Another oversimplification of the model discussed here is the neglect of the motion of the "target pion" with respect to its "parent" nucleon. This effect has been shown to be of little importance at accelerator energies, but may gain importance at very high cosmic-ray energies.[17]

Considerably more refined measurements than those available at present are necessary[30] before one can assert knowledge about anything like the cloud's "energy spectrum,"[31] its "temperature," etc.

[30] Accurate measurements of $\bar{\gamma}'$ by the $p_{\parallel}'$ vs $p_{\perp}$ method seem the most promising, especially if coupled with measurements of Δ_r on the same event.

[31] T. Yajima, S. Takagi, and G. Kobayakawa, Progr. Theoret. Phys. (Kyoto) 24, 59 (1960).

Finally, it is important to stress that all the classical notions and the picture developed here need not, and must not, be taken too literally. Probably some new physical concepts will have to be developed before deeper insight becomes possible.

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Extrapolation of Proton-Proton Scattering Data to the One-Pion Exchange Pole*

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The dependence of various p-p scattering parameters on the one-pion exchange contribution and the Coulomb scattering amplitude is investigated. Two parameters, $I_0(1-D)$ and I_0Y, have second-order one-pion poles and no Coulomb singularity in the forward direction. The extrapolation procedure frequently applied to n-p cross sections to determine the pion-nucleon coupling constant can be applied to these parameters. This is done with measurements of $I_0(1-D)$ at 142 MeV, giving a value of $8.8_{-2.6}^{+2.0}$ for g^2, compared with the currently accepted value of 14. The relative sensitivity of the various p-p parameters to the one-pion exchange contribution is discussed.

INTRODUCTION

In the past, proton-proton scattering data have been used to determine the pion-nucleon coupling constant by means of the "modified phase shift analysis" of Moravcsik *et al.*[1] In a phase-shift search, the higher angular momentum phase shifts are given by the one-pion exchange contribution, and the coupling constant is varied to obtain the best fit to the data.

Neutron-proton data have been used in a much more direct procedure.[2] The cross section is expected to have a second-order pole, due to the one-pion exchange contribution. The coefficient of the second-order term at this pole, simply related to the pion-nucleon coupling constant, is obtained by an extrapolation in $\cos\theta$, where θ is the center-of-mass scattering angle.

This simple extrapolation procedure cannot be used for the proton-proton cross section, because the Coulomb amplitude completely distorts the cross section at small angles. In this note, the possibility of extrapolating some parameter other than the cross section is investigated. Two suitable parameters are found: $I_0(1-D)$ and I_0Y. The extrapolation procedure is applied to the parameter $I_0(1-D)$, giving a reasonable value for the coupling constant.

SINGULARITIES OF p-p PARAMETERS

Following Wolfenstein,[3] we write the p-p scattering matrix as

$$M=B(\theta)S+C(\theta)[\sigma_{1n}+\sigma_{2n}]+\tfrac{1}{2}G(\theta)[\sigma_{1k}\sigma_{2k}+\sigma_{1p}\sigma_{2p}]T +\tfrac{1}{2}H(\theta)[\sigma_{1k}\sigma_{2k}-\sigma_{1p}\sigma_{2p}]T+N(\theta)[\sigma_{1n}\sigma_{2n}]T, \quad (1)$$

where

$$S=\tfrac{1}{4}(1-\sigma_1\cdot\sigma_2), \quad T=\tfrac{1}{4}(3+\sigma_1\cdot\sigma_2),$$

are the singlet and triplet projection operators. The amplitudes B, C, G, H, and N, considered as functions of $x=\cos\theta$, have singularities in the complex x plane located as follows: simple poles at $x=\pm(1+\mu^2/MT)$, from one-pion exchange, the singularities we wish to make use of; singularities at $x=\pm1$, from the Coulomb amplitude $f_c(\pm x)$, the singularities we wish to avoid; and branch cuts for $x>1+4\mu^2/MT$ and $x<-(1+4\mu^2/MT)$, from exchange of two or more pions, the singularities we wish to ignore. (T is the kinetic energy

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[1] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **116**, 1248 (1959).

[2] P. Cziffra and M. J. Moravcsik, Phys. Rev. **116**, 226 (1959).

[3] L. Wolfenstein, Ann. Rev. Nuclear Sci. **6**, 43 (1956).

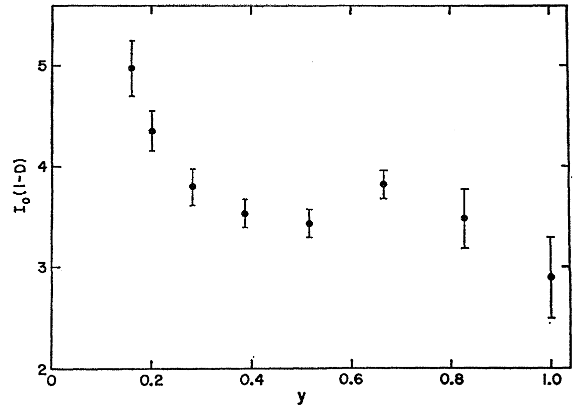

FIG. 1. The quantity $I_0(1-D)$, obtained from the measurements of references 7 and 9, plotted against $y=1+\mu^2/MT-\cos\theta$.

of the incident proton in the laboratory system, μ, the pion mass, and M, the proton mass.)

We write $B=B_0+b$, etc. where B_0 includes the Coulomb amplitude and one-pion exchange contribution, and b includes the effects of singularities for $|x|\geqslant 1+4\mu^2/MT$. Then, as shown for example, by Moravcsik[4]:

$$\begin{aligned} B_0&=-(g^2/4E)(\alpha+\beta)+f_c(x)+f_c(-x),\\ C_0&=0,\\ G_0&=-(g^2/4E)(\alpha-\beta)+2f_c(x)-2f_c(-x),\\ H_0&=(g^2/4E)(\alpha+\beta),\\ N_0&=f_c(x)-f_c(-x), \end{aligned} \tag{2}$$

where g^2 is the pion-nucleon coupling constant, ≈ 14; E is the total energy of one proton in the c.m.; $\alpha=(1+x)/(x_0+x)$; $\beta=(1-x)/(x_0-x)$; $x_0=1+\mu^2/MT$; and $f_c(x)=[-n/k(1-x)]\exp\{in\ln[(1-x)/2]\}$ is the Coulomb scattering amplitude.

An experimental quantity, which is a bilinear combination of the amplitudes, may have one-pion poles of second order, or only first order, in either the forward ($x>1$) or backward ($x<-1$) directions. Second-order poles have leading coefficients that are known (in terms of the coupling constant); first-order poles have coefficients that depend on the cuts, and hence are not known. Similarly, an experimental quantity may contain the forward Coulomb amplitude $f_c(x)$ or backward Coulomb amplitude $f_c(-x)$ to second or only first powers. Using the relations between amplitudes and experimental quantities given, for example, by Stapp,[4] we obtain the results given in Table I.

For an experimental quantity to be useful for obtaining the coupling constant by extrapolation, it should have a second-order one-pion pole and no Coulomb singularity, in at least one of the two directions. Table I shows two such quantities, $I_0(1-D)$ and I_0Y, both with the coupling constant at their forward pole amenable to determination.

[4] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nuclear Sci. **10**, 291 (1960).

Explicitly, we can write

$$I_0Y=(g^2/4E)^2\beta^2+A(x)\beta+\text{ nonsingular terms,} \tag{3}$$

$$I_0(1-D)=2(g^2/4E)^2\beta^2+B(x)\beta+\text{ nonsingular terms.} \tag{4}$$

$A(x)$, $B(x)$, and the nonsingular terms are unknown functions of x, assumed slowly varying in the neighborhood of the one-pion pole, since they are due to singularities outside the region $-1<x<1+4\mu^2/MT$. The functions $I_0Y(x_0-x)^2$ and $I_0(1-D)(x_0-x)^2$ are analytic in the region $-1<x<1+4\mu^2/MT$. If continued beyond the physical region to $x=x_0$, they will give the coupling constant

$$I_0Y(x_0-x)^2|_{x=x_0}=(g^2/4E)^2(x_0-1)^2, \tag{5}$$

$$I_0(1-D)(x_0-x)^2|_{x=x_0}=2(g^2/4E)^2(x_0-1)^2. \tag{6}$$

EXTRAPOLATION OF $I_0(1-D)$

To determine the parameter Y, one must measure three of the four triple scattering parameters: R, A, R', and A'. At present, there is no energy at which such data have been published.

On the other hand, the parameter D has been measured at 310 MeV,[5] 210 MeV,[6] 142 MeV,[7] and 98 MeV.[8] The 142-MeV measurements are most suitable for extrapolation because they extend to smaller angles (12.4° c.m.) than the others. The quantity $I_0(1-D)$, obtained from measurements of D at 142 MeV[7] and cross section at 147 MeV,[9] is plotted against $y=x_0-x$ in Fig. 1. The data strongly suggest a singularity at $y=0$.

These data, measured at 8 angles from 12.4 to 82° c.m., were extrapolated to the one-pion pole by a

TABLE I. Singularities of p-p parameters from one-pion exchange and from Coulomb scattering. β and α are the forward and backward one-pion poles, respectively; f_+ and f_- are the forward and backward Coulomb amplitudes, respectively. (See text for precise definitions.)

Parameter[a]	Forward singularities	Backward singularities
I_0	β^2, f_+^2	α^2, f_-^2
I_0 P	$0,\ f_+$	$0,\ f_-$
$I_0\ (1-D)$	$\beta^2,\ 0$	α^2, f_-^2
$I_0\ X$	$0,\ f_+^2$	$\alpha,\ f_-$
$I_0\ Y$	$\beta^2,\ 0$	$\alpha,\ f_-$
$I_0\ Z$	$0,\ f_+$	$\alpha,\ 0$
$I_0\ C_{kp}$	$\beta,\ 0$	$\alpha,\ 0$
$I_0\ (1-C_{nn})$	β^2, f_+^2	α^2, f_-^2

[a] $R=(X+Y)\cos(\theta/2)+Z\sin(\theta/2)$; $A=-(X+Y)\sin(\theta/2)+Z\cos(\theta/2)$; $R'=(X-Y)\sin(\theta/2)-Z\cos(\theta/2)$; $A'=(X-Y)\cos(\theta/2)+Z\sin(\theta/2)$.

[5] O. Chamberlain, E. Segrè, R. Tripp, C. Wiegand, and T. Ypsilantis, Phys. Rev. **105**, 288 (1957).
[6] K. Gotow, F. Lobkowicz, and E. Heer (to be published).
[7] C. F. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960).
[8] E. H. Thorndike and T. R. Ophel, Phys. Rev. **119**, 362 (1960).
[9] J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and R. Wilson, Ann. Phys. **5**, 299 (1958).

TABLE II. Determination of the coupling constant g^2, for extrapolations with polynomials of order N. The goodness of the fit, χ^2, and the quantity ρ^2, defined in the text, are also given.

N	$(g^2/14)^2$	χ^2	ρ^2
1	-1.29 ± 0.33	137.4	22.9
2	0.39 ± 0.20	7.6	1.52
3	0.59 ± 0.45	7.0	1.75
4	-0.23 ± 0.51	2.6	0.87

method identical to that used by Cziffra and Moravcsik[2] in extrapolating n-p cross sections to the one-pion pole. The quantity

$$M=I_0(1-D)(x_0-x)^2 \tag{7}$$

was fitted by the series

$$F=\sum_{k=0}^{N} a_k y^k \tag{8}$$

by the method of least squares. The coefficient a_0 gives the coupling constant through the relation

$$a_0=2(g^2/4E)^2(x_0-1)^2. \tag{9}$$

Results of this extrapolation are given in Table II. χ^2 is the usual measure of goodness of fit of the power series to the measurements. $\rho^2=\chi^2/(8-N-1)$, should be near 1 for a statistical fit.

Cziffra and Moravcsik[2] suggest that the best value of N to use is the first one after ρ^2 has stopped its rapid decrease, in our case, $N=2$. This gives a value of g^2 of $8.8_{-2.6}^{+2.0}$, rather lower than the value of 14 currently accepted.

CONCLUSIONS

The method described for obtaining the pion-nucleon coupling constant from proton-proton data does not compare favorably with that of the modified phase-shift analysis.[1] (Cziffra and Moravcsik[2] discuss the relative merits of modified phase-shift analysis and extrapolation of n-p cross sections; their discussion is applicable here.) However, the following conclusions are warranted.

(1) The measurements of cross section and depolarization at 142 MeV, by themselves, suggest that the proton-proton scattering amplitudes have a pole with residue near that given by one-pion exchange. (While the results for $N=2$ are barely compatible with $a_0=0$, the data exclude a nonsingular three-parameter fit. For example, a fit with $N=4$, but $a_0=a_1=0$, had a $\chi^2=20.9$, compared with $\chi^2=7.6$ for the singular, $N=2$ fit.)

(2) Inclusion of small-angle measurements of the D parameter in a modified phase-shift analysis search should materially increase the sensitivity of χ^2 to variations in coupling constant or pion mass, since the small-angle D values depend on one-pion exchange in such a direct way. Conversely, it will be difficult to fit the small-angle measurements by phase-shift sets that do not include one-pion exchange contributions.

(3) From Table I one can infer the relative sensitivity of the various parameters to the one-pion exchange contribution. At forward angles, after I_0 $(1-D)$, come I_0R and I_0A' (which essentially determine I_0Y at small angles), and then I_0. I_0A and I_0R' (which essentially determine I_0Z at small angles) are insensitive, as is I_0P. (Measurement of the correlation coefficients C_{nn} and C_{kp}, at small angles, is sufficiently difficult that they need not be considered.)

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