the configurations which can be reached in a finite system, that is, for example, a fluid in equilibrium with crystallites of average size greater than the number of particles dealt with is impossible to achieve. This constraint for the small systems previously investigated resulted in stabilization of the predominant phase. Thus, the system was either all solid and when a rare fluctuation disordered enough of the system, it became completely fluid. For the 870-particle system the constraint again stabilizes the more abundant phase causing the pressure to be high on the fluid side and low on the crystal side. It thus seems that the phase separation which might occur in infinite systems is not complete in finite systems, since a sizable portion of the system lies in the fluid-crystal boundary region and this region is of intermediate density and evidently takes on more of the character of the predominant phase.

The horizontal line in Fig. 1 drawn at $pA_0/NkT=7.72$ and extending from A/A_0 of 1.266 to 1.312 corresponds to the usual "equal area" rule. If the phase transition for an infinite system is of first order at the pressure indicated by this straight line, then the resulting entropy change across the transition $\Delta S/Nk$ is $p\Delta A/NkT=0.36$. The change of entropy across the same density interval corresponding to the expansion of the one particle cell as calculated by the free volume theory is 0.30. This indicates that the change of communal entropy (0.06) across the transition is very much smaller than unity. This is hardly in accord with the view[5] that the difference between a dense fluid and a solid is one of the accessibility of the entire space in the fluid and localization of a molecule in a solid.

The complete equation of state and comparisons of it with the predictions of various theories will be the subject of further publications.

We are deeply indebted to Mary Ann Mansigh and Norman Hardy for their invaluable help in programming, and to Dr. Sidney Fernbach of the Livermore Computing Division for his cooperation.

[5] J. O. Hirschfelder, D. P. Stevenson, and H. Eyring, J. Chem. Phys. **5**, 896 (1937); however, see also O. K. Rice, J. Chem. Phys. **6**, 476 (1938).

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Thermal Conduction in Liquid Helium II. I. Temperature Dependence

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The thermal conductivity of liquid helium contained in a cylindrical stainless-steel capillary 0.080 cm in diam by 5.16 cm long has been studied between 0.9°K and the λ point. The relation between temperature gradient and heat current density W for heat currents greater than the critical heat current density W_c is best expressed in the form $\text{grad}T=DW^n$, where D is a temperature-dependent constant and n varies from about 3.0 at low temperatures to about 3.5 above 1.7°K. Below W_c the temperature gradient is much smaller, and is determined entirely by the viscosity of the normal component. W_c was measured over the entire temperature range by a combination of two methods, which are in complete agreement in the region of overlap. The results suggest that, below about 1.7°K, W_c is the result of some sort of normal turbulence describable by a Reynolds number involving the normal fluid velocity but the *total* density. At higher temperatures such an explanation is no longer adequate, and some other type of critical velocity must be invoked.

I. INTRODUCTION

LIQUID helium II has an unusually high thermal conductivity, which, in terms of the two-fluid model, can be explained by an internal convection of the normal and superfluid components. According to this model, normal fluid flows away from the source of heat with a velocity $\mathbf{v}_n$ which is related to the heat current density $\mathbf{W}$ by the equation

$$\mathbf{v}_n=\mathbf{W}/\rho ST, \tag{1}$$

where ρ is the density, S the entropy per unit mass, and T the temperature. The superfluid flows in the opposite direction with a velocity $\mathbf{v}_s$ which is determined by the additional condition that there be no net mass flow,

$$\rho_n\mathbf{v}_n+\rho_s\mathbf{v}_s=0, \tag{2}$$

where ρ_n and ρ_s are, respectively, the normal and superfluid densities.

As long as $\mathbf{W}$ is sufficiently small, the only dissipative mechanism present is the viscosity of the normal component η_n. For a cylindrical channel of radius r, the relation between temperature gradient and heat current density is then given by [1,2]

$$\mathbf{W}=-(r^2\rho^2S^2T/8\eta_n)\,\text{grad}T. \tag{3}$$

* Operated with support from the U. S. Army, Navy, and Air Force.

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[2] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).

When $\mathbf{W}$ is increased, a point is eventually reached at which grad$T$ suddenly rises above the value given by Eq. (3). This value of $\mathbf{W}$, the magnitude of which will be referred to as the critical heat current density W_c, is a function of temperature, channel geometry, and past history of the helium; it is also sometimes subject to large hysteresis. A further increase in $\mathbf{W}$ results in a rapid increase in temperature gradient; in this supercritical region the observed behavior can often be satisfactorily described by the relation[2]

$$|\mathrm{grad}T| = -D|\mathbf{W}-\mathbf{W}_0|^3, \tag{4}$$

where D and $\mathbf{W}_0$ are temperature-dependent constants. It has not been firmly established whether the thermal resistance described by Eq. (4) should be added to the viscous term given by Eq. (3) or whether it replaces it. In the present paper it will be assumed that the viscous term is always present, and the results will be discussed in terms of the quantity (grad$T$)*, defined as the observed temperature gradient minus the viscous term calculated according to Eq. (3). Except at the lowest temperatures, the viscous term is negligible. Since only one-dimensional flow is to be considered here, W will henceforth be considered a scalar and the minus sign in Eqs. (3) and (4) will be ignored.

The high thermal conductivity and nonlinear behavior of liquid helium II at large heat current densities were studied by a number of early investigators.[3–8] The linear behavior at lower heat-current densities was subsequently found in fine slits and capillaries by Keesom and Duyckaerts,[9] Mellink,[10] and Meyer and Mellink.[11] Critical heat currents were observed by several authors,[11–13] but until recently no precise measurements of W_c have been carried out. Critical velocities have been studied in a wide variety of other experiments, but, except for a few measurements particularly relevant to the present work, these will not be discussed here.

The approximately cubic dependence of temperature gradient upon heat current in the supercritical region was explained phenomenologically by Gorter and Mellink[2] in terms of a proposed mutual friction force between the normal and superfluid components of the form

$$F_{sn} = A\rho_s\rho_n(|\mathbf{v}_s-\mathbf{v}_n|)^3, \tag{5}$$

where A is a constant depending somewhat upon temperature. Vinen[14] later proposed that the mutual friction should really be given by the slightly modified equation

$$\mathbf{F}_{sn} = A\rho_s\rho_n(|\mathbf{v}_s-\mathbf{v}_n|-v_0)^2(\mathbf{v}_s-\mathbf{v}_n); \tag{6}$$

in practice, however, the difference between the two forms is negligible. The mutual friction force is supposed to vanish below W_c.

London and Zilsel[1] first explained the linear region in terms of the viscosity of the normal component, and a number of investigators[15–19] have subsequently used measurements in this region to determine the viscosity. The behavior there is, therefore, well established. The situation near W_c and in the supercritical region is, however, not so well understood. A thorough study of thermal conduction in wide rectangular channels (2.40 $\times$6.45 and 4.00$\times$7.83 mm) was carried out by Vinen,[14] who also developed methods for observing W_c in such channels, where the associated temperature gradient is too small to observe directly. Vinen reported a few values of W_c below 1.5°K; at higher temperatures the transition was no longer sharp and no results were given. Additional measurements in a channel of identical cross-sectional area were made by Careri and collaborators,[20,21] who observe a reduction in the mobility of negative ions in the supercritical range. Their results agreed well with Vinen's and extended the measurements down to nearly 0.8°K.

Vinen developed a detailed theory which accounted well for his observations, based on the idea that the superfluid component of liquid helium II at supercritical heat current densities contains a tangled mass of quantized vortex line, which interacts with the phonons and rotons making up the normal component and thus gives rise to the Gorter-Mellink mutual friction. This theory correctly predicted the dependence of W_c upon channel diameter (at least for channels of similar shape) and also suggested a possible origin for the hysteresis observed in narrower channels.[18,22–24] It

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[3] W. H. Keesom and Miss A. P. Keesom, Physica **3**, 359 (1936).
[4] W. H. Keesom, Miss A. P. Keesom, and B. F. Saris, Physica **5**, 281 (1938).
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[11] L. Meyer and J. H. Mellink, Physica **13**, 197 (1947).
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[16] A. Broese van Groenou, J. D. Poll, A. M. G. Delsing, and C. J. Gorter, Physica **22**, 905 (1956).
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[22] D. F. Brewer, D. O. Edwards, and K. Mendelssohn, Phil. Mag. **1**, 1130 (1956).
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[24] D. F. Brewer and D. O. Edwards, Phil. Mag. **6**, 775 (1961).

was, however, not successful in predicting quantitatively the temperature dependence of W_c or the rapid disappearance of W_c as the temperature was raised.

Brewer, Edwards, and Mendelssohn[22] have detected critical heat currents, accompanied by considerable hysteresis, in a channel of diam 5×10^{-3} cm. Similar results have been obtained in larger channels (up to 0.17-cm diam) by Mendelssohn and Steele.[23] Brewer and Edwards[24] have recently reported more detailed studies, including values of W_c as a function of temperature and pressure. The consensus of these measurements seems to be that W_c is associated with some form of turbulence in the superfluid, possibly involving vortex lines as suggested by Vinen, and leading to a critical value of the *relative* velocity, $\mathbf{v}_n-\mathbf{v}_s$.

Staas and Taconis[25] studied gravitational flow through a 0.026-cm diam capillary connected in series with a superleak, and reported the temperature dependence of the resulting critical flow rate. Since only the superfluid can flow in this experiment, it is clear that the critical velocity which they observed must be $\mathbf{v}_s$ (or possibly $\mathbf{v}_s-\mathbf{v}_n$, which is the same since $\mathbf{v}_n=0$). Comparable critical velocities, but with an entirely different temperature dependence, were observed by Kidder and Fairbank[26] in a channel blocked with superfluid filters at either end, the flow in this case being produced by the thermomechanical effect. Heat-flow experiments in small channels[15] also indicate that it is $\mathbf{v}_s$ that is the critical quantity.

Another type of experiment was performed by Staas, Taconis, and van Alphen.[19] These authors made simultaneous measurements of pressure difference and temperature difference across a capillary connected in parallel with a superleak so arranged that when a heat current flowed in the capillary $|\mathbf{v}_n-\mathbf{v}_s|\approx0$. These measurements were designed to study the behavior of the liquid in the absence of mutual friction. In these experiments it was found that the proportionality between pressure gradient and heat-current density given by Allen and Reekie's rule[27] breaks down at a critical value of the Reynolds number $\mathcal{R}=\rho d v_n/\eta_n$ of about 1300, where ρ is the total density, d the channel diameter, and v_n the normal fluid velocity. Moreover, an identical critical Reynolds number was observed in a similar experiment involving pure heat conduction with no net mass flow. At higher Reynolds numbers in both cases, the pressure gradient obeyed Blasius' rule,[28] as is the case for ordinary liquids. Although it is not obvious why the relevant Reynolds number involves ρ instead of ρ_n in the case of pure heat conduction, it is clear that some sort of normal-fluid turbulence will occur at suitably high flow velocities, and may be responsible for some of the previously observed critical phenomena. No evidence of superfluid interaction directly with the walls was observed in these experiments.

It thus appears that critical velocities may be associated with a variety of causes, depending on the circumstances of the experiment. In particular, although there is abundant evidence that a critical superfluid velocity or relative velocity exists, it appears that it may often be obscured by some sort of turbulence associated with the normal fluid, which is probably of a nearly classical nature. A similar suggestion has recently been made by Meservey.[29] The present measurements were designed to elucidate this problem by studying in detail the nature of the thermal conduction process in moderately wide channels in the neighborhood of W_c and in the supercritical region. This paper reports measurements, in a cylindrical channel 0.080 cm in diam by 5.16 cm long, of W_c and the dependence of temperature gradient upon W between 0.9°K and the λ point. A second paper in this series, subsequently referred to as II, will describe measurements of W_c in channels of various geometries; Paper III will deal with the effects of rotation upon W_c in some of the same channels. Preliminary results of some of these experiments have previously been reported.[30,31]

II. EXPERIMENTAL METHOD

A. Apparatus

The experimental apparatus is similar to that described in an earlier publication.[32] The channel in which the heat current flows consists of nine identical stainless steel tubes connected in parallel, 0.080 cm in diam by 5.16 cm long, with a wall thickness of 0.025 cm. These tubes are connected at one end to a chamber containing a wire-wound heater and a resistance thermometer consisting of a 100-Ω, one-half W, Allen Bradley carbon resistor. The other end of the channel opens into the helium bath, where a similar thermometer is located. The entire apparatus is surrounded by a vacuum jacket and is mounted horizontally along the diameter of an aluminum rotor immersed in the helium bath, which could be rotated about a vertical axis during the experiments described in III. The parts of the apparatus connected with rotation will be described in that paper. Nine parallel channels were used instead of a single one to increase the total cross-sectional area, thus minimizing the stray heat-current density in the channel re-

[25] F. A. Staas and K. W. Taconis, Physica **27**, 924 (1961).

[26] J. N. Kidder and W. M. Fairbank, *Proceedings of Seventh International Conference on Low-Temperature Physics, Toronto, 1960* (University of Toronto Press, Toronto, 1961) p. 560, and (to be published).

[27] J. F. Allen and J. Reekie, Proc. Cambridge Phil Soc. **35**, 114 (1939).

[28] Blasius' rule states that the pressure drop in turbulent flow is proportional to the seven-fourths power of the velocity, for Reynolds numbers less than 10^5. See, for example, H. Schlichting, *Boundary Layer Theory* (McGraw-Hill Book Company, Inc., New York, 1960), 4th ed., p. 503.

[29] R. Meservey, Bull. Am. Phys. Soc. **6**, 64 (1961); and (to be published).

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sulting from small variations in bath temperature. In this way the necessity for precise temperature regulation was avoided. Leads for the internal thermometer and heater were led through closely fitting capillary tubes parallel to the channel, into which they were sealed with glyptal. Possible errors arising from the flow of superfluid through the resulting annular gaps are discussed in Sec. II C; in II this problem is investigated experimentally by introducing the leads in another manner. In II and III the channel used in the present experiments is designated C6.

Heater power was supplied from a battery and potentiometer. Separate current and potential leads were provided so that both heater voltage and current could be measured, but since the heater resistance was found to be independent of temperature and not to change from run to run, in general only the current was measured. Temperature differences between the ends of the channel were measured with the two resistance thermometers, which were connected differentially. A decade resistor was placed in series with the internal thermometer, so that temperature differences could be measured directly; the bridge was adjusted to compensate approximately for the slightly different temperature coefficients of the two thermometers, so that the balance was insensitive to small variations in bath temperature. The output of the bridge, which was operated at 33 cps, was amplified by a narrow-band amplifier which has been previously described.[33] The power dissipated in each thermometer varied between roughly 10^{-5} and 2×10^{-6} W. Under these conditions temperature differences of about 1 μdeg could readily be observed. During transient measurements, the bandwidth of the amplifier was increased to about 20 cps, with some loss in sensitivity, so that delay times as short as 0.1 sec could be measured. The heater voltage and the output of the thermometer bridge were simultaneously recorded on a dual-channel Brush recorder operated at a tape speed of 25 mm/sec. Delay times were read directly from this record.

The bath temperature was normally determined from the vapor pressure of the helium, which was measured on mercury and butyl phthalate manometers. The 1958 scale[34] was used. Between 0.9 and 1.2°K, where a few additional measurements were made with the aid of a large booster diffusion pump, temperatures were estimated from the extrapolated calibration curve of a 33-Ω carbon resistance thermometer, and are probably less accurate. During most of the measurements above about 1.3°K the bath pressure was regulated by means of a pressure regulator described by Walker,[35] which was capable of holding the pressure constant to within ±0.02 mm Hg. For measurements at lower temperatures the regulator introduced too great a constriction in the pumping line, so it was removed and the pressure was regulated manually.

B. Experimental Procedure

The temperature gradient was measured as a function of heat-current density at a number of temperatures between 0.9°K and the λ point. For heat currents near the critical value more than a minute was required for the equilibrium temperature gradient to be established; in order to avoid possible errors resulting from drift of the thermometer balance during this period, the bridge was balanced after the heat current had been flowing for some time, and the change in resistance when the heat current was turned off was then measured. In this way measurements could be made very quickly, and drift was unimportant. At larger heat currents, where equilibrium was reached rapidly, the bridge was first unbalanced by a given amount and the heat current then increased until a null was obtained. After every such reading the null balance was checked to verify that drift had not occurred. Measurements were made with increasing and decreasing heat currents and the results averaged; no hysteresis was ever observed in these experiments, and values obtained for increasing and decreasing heat currents generally agreed within the combined experimental errors of the measuring equipment. The maximum temperature gradients ordinarily employed were about 3×10^{-3} °K/cm, although larger gradients were occasionally used.

During each run the internal thermometer was calibrated against the vapor pressure of the bath at a number of temperatures, and the logarithm of the resistance plotted against $1/T$. The results usually fell on a straight line except at the lowest temperatures, where some curvature was observed. The temperature coefficient of the thermometer occasionally varied as much as 10% between runs, but the resulting curves of $\mathrm{grad}T(W)$ found on different days were reproducible within experimental error. This behavior was therefore not objectionable, as long as the thermometer was calibrated during every run. The thermometer characteristic did not change noticeably upon repeated cycling over the helium range of temperatures. Values of temperature difference were found from ΔR by fitting the calibration data over a limited temperature range to an equation of the form $\log R=a+b/T$. After the coefficients were evaluated, ΔT was calculated including both linear and quadratic terms in a Taylor expansion in ΔR; at the largest values of $\mathrm{grad}T$ the quadratic term introduced a correction of nearly 10%, and therefore cannot be neglected.

Because of the rapid temperature dependence of the thermal conductivity, the temperature gradient at the cold end of the channel, which is the quantity of interest, is not the same as the average gradient $\Delta T/l$.

[33] C. Blake, C. E. Chase, and E. Maxwell, Rev. Sci. Instr. **29**, 715 (1958).
[34] H. van Dijk, M. Durieux, J. R. Clement, and J. K. Logan, Physica **24**, S129 (1958).
[35] E. J. Walker, Rev. Sci. Instr. **30**, 834 (1959).

If the temperature gradient can be written $\text{grad}T = D(W-W_0)^n$, it can easily be shown that, for a channel of length l

$$\text{grad}T \approx \frac{\Delta T}{l} - \frac{(\Delta T)^2}{2l}\frac{\partial \ln D}{\partial T}, \tag{7}$$

provided the temperature dependence of W_0 and n can be neglected. This correction was evaluated by determining D from the uncorrected data, finding $\partial \ln D/\partial T$, and then verifying that a further stage of approximation was unnecessary. It is important both near the λ point and at the lowest temperatures, amounting to more than 10% at the largest heat-current densities used.

Critical heat-current densities were measured by two independent methods. Below about 1.7°K and within a few hundredths of a degree of the λ point, W_c could be observed directly as a sharp break in the curves of $\text{grad}T$ as a function of W. Measurements were also made over the entire temperature range by a method due to Vinen.[14] This method is based on the observation that, when a heat current is suddenly switched on, the equilibrium temperature gradient is established only after a delay time τ, of the order of 0.1 to 60 sec, which is a function of heat current and temperature. Vinen ascribed this delay time to the slow growth of turbulence in the superfluid, taking the form of a tangled mass of vortex line, and was able to write equations satisfactorily describing the process. Now, if there is initially present a small amount of such turbulence, perhaps as a result of the presence of a small heat current, τ is greatly reduced because the slow, early stages of growth are eliminated. τ thus serves as a sensitive indicator of the presence of amounts of turbulence too small to detect by ordinary methods. For convenience, τ is defined as the time required for the temperature gradient to attain one-half its final value, a convention originated by Vinen and retained here.

The experiment consists of the following sequence of operations, which were carried out automatically with a simple timing device and relays: (i) the helium is left completely undisturbed for about three minutes; (ii) a small heat current W_1 is switched on; (iii) about ninety seconds later, the heat current is increased to a moderately large value W_2, and τ is measured. If these operations are repeated for a number of values of W_1 and with the same value of W_2 in each case, a curve is obtained like that shown in Fig. 1. Sufficiently small values of W_1 have little or no effect upon τ, but when a certain value of W_1 is reached, τ suddenly drops to a much smaller value. This drop in τ is interpreted as an increase in the turbulence of the liquid, and W_c is defined as the value of W_1 at which it occurs. The drop in τ at W_c is sharpest at the lowest temperatures. As the temperature is raised, τ starts to fall gradually at values of $W_1 < W_c$, until eventually the sharp break disappears altogether and τ becomes a smoothly decreasing function of W_1. This behavior is in complete agreement with results previously reported by Vinen.[14] However, one additional fact was noted during the present work, which was apparently not observed by Vinen: For nonzero values of W_1 appreciably less than W_c, τ is usually larger than it is for $W_1=0$. This is evident in Fig. 1. The reason for this behavior is unknown. It was not usually necessary to measure the complete dependence of τ on W_1 to find W_c, and often only a few points were taken in the transition region.

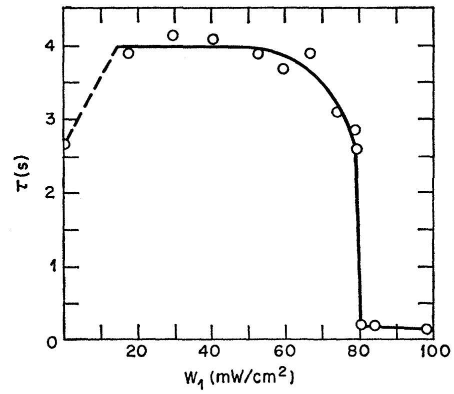

FIG. 1. Delay time as a function of W_1 at 1.40°K; $W_2 = 160$ mW/cm².

According to Vinen's point of view, a true critical heat current is present only when the behavior of $\tau(W_1)$ is actually discontinuous; accordingly, he reported values of W_c only below about 1.5°K. In the present channel, which was smaller than Vinen's, the critical heat current remained sharp up to much higher temperatures. Moreover, direct observations showed that the rise in $\text{grad}T$ at W_c is never actually discontinuous, even at the lowest temperatures, so there appears to be no reason to maintain this distinction. Values of W_c reported here below about 1.5°K correspond to a sharp break in $\tau(W_1)$, although even below this temperature some decrease in τ was noted before the sharp break occurred. At higher temperatures, where τ decreased smoothly over a considerable range of values of W_1, it was found experimentally that the critical heat current determined by direct observation of $\text{grad}T$ corresponded to the *bottom* of this decline, where τ generally leveled off at a value of about 0.2 sec. This point has therefore been called W_c in the present experiments, although it is realized that consistency requires the assumption of a considerable amount of subcritical turbulence in such a case. It appears from this that turbulence must set in gradually, perhaps starting at a very small heat current, and only has a measurable effect on the temperature gradient when it is almost fully developed.

In the above discussion the term "turbulence" has been used loosely to describe whatever process may be responsible for the behavior of τ and the sharp appearance of additional thermal resistance at W_c. In what follows it will be suggested that, at least at some

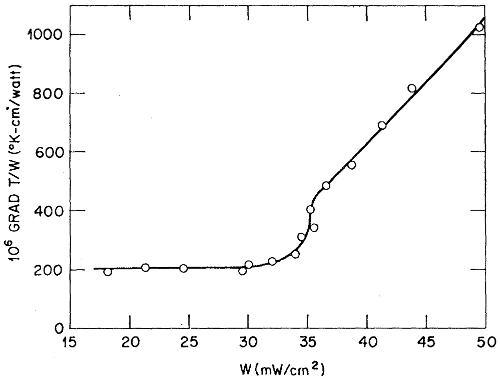

FIG. 2. Thermal resistance, (grad$T$)/W, as a function of heat current density at 1.15°K.

temperatures, this turbulence may occur in the normal fluid or the whole liquid instead of in the superfluid as Vinen supposes. It is, therefore, worthwhile to emphasize that Vinen's method of determining W_c is valid whatever the nature of the underlying mechanism, as is demonstrated by comparison with the results of direct observations. Also, although the present measurements suggest that the transition observed by Vinen is associated with some form of normal-fluid turbulence, they do not exclude the possibility that a smaller critical heat current also exists, associated perhaps with the first appearance of what has been referred to above as subcritical turbulence.

C. Sources of Error

Heat-current densities were measured with an estimated relative accuracy of $\pm 1\%$, but absolute values may be subject to an additional error of 2 or 3% as a result of errors in measuring the diameter of the channel. Values of grad$T$ were determined within about $\pm 10^{-7}$ °K/cm or $\pm 2\%$, whichever is greater; the latter figure is due chiefly to the inaccuracy of calibration of the thermometer. Less than $10^{-3}\%$ of the heat current is carried by the channel walls, even at the largest heat currents. The bath temperature was measured within about ± 0.002°K above 1.2°K. Below 1.2°K the precision of temperature measurement was much poorer, and errors may be as large as ± 0.02°K.

The precision of measurements of W_c by the Vinen technique was determined primarily by the number of points taken in the transition region, and was in general about $\pm 2\%$. In cases where the transition was very broad the uncertainties are larger, and are shown by error bars through the experimental points.

A further possible source of error arises from superfluid flow through the narrow annular gaps surrounding the heater and thermometer leads, if these gaps are not completely sealed. Since the cross-sectional area of these gaps will be negligibly small compared to that of the channel, the effect on grad$T$ at values of $W > W_c$ should be negligible; however, the observed value of W_c may be increased because the critical velocity in the narrow gap will be much larger than that in the channel proper. If $v_n = 0$ within the gap and the condition that there be no net mass transfer is applied to the parallel combination of channel and gap, it is easily shown that the observed critical heat current is greater than the true value by the factor $(1 + v_c' A'/v_c A)$, where A' and A are the cross-sectional areas of the gap and the channel, respectively, and v_c' and v_c are the corresponding critical velocities. It turns out that, for purposes of this calculation, it makes no difference whether the relevant critical velocity is v_s or $v_s - v_n$. (If v_n is the critical quantity the correction vanishes, since it has been assumed that within the gap $v_n = 0$). The magnitude of v_c' can be estimated from the measurements in channels of various geometries to be reported in II. The resulting estimated correction amounts to less than 3%, and is therefore probably negligible. In II, this question is investigated experimentally by introducing the leads in another manner.

Vinen[14] has pointed out that, because of the finite length of the channels used in his experiments, the observed value of W_c may be larger than the true value. This difficulty arises because, during the time τ taken for turbulence to build up in the liquid, the superfluid moves a distance $v_s\tau$ down the channel. If $v_s\tau$ is greater than the channel length l, superfluid turbulence will never be fully developed. Since τ apparently rises to infinity[14] at W_c, it is clear that $v_s\tau$ will always exceed l for some value of $W > W_c$. The quantity supposedly measured by the Vinen technique is thus the heat current W_c' at which $v_s\tau \approx l$. Vinen concluded that in his case ($l = 10$ cm) the difference between W_c' and W_c was always negligible. In the present experiments ($l = 5$ cm) the error might be more important, but is unlikely to amount to more than a few percent. It is impossible to assess this source of error more precisely without knowledge of the variation

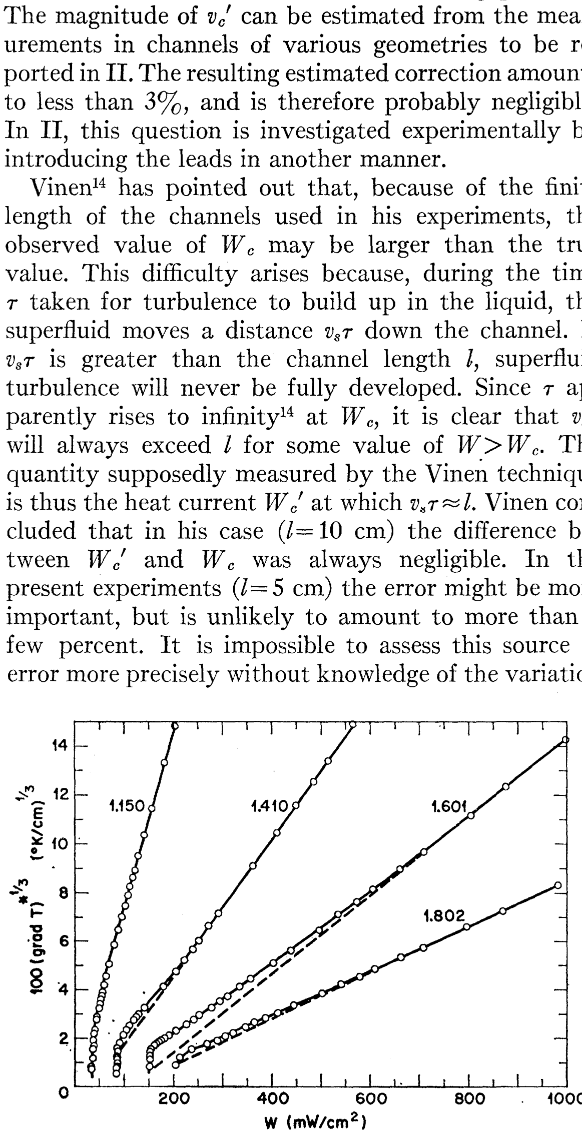

FIG. 3. (grad$T$)$^{*\frac{1}{3}}$ as a function of heat current density. (grad$T$)* is the temperature gradient minus the contribution due to the viscosity of the normal component. Broken straight lines are drawn for comparison only, and have no theoretical significance. Numbers attached to the curves are the temperature in °K.

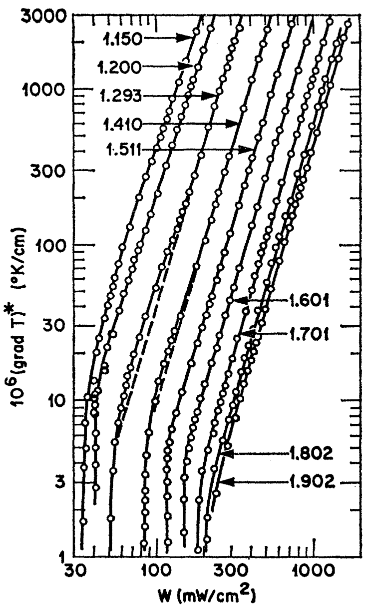

FIG. 4. (grad$T$)* as a function of heat current density. Numbers attached to the curves are the temperature in °K.

of $\tau(W)$ just above W_c. If critical velocities other than v_s are involved, it may be necessary to consider other length effects as well.[36]

III. RESULTS

A. The Thermal Conductivity

Figure 2 shows the thermal resistance (grad$T$)/W as a function of heat current density at 1.15°K, the lowest temperature at which detailed measurements were made. Up to about 33 mW/cm² the thermal resistance is constant, corresponding to the normal fluid viscosity η_n. W_c occurs near 35 mW/cm², and the thermal resistance subsequently rises rapidly. Although eventually the thermal resistance is approximately quadratic in W, this initial rise is approximately linear. It should be noted that W_c does not appear to represent an actual discontinuity in thermal resistance, for it was possible to maintain the temperature gradient in equilibrium by a suitable choice of heat current everywhere throughout the region. Moreover, no hysteresis was observed when a given value of W was approached from opposite directions. The behavior at other temperatures is similar, but the viscous contribution falls rapidly with increasing temperature and was unobservably small above 1.6°K. Between 1.7°K and a temperature a few hundredths of a degree below the λ point, the thermal resistance in the neighborhood of W_c was too small to detect, and direct observations of W_c were accordingly impossible.

These results can be most conveniently compared with the cube-law relation [Eq. (4)] by plotting (grad$T$)$^{*\frac{1}{3}}$ as a function of W, where (grad$T$)* is the temperature gradient minus the contribution due to the normal viscosity η_n. On such a plot the data should lie on a straight line of slope $D^{\frac{1}{3}}$ and intercept W_0. A few of the present data are plotted in this way in Fig. 3. It can be seen that, except at the lowest temperature, considerable departures from linearity occur. The broken straight lines in the figure are drawn for comparison only, and have no theoretical significance. Vinen[14] noted departures from linearity only near the λ point, while in the present case considerable deviations are evident down to 1.4°K. If these deviations are ignored and the best straight line is drawn through the data, the results are in approximate agreement with those of Vinen. It should be noted that most investigators[2,6,13,14] have claimed only approximate agreement with Eq. (4).

The failure of Eq. (4) to describe the results prompted a search for other empirical formulas which might fit the data better. The simplest suitable expression is of the form

$$(\mathrm{grad}T)^* = DW^n, \tag{8}$$

which is illustrated by the logarithmic plot shown in Figs. 4 and 5. All the data above W_c fit this expression within experimental error except between about 1.3 and 1.5°K, where small "bumps" are evident, and at 1.15°K, where the points just above W_c are low. It is particularly striking that this expression holds up to at least 2.162°K, only 0.011°K below T_λ. The "bumps" on the curves at 1.293 and 1.410°K might be interpreted as a change in slope at a heat current density of about 170 mW/cm²; this behavior will be discussed in more detail later.

The temperature dependence of the exponent n in Eq. (8) is shown in Fig. 6. This exponent is nearly constant above 1.7°K and roughly equal to 3.5; at lower temperatures it falls gradually to a value near 3. Detailed measurements were not made below 1.15°K, but it appears qualitatively that n remains in the vicinity of 3 there.

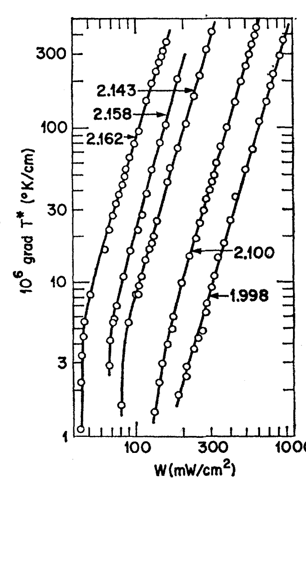

FIG. 5. (grad$T$)* as a function of heat current density near the λ point. Numbers attached to the curves are the temperature in °K.

[36] The importance of length effects was pointed out to the author independently by R. Meservey and S. M. Bhagat.

B. The Critical Heat Current

The temperature dependence of the critical heat-current density W_c is shown in Fig. 7. The results of direct measurements (shown by squares) are in complete agreement with those obtained by the Vinen method (circles). This justification of the latter method is important, because in larger channels W_c is so small that it cannot be observed directly. In II and III the Vinen method is used exclusively for measuring W_c. The points below 1.15°K were obtained in a single run using a booster diffusion pump; in this region the temperature measurements are rather less reliable.

If W_c is plotted against ρ_n instead of against T, the resulting graph is a straight line below 1.4°K intersecting the ordinate at $W_{c0}\approx 17$ mW/cm². While this may be evidence for a nonzero critical heat-current density at absolute zero, such an extrapolation should be regarded with considerable caution. It may be of interest to point out that W_c cannot remain nonzero right down to 0°K, because the normal-fluid velocity increases with decreasing ρ_n and would eventually be limited by the velocity of sound. At a heat current density of 17 mW/cm², this would occur at $T\approx 0.16$°K. Mean-free-path effects would further complicate the situation at these temperatures.

Figure 7 shows a pronounced "knee" at 1.7°K, about the same temperature as the change in the exponent n shown in Fig. 6. These characteristics suggest that some fundamental change in the thermal conduction process may occur at this temperature. This suggestion is not a new one. As early as 1939, Allen and Ganz[8] discovered that the pressure dependence of the thermal conductivity (measured at a constant temperature gradient of 10^{-3} °K/cm) changed sign at 1.63°K. Keesom and Saris[5] and Keesom, Saris, and Meyer[6] further found that the temperature dependence of the thermal conductivity (again for a constant temperature gradient) changed markedly at the same temperature, and that above about 1.6°K the thermal conductivity depended slightly on the diameter of the channel, being somewhat smaller in larger capillaries. This remarkable result appears to have received little

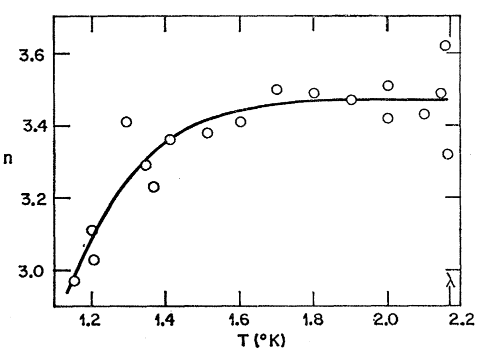

FIG. 6. Temperature dependence of the exponent n in the relation $(\mathrm{grad}T)^*=DW^n$.

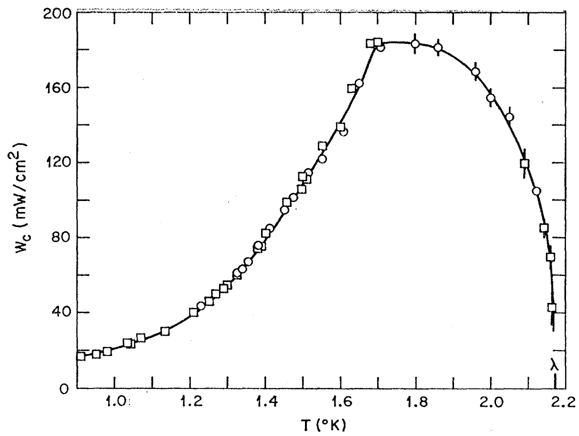

FIG. 7. Critical heat current as a function of temperature. □ Points observed directly; ○ Points obtained by the Vinen technique.

further attention, although Vinen[14] noted that the value of the mutual friction constant A increased more rapidly near the λ point in his experiments (which were carried out in very large channels) than it did in Gorter and Mellink's.[2] The present results shed additional light on the nature of the change taking place at this temperature; this will be discussed in the following section.

IV. DISCUSSION

A. Critical Heat Current Below 1.7°K

Let us first consider the possibility that the critical heat current is the result of some sort of turbulence of an essentially classical nature. Similar ideas have often been suggested,[14,20–24,26,29,37] but it has usually been assumed that the turbulence is associated with the superfluid component, perhaps taking the form of a tangled mass of vortex lines.[14] In ordinary liquids the transition from laminar to turbulent flow, which has been studied in great detail,[38] can be characterized by the dimensionless Reynolds number $\mathcal{R}=\rho vd/\eta$, where ρ is the density, v the velocity averaged over the channel cross section, d the channel diameter, and η the viscosity. When $\mathcal{R}\lesssim 2300$ the flow is always laminar, but for higher values of $\mathcal{R}$ turbulence will usually develop if the liquid is disturbed in any way. By taking great precautions to avoid vibration or stirring of the liquid it is possible to maintain laminar flow up to much higher Reynolds numbers, and no upper limit to this process has yet been found. The critical Reynolds number $\mathcal{R}_c\approx 2300$ therefore corresponds to the lower limit below which laminar flow always occurs, but turbulence will not necessarily develop when $\mathcal{R}_c$ is exceeded.

In liquid helium it is formally possible to define several different Reynolds numbers. If the superfluid

[37] J. G. Dash, Phys. Rev. **94**, 825, 1091 (1954).

[38] See, for example, H. Schlichting, *Boundary Layer Theory* (McGraw-Hill Book Company, Inc., New York, 1960), 4th ed.

viscosity $\eta_s=0$, the superfluid Reynolds number $\mathcal{R}_s=\rho_s v_s d/\eta_s$ must be infinite whenever $v_s\neq 0$, and one would expect superfluid turbulence to develop immediately if it were not for the restriction $\mathrm{curl}\mathbf{v}_s=0$, which prohibits the development of rotational motion in the superfluid unless a large excitation energy is available.[39] It has long been known[1] that the usual normal-fluid Reynolds number $\mathcal{R}_n=\rho_n v_n d/\eta_n$ is much too small to account for the observed W_c. The two most obvious quantities that are of the correct order of magnitude are therefore the numbers

$$\mathcal{R}_1=\rho d|\mathbf{v}_n-\mathbf{v}_s|/\eta_n \quad \text{and} \quad \mathcal{R}_2=\rho d|\mathbf{v}_n|/\eta_n \tag{9}$$

involving the normal-fluid velocity or the relative velocity but the *total* density. Staas, Taconis, and van Alphen[19] were able to describe their results in terms of a critical value of $\mathcal{R}_2$ equal to 1300, both in the case of pure heat conduction and when the normal and superfluid moved together. Donnelly and Hollis Hallett[40] have analyzed a number of experiments with oscillating systems, in which two critical amplitudes are found; in every case the larger critical velocity could be described in terms of $\mathcal{R}_2$. The use of such a quantity to describe critical velocities has also been discussed by Meservey.[29] At first glance it is puzzling that a Reynolds number involving the total density can be involved when the normal and superfluid components are flowing in opposite directions; the success of this quantity in explaining the results of Staas *et al.*[19] presumably implies that the two components are coupled together in some way even though their average velocities are oppositely directed. The resulting turbulence probably involves the entire liquid, but it is not inconceivable that only the normal fluid becomes turbulent even though the inertial term in the Reynolds number involves the total density. For convenience, this condition will be referred to as normal turbulence, although it is understood that the whole fluid may be involved.

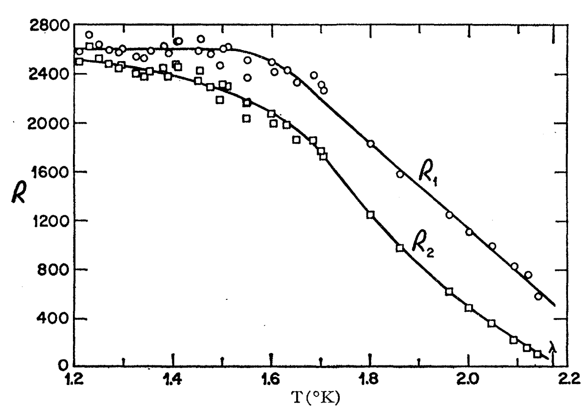

FIG. 8. Temperature dependence of the critical Reynolds numbers. $\mathcal{R}_1=\rho d|\mathbf{v}_n-\mathbf{v}_s|/\eta_n$; $\mathcal{R}_2=\rho d|\mathbf{v}_n|/\eta_n$.

[39] L. D. Landau, J. Phys. U.S.S.R. **5**, 71 (1941).

[40] R. J. Donnelly and A. C. Hollis Hallett, Ann. Phys. **3**, 320 (1958).

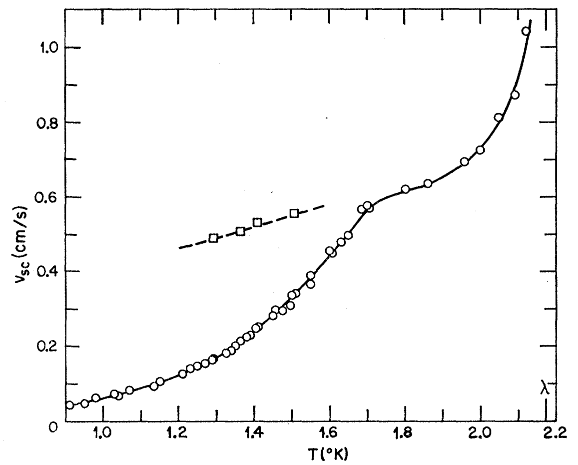

FIG. 9. Temperature dependence of the critical superfluid velocity. ○ Obtained from the results of Fig. 7; □ Locus of break in curves of Fig. 4.

Values of $\mathcal{R}_1$ and $\mathcal{R}_2$ calculated from the present data using the viscosity measurements of Brewer and Edwards[17] are shown as a function of temperature in Fig. 8. Data below 1.2°K have been omitted because the large uncertainty in the temperature measurements makes the results unreliable there. Below about 1.6°K, $\mathcal{R}_1$ is remarkably constant at a value close to 2600; $\mathcal{R}_2$ is also reasonably constant there, although there is a noticeable decrease with increasing temperature. The close resemblance of this behavior to that of ordinary liquids suggests strongly that the critical heat current in this region is associated with the normal turbulence described above. Moreover, measurements of $\mathrm{grad}T$ in the subcritical region showed that there is no appreciable increase in temperature gradient above that due to viscosity until W_c is reached. It therefore appears that, if any additional frictional forces appear at heat currents lower than that corresponding to this turbulent transition, their effects on the temperature gradient must be small.

It may be noted that the condition $\mathcal{R}=\text{const}$ leads to a critical velocity varying as $1/d$, as is found in a variety of experiments in channels wider than about 10^{-3} cm.[41] It is tempting to suppose that many of the critical heat currents observed in such wide channels may in fact be due to normal turbulence, at least at low temperatures, although most investigators have previously interpreted their results in terms of *superfluid* turbulence of some kind.[14,20–24,26]

B. Critical Heat Current Above 1.7°K

Above 1.7°K both $\mathcal{R}_1$ and $\mathcal{R}_2$ decrease rapidly with increasing temperature, and it is clear that some other mechanism must be invoked to explain the observed critical heat current. This may in fact be a critical

[41] See, e.g., K. R. Atkins, *Liquid Helium* (University Press, Cambridge, England, 1959), p. 199.

superfluid velocity,[42] which is shown as a function of temperature in Fig. 9. (Here v_{sc} is defined as the superfluid velocity averaged over the channel cross section.) If the rapid drop below 1.7°K, which presumably results from the intervention of normal turbulence, is ignored, it appears that v_{sc} falls rather rapidly as the temperature is lowered from the λ point and then levels off around 1.8°K at a value of approximately 0.6 cm/sec. It is possible that v_{sc} remains near this value at lower temperatures, and would be observed there if the onset of normal turbulence could be postponed to sufficiently high Reynolds numbers. In fact, this is similar to the behavior observed by Brewer and Edwards[24] and ascribed by them to hysteresis. It is significant that they observed this hysteresis only below 1.7°K.

In Sec. III A it was noted that the curves of $(\mathrm{grad}T)^*$ as a function of W between 1.3 and 1.5°K showed a small change in slope at about 170 mW/cm^2. The superfluid velocity corresponding to the locus of this change in slope is shown in Fig. 9 by the squares and the broken line. (Two of these points were obtained from similar curves which were omitted from Fig. 4 for clarity.) Although these data are not very accurate, the fact that they lie on a plausible extrapolation of the upper part of the curve of $v_{sc}(T)$ suggests that they might represent the critical superfluid velocity in the region below 1.7°K. This possibility, however, is suggested only tentatively, because it is not entirely clear why such an effect should be observed. Indeed, it appears likely that the whole liquid is always turbulent in the supercritical region, and that it is only the mechanism governing the initiation of turbulence which is different at high and low temperatures. This viewpoint is not necessarily inconsistent with the ideas that turbulence in the superfluid involves a tangle of vortex lines and that mutual friction arises from the interaction of these vortex lines with the normal fluid.

V. CONCLUSIONS

The thermal conductivity of liquid helium II contained in a channel 0.080 cm in diameter has been shown to be well described by the relation $\mathrm{grad}T = DW^n$, where D is a temperature-dependent constant, at heat current densities greater than the critical heat current density W_c. The exponent n is approximately 3.5 above 1.7°K, and below that temperature falls to the vicinity of 3.0. The critical heat current passes through a broad maximum just above 1.7°K, and has a sharp change in slope at that temperature.

These results suggest that the mechanism responsible for W_c may be different at temperatures above and below 1.7°K. At temperatures below this value, the Reynolds numbers involving the normal-fluid velocity or the relative velocity together with the total fluid density are nearly independent of temperature and are about equal to 2600; this fact suggests that W_c in this region may be due to normal-fluid turbulence. At higher temperatures, on the other hand, these Reynolds numbers are much smaller, and some other explanation is required. It is suggested that the results in this region may be explained by a critical superfluid velocity v_{sc} such as is usually assumed to apply at lower temperatures as well. This critical superfluid velocity may persist to lower temperatures, but if so, it is normally obscured by the transition occurring at $\mathcal{R} = 2600$. Some support for this suggestion is offered by the slight change in slope of the curves of $\mathrm{grad}T$ as a function of W which is observed between roughly 1.3 and 1.5°K at a superfluid velocity of about 0.6 cm/sec. Presumably, the whole fluid becomes turbulent once W_c is exceeded at all temperatures, and it is only the mechanism responsible for the initiation of turbulence which is different above and below 1.7°K.

These measurements do not exclude the possibility of a smaller critical velocity than that reported here (perhaps associated with the initial decrease in the equilibrium time τ at heat current densities less than W_c, which is observed at all but the lowest temperatures). However, below 1.6°K the temperature gradient near W_c was directly observed, and was found not to rise significantly above that to be expected from the known viscosity of the normal component until W_c was reached. Therefore, if such a lower critical velocity exists, no appreciable increase in thermal resistance occurs when it is exceeded.

ACKNOWLEDGMENTS

The author is grateful to Dr. E. Maxwell, Dr. R. Meservey, and J. C. Fineman for many useful discussions. The importance of the Reynolds number in the present connection was first pointed out to the author by Dr. Meservey.

[42] Logically, it is equally possible that it is the *relative* velocity which is the critical quantity above 1.7°K. This quantity, however, has an appreciably greater temperature dependence, and is rising rapidly with decreasing temperature near 1.7°K.