(8) On the basis of the susceptibility measurements, the range of impurity concentrations in germanium may be divided into at least three parts. A close correspondence between the dependence of the low-temperature electrical resistance and the magnetic susceptibility on donor concentrations was observed. This division may be associated, at least qualitatively, with the strength of the donor interactions.

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Uniformly Moving Dislocations of Arbitrary Orientation in Anisotropic Media*

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The dynamical behavior of uniformly moving dislocations in anisotropic media is considered for the general case in which the dislocation involves three components of displacement. It is found that both edge and screw dislocations can display an anomalous behavior. It appears that, in general, the force of interaction between two parallel dislocations on the same slip plane changes sign with increasing dislocation velocity; this result obtains whether the dislocations involved be two edges, two screws, or an edge and a screw. The threshold velocity at which the force of interaction changes sign is a function of the orientation and type of dislocation. However, the limiting velocity, at which the energy of the dislocation becomes infinite, is a function only of the orientation of the dislocation and is the same whether the dislocation be pure edge, pure screw, or mixed in character. Numerical results are presented for $(\bar{1}10)$ $[11\bar{1}]$ dislocation motion in lithium.

I. INTRODUCTION

There has been a renewal of interest in the dynamical properties of dislocations since Weertman[1] pointed out that in isotropic materials high-speed dislocations of like sign attract rather than repel one another. The velocity range for this anomalous behavior extends from the Rayleigh wave velocity to the shear wave velocity; the latter is the limiting velocity of the edge dislocation since its energy becomes infinite at that velocity. Screw dislocations, however, behave "normally" at all velocities up to their limiting velocity, which also is the velocity of shear sound.

The dynamical behavior of dislocations in anisotropic media was considered previously by this author[2,3] for those orientations of the dislocation for which the problem could be treated as one of plane strain, i.e., for those orientations for which a pure edge dislocation requires only two components of elastic displacement, a pure screw dislocation only one. Again it was found that the force field of an edge dislocation changes sign at some velocity below its limiting velocity, whereas a screw dislocation is well-behaved at all possible velocities. However, in the anisotropic case the threshold velocity for the anomalous behavior of edge dislocations (the generalized Rayleigh wave velocity) can be any velocity from zero to the limiting velocity, depending on the elastic constants of the material and the orientation considered. The limiting velocity of a screw dislocation is different from that of an edge for these orientations, and it is possible for the limiting velocity of an edge dislocation to be less than the corresponding shear sound velocity.

Specific orientations of moving edge dislocations in cubic materials for which the plane strain analysis is not applicable have been treated by Weertman and coworkers.[4,5,6,7] This paper presents an analysis of the dynamical behavior of a uniformly moving dislocation of arbitrary orientation in any anisotropic elastic medium.

II. UNIFORMLY MOVING DISLOCATIONS

The equations of equilibrium for an anisotropic elastic medium are

$$F_{ijkl}u_{k\cdot jl}=\rho\ddot{u}_i, \tag{1}$$

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[1] J. Weertman, *Response of Metals to High-Velocity Deformation*, edited by P. G. Shewmon and V. F. Zackay (Interscience Publishers, Inc., New York, 1961).

[2] L. J. Teutonico, Phys. Rev. **124**, 1039 (1961).

[3] L. J. Teutonico, Phys. Rev. **125**, 1530 (1962).

[4] J. Weertman, Phil. Mag. **7**, 617 (1962).

[5] J. Cotner and J. Weertman, Acta Met. **10**, 515 (1962).

[6] J. Weertman, J. Appl. Phys. **33**, 1631 (1962).

[7] A. Van Hull and J. Weertman, J. Appl. Phys. **33**, 1636 (1962).

where u_i is the displacement referred to Cartesian coordinates x_i, ρ is the density of the medium, F_{ijkl} the elastic constant tensor, and all subscripts to the right of the dot represent differentiation with respect to the appropriate space coordinate. We assume the medium contains a dislocation which is parallel to x_3 and moving with velocity c in the x_1 direction on the slip plane $x_2=0$. This problem has been considered by Bullough and Bilby,[8] but they restrict their analysis to those orientations for which the edge and screw components can be considered separately, i.e., to those orientations for which plane strain applies. For an arbitrary orientation in an anisotropic medium both a pure edge and a pure screw involve three components of displacement. The solution of (1) can be found by extending the surface wave method of Bullough and Bilby. However, since time enters the problem only through $x_1'=x_1-ct$, (1) can be rewritten in the form of the equations of static equilibrium:

$$G_{ijkl}u_{k\cdot jl}=0, \tag{2}$$

where

$$G_{ijkl}=F_{ijkl}-\rho c^2\delta_{ik}\delta_{1j}\delta_{1l} \tag{3}$$

and δ_{ik} is the Kronecker delta. Hence, it was found simpler to extend to the dynamic case the analysis of stationary dislocations by Eshelby, Read, and Shockley.[9] From the latter work it is evident that the solution of (1) can be written in the form

$$u_k=\frac{b_0}{2\pi}\sum_{n=1}^{6} B_{kn}\ln z_n, \tag{4}$$

where

$$z_n=x_1'+i\lambda_n x_2. \tag{5}$$

B_{kn} and λ_n are complex constants to be determined, and b_0 is the magnitude of the Burgers vector. Substitution of (4) into (1) yields[10]

$$\sum_{n=1}^{6} z_n^{-2}B_{kn}\Phi_{ikn}=0, \tag{6}$$

where

$$\Phi_{ikn}=\Phi_{kin}=-F_{i2k2}\lambda_n^2+i\lambda_n(F_{i1k2}+F_{i2k1})+F_{i1k1}-\rho c^2\delta_{ik}. \tag{7}$$

In order that (6) be true for all (x_1',x_2), we must have that

$$B_{kn}\Phi_{ikn}=0 \quad \text{(no sum on } n\text{)}. \tag{8}$$

The set of equations given by (8) will have a nontrivial solution for the B_{kn} only if

$$\det\Phi_{ikn}=0. \tag{9}$$

From this last equation the various λ_n's are determined. Setting $y_n=-i\lambda_n$, one obtains a sextic,

$$\sum_{r=0}^{6} K_r y_n^r=0, \tag{10}$$

in which

$$\begin{aligned}
K_6={}&F_{66}F_{22}F_{44}+2F_{26}F_{46}F_{24}-F_{66}F_{24}^2-F_{22}F_{46}^2-F_{44}F_{26}^2,\\
-\tfrac{1}{2}K_5={}&F_{66}F_{22}F_{45}+F_{26}F_{46}F_{25}+F_{26}F_{24}F_{14}+F_{26}F_{24}F_{56}+F_{16}F_{22}F_{44}+F_{46}F_{24}F_{12}\\
&-F_{66}F_{24}F_{25}-F_{22}F_{46}F_{14}-F_{22}F_{46}F_{56}-F_{44}F_{26}F_{12}-F_{16}F_{24}^2-F_{45}F_{26}^2,\\
K_4={}&F_{66}F_{22}F_{55}+F_{11}F_{22}F_{44}-F_{66}F_{25}^2-F_{22}F_{14}^2-F_{22}F_{56}^2-F_{44}F_{12}^2-F_{11}F_{24}^2-F_{55}F_{26}^2\\
&+2(F_{26}F_{14}F_{25}+F_{26}F_{56}F_{25}+F_{26}F_{15}F_{24}+F_{46}F_{12}F_{25}+F_{16}F_{26}F_{44}+F_{24}F_{12}F_{14}+F_{24}F_{12}F_{56}\\
&+F_{24}F_{66}F_{14}+F_{12}F_{46}^2-F_{22}F_{14}F_{56}-F_{44}F_{12}F_{66}-F_{22}F_{15}F_{46}-F_{16}F_{24}F_{46}-F_{26}F_{46}F_{14})\\
&+4(F_{16}F_{22}F_{45}-F_{16}F_{24}F_{25}-F_{45}F_{26}F_{12})-\mu(F_{66}F_{22}+F_{44}F_{66}+F_{22}F_{44}-F_{24}^2-F_{46}^2-F_{26}^2),\\
-\tfrac{1}{2}K_3={}&F_{26}F_{15}F_{25}+F_{16}F_{22}F_{55}+F_{15}F_{24}F_{12}+F_{15}F_{24}F_{66}+F_{11}F_{22}F_{45}+F_{11}F_{26}F_{44}+F_{16}F_{24}F_{14}+F_{12}F_{14}F_{46}\\
&+F_{12}F_{14}F_{25}+F_{12}F_{56}F_{25}+F_{66}F_{14}F_{25}-F_{22}F_{15}F_{14}-F_{22}F_{15}F_{56}-F_{44}F_{16}F_{12}-F_{16}F_{56}F_{24}-F_{26}F_{15}F_{46}\\
&-F_{11}F_{24}F_{46}-F_{11}F_{24}F_{25}-F_{55}F_{26}F_{12}-F_{16}F_{46}F_{25}-F_{16}F_{25}^2-F_{26}F_{14}^2-F_{26}F_{14}F_{56}-F_{45}F_{12}^2\\
&+2(F_{46}F_{56}F_{12}+F_{16}F_{26}F_{45}-F_{45}F_{12}F_{66})-\mu(F_{66}F_{45}+F_{16}F_{22}+F_{44}F_{16}+F_{22}F_{45}\\
&+F_{26}F_{44}-F_{24}F_{46}-F_{24}F_{25}-F_{46}F_{14}-F_{46}F_{56}-F_{26}F_{12}),\\
K_2={}&F_{11}F_{22}F_{55}+F_{11}F_{44}F_{66}-F_{22}F_{15}^2-F_{44}F_{16}^2-F_{11}F_{46}^2-F_{11}F_{25}^2-F_{66}F_{14}^2-F_{55}F_{12}^2+2(F_{16}F_{26}F_{55}\\
&+F_{56}F_{12}F_{14}+F_{15}F_{12}F_{46}+F_{15}F_{12}F_{25}+F_{15}F_{66}F_{46}+F_{15}F_{66}F_{25}+F_{16}F_{14}F_{46}+F_{16}F_{14}F_{25}+F_{16}F_{15}F_{24}\\
&+F_{12}F_{56}^2-F_{16}F_{56}F_{25}-F_{15}F_{26}F_{56}-F_{24}F_{56}F_{11}-F_{46}F_{15}F_{66}-F_{46}F_{25}F_{11}-F_{12}F_{66}F_{55})\\
&+4(F_{26}F_{45}F_{11}-F_{15}F_{26}F_{14}-F_{16}F_{45}F_{12})-\mu[(F_{55}F_{66}+F_{22}F_{55}+F_{11}F_{22}+F_{44}F_{66}\\
&+F_{11}F_{44}-F_{46}^2-F_{25}^2-F_{14}^2-F_{56}^2-F_{12}^2)+4F_{45}(F_{16}+F_{26})+2(F_{16}F_{26}-F_{24}F_{56}-F_{46}F_{15}-F_{46}F_{25}\\
&-F_{14}F_{56}-F_{12}F_{66})]+\mu^2(F_{66}+F_{22}+F_{44}),\\
-\tfrac{1}{2}K_1={}&F_{15}F_{56}F_{12}+F_{11}F_{55}F_{26}+F_{16}F_{56}F_{14}+F_{45}F_{11}F_{66}+F_{15}F_{16}F_{46}+F_{15}F_{16}F_{25}-F_{26}F_{15}^2-F_{45}F_{16}^2-F_{56}F_{11}F_{46}\\
&-F_{56}F_{11}F_{25}-F_{15}F_{66}F_{14}-F_{16}F_{55}F_{12}-\mu(F_{55}F_{16}+F_{55}F_{26}+F_{11}F_{26}+F_{45}F_{66}+F_{45}F_{11}-F_{56}F_{46}-F_{56}F_{25}\\
&-F_{15}F_{14}-F_{15}F_{56}-F_{16}F_{12})+\mu^2(F_{16}+F_{26}+F_{45}),\\
K_0={}&F_{11}F_{66}F_{55}+2F_{16}F_{56}F_{15}-F_{11}F_{56}^2-F_{66}F_{15}^2-F_{55}F_{16}^2-\mu(F_{11}F_{66}+F_{11}F_{55}+F_{55}F_{66}-F_{56}^2-F_{15}^2-F_{16}^2)\\
&+\mu^2(F_{11}+F_{66}+F_{55})-\mu^3;
\end{aligned} \tag{11}$$

[8] R. Bullough and B. A. Bilby, Proc. Phys. Soc. (London) **B67**, 615 (1954).
[9] J. D. Eshelby, W. T. Read, and W. Shockley, Acta Met. **1**, 251 (1953).
[10] Summation is implied if the subscript i, j, k, or l is repeated; summation over n will be indicated explicitly.

$\mu=\rho c^2$, and we have used the contracted notation for the subscripts on F_{ijkl}, i.e., $ij\rightarrow i$, $i=j$; $ij\rightarrow k+3$, $i\neq j$. Since all the K_r are real, the roots are of the form $y_n=q_n\mp ip_n$ ($n=1, 2, 3$), where p_n and q_n are real. The vector B_{kn} corresponding to each y_n is in general complex. In order that the displacements u_k given by (4) be real, it is necessary that the imaginary parts of corresponding pairs of solutions shall cancel. Therefore, we need take only three roots, no two of which are complex conjugates, i.e.,

$$y_n=q_n-ip_n,\quad n=1, 2, 3 \tag{12}$$

and write

$$u_k=R\left[\frac{b_0}{2\pi}\sum_{n=1}^{3} B_{kn}\ln z_n\right]. \tag{13}$$

The stress components associated with these displacements are given by

$$\sigma_{ij}=F_{ijkl}u_{k\cdot l}=R\left[\frac{b_0}{2\pi}\sum_{n=1}^{3}\frac{B_{kn}}{z_n}(F_{ijk1}-y_nF_{ijk2})\right]. \tag{14}$$

To determine the B_{kn} we start with Eq. (8). Solving (for a fixed n) for two of them (B_{2n},B_{3n}) in terms of the third (B_{1n}), we obtain

$$B_{2n}/B_{1n}=\Phi_{21n}\Phi_{13n}-\Phi_{11n}\Phi_{23n}/\Phi_{21n}\Phi_{23n}-\Phi_{22n}\Phi_{13n}\equiv\omega_n+i\nu_n, \tag{15}$$

$$B_{3n}/B_{1n}=\Phi_{11n}\Phi_{22n}-\Phi_{21n}{}^2/\Phi_{21n}\Phi_{23n}-\Phi_{22n}\Phi_{13n}\equiv f_n+id_n, \tag{16}$$

where ω_n, ν_n, f_n, d_n are real. The solution of (1) thus reduces to a determination of the real and imaginary parts of the B_{1n}. The six equations required for this are supplied by the conditions for a dislocation: (i) the displacement changes by a constant vector **b** on traversing any circuit enclosing the dislocation, (ii) the resultant force **F** on any surface surrounding the dislocation must vanish.[11] These conditons read

$$\oint u_{i\cdot j}dx_j=b_i, \tag{17}$$

$$F_i=\oint(\sigma_{ij}-\rho c^2u_{i\cdot 1}\delta_{1j})dx_j=0. \tag{18}$$

Setting

$$B_{1n}=\alpha_n+i\epsilon_n \tag{19}$$

(α_n, ϵ_n real), and using Eqs. (12)–(16), we find that conditions (17) and (18) become the following set of six linear equations for the determination of α_n and ϵ_n:

$$\begin{aligned}&\sum_{n=1}^{3}(T_{in}\alpha_n+S_{in}\epsilon_n)=0,\quad i=1,2,3\\&\sum_{n=1}^{3}\epsilon_n=a_1,\\&\sum_{n=1}^{3}(\nu_n\alpha_n+\omega_n\epsilon_n)=a_2,\\&\sum_{n=1}^{3}(d_n\alpha_n+f_n\epsilon_n)=a_3,\end{aligned} \tag{20}$$

where

$$\begin{aligned}T_{in}&=p_n(F_{i212}+\omega_nF_{i222}+f_nF_{i232})\\&\quad+\nu_n(F_{i221}-q_nF_{i222})+d_n(F_{i231}-q_nF_{i232}),\\S_{in}&=(F_{i211}-q_nF_{i212})+\omega_n(F_{i221}-q_nF_{i222})\\&\quad+f_n(F_{i231}-q_nF_{i232})-p_n(\nu_nF_{i222}+d_nF_{i232}),\\a_k&=-b_k/b_0,\quad\sum_{k=1}^{3}a_k{}^2=1.\end{aligned} \tag{21}$$

With α_n, ϵ_n determined from (20) the problem is, in principle, solved.

III. CRITICAL VELOCITIES

Our main interest in considering uniformly moving dislocations in anisotropic media is the determination of certain critical velocities: (i) the limiting velocity, at which the energy of the dislocation becomes infinite, and (ii) the threshold velocity, at which the force between two parallel dislocations on the same slip plane changes sign. Formulas from which these velocities can be obtained are now derived.

A. Limiting Velocity

The energy per unit volume, dE_t, associated with the moving dislocation consists of two parts, a potential energy density $\frac{1}{2}(F_{ijkl}u_{k\cdot l}u_{i\cdot j})$ and a kinetic energy density $\frac{1}{2}(\rho\dot{u}_k\dot{u}_k)$. Utilizing (13) and setting

$$B_{kn}=\alpha_{kn}+i\epsilon_{kn}\quad(\alpha_{kn},\ \epsilon_{kn}\text{ real}),$$

we obtain that

$$dE_t=\frac{b_0{}^2}{8\pi^2}\sum_{n=1}^{3}\sum_{m=1}^{3}H_{ijkl}R_{kln}R_{ijm}, \tag{22}$$

where

$$\begin{aligned}R_{kln}&=x_1'\varphi_{kln}+x_2\psi_{kln}/(x_1'-q_nx_2)^2+(p_nx_2)^2,\\\varphi_{kln}&=\alpha_{kn}(\delta_{1l}-q_n\delta_{2l})-\epsilon_{kn}p_n\delta_{2l},\\\psi_{kln}&=\alpha_{kn}[\delta_{2l}(p_n{}^2+q_n{}^2)-q_n\delta_{1l}]+\epsilon_{kn}p_n\delta_{1l},\end{aligned} \tag{23}$$

and

$$H_{ijkl}=F_{ijkl}+\rho c^2\delta_{ik}\delta_{1l}\delta_{1j}.$$

Integrating (22) over an area normal to x_3, we find that E_t, the energy per unit length of the moving dislocation,

[11] The couple on this surface vanishes for any B_{1n}.

can be written as

$$E_t=\frac{b_0^2}{4\pi}\ln\frac{R_2}{R_1}$$

$$\times\sum_{n=1}^{3}\sum_{m=1}^{3}\frac{H_{ijkl}[\varphi_{kln}\varphi_{ijm}\{p_n(p_m^2+q_m^2)+p_m(p_n^2+q_n^2)\}+\psi_{kln}\psi_{ijm}(p_n+p_m)+(\varphi_{kln}\psi_{ijm}+\varphi_{ijm}\psi_{kln})(p_nq_m+p_mq_n)]}{p_np_m[(p_n+p_m)^2+(q_n-q_m)^2]}, \quad (24)$$

where R_2 represents a dimension of the medium, R_1 a radius of order b_0. From this formula we see that the energy of the dislocation becomes infinite when $p_n=0$ for any n. Therefore, the limiting velocity of the dislocation is the first velocity for which one of the roots of Eq. (10) becomes real. The coefficients in (10) are functions of ρc^2 and the F_{ijkl} only; hence, the limiting velocity is a function only of the orientation of the dislocation, and not the type of dislocation. In the general anisotropic case for which a dislocation involves three components of displacement, its limiting velocity is the same whether it be pure edge, pure screw, or mixed in character.

B. Threshold Velocity

The force per unit length on a dislocation due to a stress field σ_{ij} is[12]

$$F_l=\epsilon_{kjl}b_i\sigma_{ij}t_k, \quad (25)$$

where ϵ_{kjl} is the permutation tensor, b_i are the components of the Burgers vector, and t_k the components of the unit vector tangent to the dislocation. For an infinite dislocation parallel to x_3 we have that

$$F_1=b_i\sigma_{i2},\quad F_2=-b_i\sigma_{i1},\quad F_3=0. \quad (26)$$

We now consider those cases for which the stress field σ_{ij} arises from a uniformly moving dislocation which is parallel to the first dislocation. Specifically we are interested in finding that velocity at which the force between two parallel dislocations on the same slip plane changes sign.

1. *Edge-Edge Interaction*

The force exerted in the x_1 direction by one edge dislocation on another parallel edge is[13] $b_1\sigma_{12}^E(x_1',x_2)$. From (14), we obtain that the shear stress component σ_{12}^E on the slip plane is given by

$$\sigma_{12}^E(x_1',0)=R\left[\frac{b_0}{2\pi x_1'}\sum_{n=1}^{3}B_{kn}^E(F_{12k1}-y_nF_{12k2})\right]. \quad (27)$$

Using (12), (15), (16), and (21), this can be written in the form

$$\sigma_{12}^E(x_1',0)=(b_0/2\pi x_1')W_6^E, \quad (28)$$

where

$$W_6^E=\sum_{n=1}^{3}(\alpha_n^ES_{1n}-\epsilon_n^ET_{1n}) \quad (29)$$

and α_n^E, ϵ_n^E are the solutions of (20) for an edge dislocation, i.e., for $a_1=-1$, $a_2=a_3=0$. The threshold velocity at which the force between two edge dislocations on the same slip plane changes sign is that velocity for which $W_6^E=0$. If this velocity is less than the limiting velocity, then edge dislocations will display an anomalous dynamical behavior.

2. *Screw-Screw Interaction*

The force exerted in the x_1 direction by one screw dislocation on another parallel screw is $b_3\sigma_{23}^S(x_1',x_2)$. From (14), we have that the stress component σ_{23}^S on the slip plane is

$$\sigma_{23}^S(x_1',0)=R\left[\frac{b_0}{2\pi x_1'}\sum_{n=1}^{3}B_{kn}^S(F_{23k1}-y_nF_{23k2})\right]. \quad (30)$$

Using (12), (15), (16), and (21), we can rewrite this last formula as

$$\sigma_{23}^S(x_1',0)=(b_0/2\pi x_1')W_4^S, \quad (31)$$

where

$$W_4^S=\sum_{n=1}^{3}(\alpha_n^SS_{3n}-\epsilon_n^ST_{3n}) \quad (32)$$

and α_n^S, ϵ_n^S are the solutions of (20) for a screw dislocation, i.e., for $a_1=a_2=0$, $a_3=-1$. The threshold velocity for screw dislocations is that velocity for which $W_4^S=0$. In all cases treated previously[1,2] this threshold velocity was found to be equal to the limiting velocity and screw dislocations are always well-behaved; in isotropic materials and for those orientations in anisotropic materials for which plane strain applies, W_4^S does not change sign with dislocation velocity but goes to zero at the limiting velocity. However, in the present analysis the formulas for screw dislocations are similar to those for edge dislocations; since the latter behave anomalously, it seems reasonable to expect that in general the threshold velocity for screws will be different from the limiting velocity, i.e., for those orientations in an anisotropic medium for which a screw dislocation requires three components of displacement it is possible that screw dislocations will also display an anomalous dynamical behavior.

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[12] M. O. Peach and J. S. Koehler, Phys. Rev. **80**, 436 (1950).
[13] Superscript E refers to an edge dislocation, superscript S to a screw dislocation.

3. *Edge-Screw Interactions*

The force exerted in the x_1 direction by an edge dislocation on a parallel screw dislocation is $b_3\sigma_{23}{}^E(x_1',x_2)$; similarly the force exerted by a screw dislocation on a parallel edge is $b_1\sigma_{12}{}^S(x_1',x_2)$. Proceeding as above, we find that on the slip plane,

$$\begin{aligned}\sigma_{23}{}^E(x_1',0)&=(b_0/2\pi x_1')W_4{}^E,\\ \sigma_{12}{}^S(x_1',0)&=(b_0/2\pi x_1')W_6{}^S,\end{aligned}\tag{33}$$

where

$$\begin{aligned}W_4{}^E&=\sum_{n=1}^{3}(\alpha_n{}^E S_{3n}-\epsilon_n{}^E T_{3n}),\\ W_6{}^S&=\sum_{n=1}^{3}(\alpha_n{}^S S_{1n}-\epsilon_n{}^S T_{1n}).\end{aligned}\tag{34}$$

In isotropic materials, $\sigma_{23}{}^E$ and $\sigma_{12}{}^S$ vanish and there is no interaction between parallel edge and screw dislocations. The same is true for those orientations in an anisotropic material for which the edge and screw components can be treated separately. However, for the general anisotropic case in which the dislocation has three components of displacement these stress components do not (in general) vanish and there are interactions between parallel edge and screw dislocations. Also, it appears possible that the interaction forces will change sign with increasing dislocation velocity.

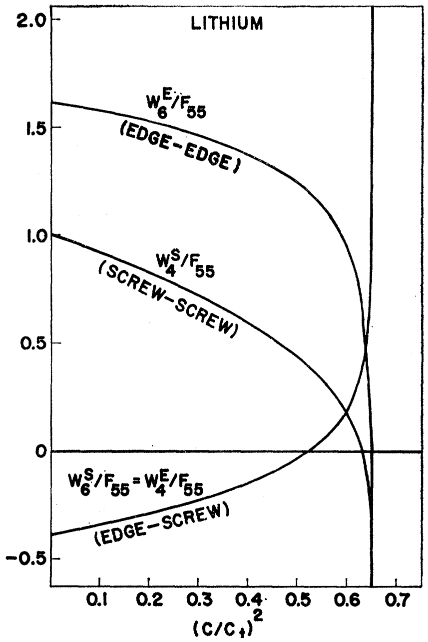

FIG. 2. Various (normalized) shear stress components on the slip plane of a uniformly moving dislocation are plotted as a function of dislocation velocity c. $W_6/F_{55}=(2\pi x_1'/b_0)[\sigma_{12}(x_1',0)/F_{55}]$, $W_4/F_{55}=(2\pi x_1'/b_0)[\sigma_{23}(x_1',0)/F_{55}]$, $c_t=(F_{55}/\rho)^{\frac{1}{2}}$, the superscript E refers to an edge dislocation, the superscript S to a screw dislocation. The interaction associated with each shear stress is designated.

IV. APPLICATION OF RESULTS

To discuss the dynamical behavior of an edge or screw dislocation in a given orientation, we require numerical solutions for W_6 and W_4 as functions of dislocation velocity c. We must first calculate the F_{ijkl} by the usual tensor transformation from the c_{ijkl}, the elastic constants in the crystal coordinates. Then for a given value of c, one proceeds as follows: (i) evaluate the K_r in (11), (ii) solve for y_n in (10), (iii) evaluate the Φ_{ikn} in (7), (iv) evaluate ω_n, ν_n, f_n, d_n in (15), (16), (v) evaluate S_{in}, T_{in} in (21), (vi) solve (20) for $\alpha_n{}^E$, $\epsilon_n{}^E$ and then evaluate $W_6{}^E$ in (29) and $W_4{}^E$ in (34), (vii) solve (20) for $\alpha_n{}^S$, $\epsilon_n{}^S$, and then evaluate $W_4{}^S$ in (30) and $W_6{}^S$ in (34). These seven steps are then repeated for various values of c. The first value of c for which one root of (10) becomes real is the limiting velocity of a dislocation in that orientation.

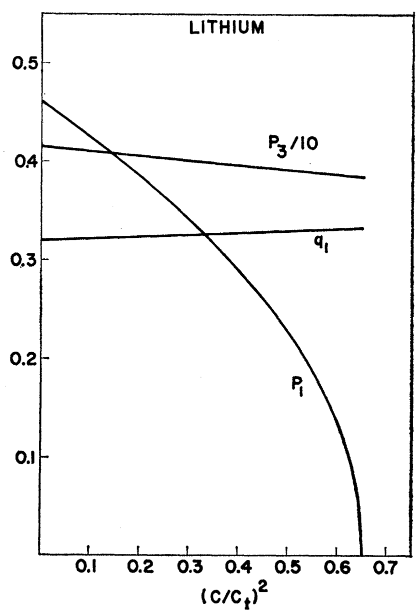

FIG. 1. For $(\bar{1}10)$ $[11\bar{1}]$ dislocation motion in lithium, the roots y_n of Eq. (10) are of the form $\pm q_1\pm ip_1$, $\pm ip_3$. The real and imaginary parts of y_n are shown as functions of $(c/c_t)^2$; c is the dislocation velocity and $c_t=(F_{55}/\rho)^{\frac{1}{2}}$.

As an example of the procedure we consider a body-centered cubic material with a dislocation lying along $[112]$ in the $(\bar{1}10)$ plane, and moving in the $[11\bar{1}]$ direction. Using the symmetry arrays given by Waterman,[14] we see that the elastic constant matrix for this orientation is of the form

$$\begin{bmatrix}11 & 12 & 12 & 0 & 0 & 0\\ 12 & 22 & 23 & 0 & 25 & 0\\ 12 & 23 & 22 & 0 & -25 & 0\\ 0 & 0 & 0 & \frac{1}{2}(22-23) & 0 & 25\\ 0 & 25 & -25 & 0 & 55 & 0\\ 0 & 0 & 0 & 25 & 0 & 55\end{bmatrix}. \tag{35}$$

[14] P. C. Waterman, Phys. Rev. **113**, 1240 (1959).

The six constants required are

$$\begin{aligned} F_{11}&=\tfrac{1}{3}(C_{11}+2C_{12}+4C_{44}),\\ F_{22}&=\tfrac{1}{2}(C_{11}+C_{12}+2C_{44}),\\ F_{12}&=\tfrac{1}{3}(C_{11}+2C_{12}-2C_{44}),\\ F_{23}&=\tfrac{1}{6}(C_{11}+5C_{12}-2C_{44}),\\ F_{25}&=\tfrac{1}{6}\sqrt{2}(C_{11}-C_{12}-2C_{44}),\\ F_{55}&=\tfrac{1}{3}(C_{11}-C_{12}+C_{44}). \end{aligned} \tag{36}$$

Since nine of the F_{ij}'s are equal to zero, the coefficients K_1, K_3, K_5 in (10) vanish and the sextic equation for y_n reduces to a bicubic.

The material considered here is lithium; its elastic constants[15] at 78°K are (in units of 10^{11} dyn/cm^2)

$$C_{11}=1.48,\quad C_{44}=1.08,\quad C_{12}=1.25. \tag{37}$$

Lithium was chosen since its anisotropy factor $A=2C_{44}/(C_{11}-C_{12})$ is the largest encountered (9.4) among all bcc elements for which elastic constant data are available. The roots of Eq. (10) for lithium are of the form

$$y_1=q_1-ip_1,\quad y_z=-q_1-ip_1,\quad y_3=-ip_3, \tag{38}$$

at all permissible velocities. In Fig. 1 are plotted q_1, p_1, and $p_3/10$ as functions of $(c/c_t)^2$, where $c_t=(F_{55}/\rho)^{\frac{1}{2}}$ is is the appropriate shear sound velocity for this orientation. q_1 and p_3 vary only slightly with velocity, whereas p_1 decreases sharply and goes to zero at $(c/c_t)^2\simeq 0.654$. Hence, the limiting velocity c_∞ for a dislocation in this orientation is given by $c_\infty\simeq 0.809c_t$.

In Fig. 2 are plotted the normalized shear stresses W_6/F_{55} and W_4/F_{55} for both edge and screw dislocations as functions of $(c/c_t)^2$. The features to be noted are the following: (i) As in all previous cases studied, it is found that $W_6{}^E$ decreases with velocity and changes sign at high velocities, i.e., there is a range of velocities near the limiting velocity for which two parallel (like) edge dislocations on the same slip plane will attract. (ii) $W_4{}^S$ decreases with velocity and likewise becomes negative at high velocities. This is a new result and shows that screw dislocations as well display an anomalous dynamical behavior; i.e., at high velocities, like screw dislocations on the same slip plane will attract. Furthermore, for this orientation in lithium, the threshold velocity $(c_r{}^S)$ for the anomalous behavior of screw dislocations is lower than the corresponding threshold velocity $(c_r{}^E)$ for edge dislocations. (iii) The shear stresses $W_6{}^S$, $W_4{}^E$ turn out to be same at all velocities. The magnitude of the edge-screw interaction decreases with velocity and then changes sign at $c=c_r{}^{SE}$. The edge-screw interaction is the first to change sign with increasing dislocation velocity for this orientation in lithium. We can summarize our numerical results as follows:

	$c_r{}^{SE}/c_t$	$c_r{}^S/c_t$	$c_r{}^E/c_t$	c_∞/c_t	$c_r{}^{SE}/c_\infty$	$c_r{}^S/c_\infty$	$c_r{}^E/c_\infty$
Li	0.723	0.794	0.804	0.809	0.894	0.983	0.995

(39)

Hence, it appears that in the general anisotropic case for which a dislocation involves three components of displacement, both edge and screw dislocations can exhibit an anomalous dynamical behavior. The threshold velocity for this anomalous behavior depends on the type of dislocation as well as its orientation; the limiting velocity, however, is a function only of the orientation of the dislocation.

[15] H. C. Nash and C. S. Smith, J. Phys. Chem. Solids **9**, 113 (1959).