anisotropic. The effect of anisotropic emission was investigated with the aid of Eqs. (3) and (4). The effect of preferential forward-backward emission is to decrease  $R$  and increase  $\eta$ , leading to a net increase in the energy of the struck nucleus. The effect of preferential transverse emission, on the other hand, is in the opposite direction. A value of  $b/a$  of  $-1$  thus leads to a 30% lower energy for the struck nucleus than the expressions for isotropic emission. While this result refers to the expressions for  $N=1$ , a similar situation holds for  $N=2$ . We may thus conclude that Be<sup>7</sup> is emitted to a comparable extent by a direct interaction process and by isotropic evaporation from a compound nucleus, or that it is exclusively evaporated from a compound nucleus with a strong preference for transverse emission.<sup>18</sup> Even in the latter case, however, one cannot rule out a direct interaction

<sup>17</sup> H. Heckman, B. Perkins, W. Simon, F. Smith, and W. Barkas, Phys. Rev. **117**, 544 (1960).

<sup>18</sup> A preliminary report of angular distribution measurements on Be<sup>7</sup> formed in the reaction of aluminum with 42-MeV  $\alpha$  particles has appeared recently; see A. W. Fairhall, I. Halpern, and C. O. Hower, Cyclotron Research Progress Report, University of Washington, 1961 (unpublished). The angular distribution in the center-of-mass system has a large peak in the forward direction and is isotropic at large angles. These results are consistent with the first hypothesis proposed for the emission of Be<sup>7</sup>.

mechanism since it is easy to postulate angular distributions for particles emitted in direct interactions that will, when weighted by suitable relative intensity factors, lead to the same average recoil behavior as an evaporation mechanism. We merely discriminate against the most common direct process in which particles are emitted at very small angles to the beam.

The results for  $\text{Na}^{22}$  and  $\text{Na}^{24}$  analyzed with the aid of Eq. (8) are given in Table IV. The values of  $\mu$  were obtained from the reaction kinematics by the following prescription. It was assumed that  $\text{He}^3$  and  $\alpha$  particles were emitted with an energy equal to their Coulomb barrier, that  $\text{Be}^7$  was emitted with the measured average energy, that a 1-MeV neutron was emitted in the  $(\alpha, 2an)$  reaction, and that any remaining excitation energy was dissipated in the emission of gamma rays. The value of  $\mu$  for  $\text{Na}^{24}$  formation was obtained from the values of  $\mu$  for both reaction paths, weighted by their respective measured cross sections, on the assumption that only 60% of the  $\text{Be}^7$  fragments were emitted in a random direction and contributed to  $\mu$ . When several particles were emitted, the value of  $V$  was taken as the root mean square of the values of  $V$  due to each emitted particle. The calculated values of  $\mu$  for  $\text{Na}^{22}$  and  $\text{Na}^{24}$  were 0.7 and 0.6, respectively. The uncertainty in this estimate is of little importance, since the ranges obtained in this fashion are only about 5% larger than those given by the simpler expression for constant cross section in which, of course,  $\mu$  does not enter.

The kinetic energies of the fragments were obtained from the ranges with the aid of the previously mentioned range-energy data.<sup>16,17</sup> The energy of the struck nucleus was obtained from the fragment energy through multiplication by the appropriate mass ratio factor. The results are compared in Table IV with the value expected for compound-nucleus formation and it is seen that the experimental and calculated values are in agreement. The range-energy data of Poskanzer<sup>16</sup> indi-

cates that  $N=1.2$  in that particular energy range. If this value of  $N$  is used in the analysis instead of the interpolated value of 1.0, the resultant ranges are reduced by some 12% from the values quoted in Table IV, and the corresponding energies are reduced by about 25%. Even in this case, however, compound-nucleus formation would still account for most of the observed yield, since a direct interaction mechanism in which particles are emitted along the beam direction would give much smaller ranges.

Summarizing the results of the present study, it appears that  $\text{Na}^{22}$  and  $\text{Na}^{24}$  are primarily formed in processes involving compound-nucleus formation. The emission of  $\text{Be}^7$ , on the other hand, probably is associated with a direct interaction mechanism, possibly a pickup process, as well as with evaporation from a compound nucleus. The results for  $\text{Be}^7$  are not inconsistent with those for  $\text{Na}^{24}$  since  $\text{Be}^7$  formation accounts for only about 40% of the yield of  $\text{Na}^{24}$ . A 40% contribution of a direct mechanism to the formation of  $\text{Be}^7$  would thus decrease the energy of the  $\text{Na}^{24}$  recoils by somewhat less than 16% from the value expected for compound-nucleus formation. The experimental results are consistent with this estimate. It is perhaps surprising that evaporation from a compound nucleus should be an important mechanism at these low mass numbers. It should be kept in mind, however, that the present experiment samples only about 1% of the total inelastic cross section and that more probable reactions may exhibit a different behavior.

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