Significance of Spatial Isotropy*

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(Received March 14, 1962)

It has been suggested occasionally that the fine structure "constant" may not be a fixed number, but that it may be determined, in a manner not yet understood, by the distribution of mass in the universe. A simple point-particle picture is used to indicate the significance of this idea for the motion of free test particles. The possible generally covariant equations of motion for a point particle are considered, and it is shown that one can find a suitable model which is consistent with the observed structure independence of gravitational acceleration. However, it is indicated that such a model is not consistent with the precise observations, made by Hughes *et al.* and Drever, of the local isotropy of space. That is, with the observed structure independence of gravitational acceleration and local isotropy of space, and assuming general covariance, it seems to be necessary to rule out any appreciable variation with position in the value of the fine structure constant.

EVIDENCE for the local isotropic nature of space has been drawn from recent discussions of nuclear magnetic resonance experiments.[1–4] This need not conflict with Mach's Principle.[5,6] However, this evidence, along with the observation that gravitational acceleration is sensibly independent of structure[7] does have an indirect bearing on the conjecture that the values of physical numbers such as the fine structure constant may be determined by the structure of the universe.[8] The connection with the observations arises when one attempts to fit such a conjecture into a generally covariant field theory of gravity.

The possibility of a variable fine structure "constant" follows from the idea that the gravitational interaction might play an important role in the structure of elementary particles,[9,10] and that Mach's principle might imply a connection between the strength of the gravitational interaction and the structure of the universe.[11,12] By this argument $\alpha = e^2/\hbar c$ might be given by the approximate equation[10]

$$\alpha^{-1} \sim \ln(\hbar c/Gm^2), \tag{1}$$

where m is the mass of an elementary particle, and where G is given in the sense of a rough order of magnitude by an approximate equation of the general form

$$G^{-1} \sim \sum m_i/r_i c^2. \tag{2}$$

The sum is to be taken over all particles in the visible (that is, causally connected) part of the universe. A proper relativistic formulation of an equation similar to (2) has been given.[12]

Accepting for the moment the validity of these ideas, one would expect that the energy of a particle, measured with given clocks and measuring rods, depends on position. For example, with Eqs. (1) and (2), the rate of change with position of the energy of a nucleus would be

$$dE/dx \sim (E_e/\alpha)(d\alpha/dx) \sim -\alpha E_e G(dG^{-1}/dx)$$
$$\sim \alpha E_e[-G(d/dx)\sum m_i/r_i c^2], \tag{3}$$

where E is the total energy and E_e the electrostatic energy of the nucleus. Corrections to (3) due to the compressibility of the nucleus are not important here. Units of energy were chosen so that the energy of a particle which has no electromagnetic structure would be constant. Neglecting the very small variation in G with position, the term in square brackets in Eq. (3) is seen to be the gradient of the dimensionless Newtonian gravitational potential.

These ideas cannot be ruled out by the observational evidence which has been considered so far.[13] We are interested here in the problem of whether the idea of variable energy [for example, Eq. (3)] can fit into a generally covariant field theory of gravity consistent with the Eötvös experiment and the observations of spatial isotropy.

The problem is particularly simple if complex objects, such as atoms, are treated as point particles, where, to take account of the above conjecture, the mass of the particle is variable. The equations of motion of a point particle in general relativity are derived from the equation

$$0 = \delta \int m(g_{ij}u^i u^j)^{\frac{1}{2}} ds, \tag{4}$$

where $u^i = dx^i/ds$ is the velocity of the particle. If this were modified with a variable mass $m(x)$, the equation

* This work was supported in part by research contracts with the Office of Naval Research and the U. S. Atomic Energy Commission.

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of motion would be

$$(d/ds)(mg_{ij}u^j)=\tfrac{1}{2}m(\partial g_{jk}/\partial x^i)u^ju^k+(\partial m/\partial x^i). \quad (5)$$

With the example mentioned above, the size of the second term on the right-hand side of (5) would be estimated using Eq. (3). Here the difference in the gravitational accelerations of different particles may be of the order of $\alpha E_e/E$ times the gravitational acceleration. The ratio E_e/E of electrostatic to total energy may be as large as 10^{-2}, so the accelerations of different particles may differ by a part in 10^4. This is at least four orders of magnitude larger than the limit set by observations.[7]

We must seek a new gravitational force field to balance the anomalous force due to variable energy. The simplest assumption is a scalar field $\varphi(x)$. The action (4) would be modified by adding an interaction term

$$0=\delta\int[m(x)(g_{ij}u^iu^j)^{\frac{1}{2}}+\varphi(x)]ds. \quad (6)$$

This, however, adds nothing new for the velocities u_i must satisfy

$$g_{ij}u^iu^j=1, \quad (7)$$

and Eq. (6), with the constraint (7) is equivalent to Eq. (4), with m replaced by $m+\varphi$.

The next simplest scheme would be a vector field $f_i(x)$ with the action principle

$$0=\delta\int[m(x)(g_{ij}u^iu^j)^{\frac{1}{2}}+f_iu^i]ds. \quad (8)$$

This theory is invariant against gauge transformations of $f_i(x)$. With a more realistic model for test particles, this would mean that Eq. (8) corresponds to a situation where $f_i(x)$ couples with a conserved quantity, such as charge or heavy-particle number. It would be possible to change the structure of the particle appreciably, and hence to alter the anomalous gravitational force associated with variable energy, without altering the force due to $f_i(x)$. It is concluded that the vector theory is of no interest for this simplified discussion of the possible gravitational fields which might interact with real particles.

The theory with a tensor field (in addition to the metric tensor) need not conflict with the Eötvös experiment. With the field h_{ij}, the only interesting scalar is $h_{ij}u^iu^j$, and the free variational pri iciple representing an interaction with a tensor field and leading to equations of motion consistent with Eq. (7) is

$$0=\delta\int[m(x)(g_{ij}u^iu^j)^{\frac{1}{2}}+\bar{m}(x)(h_{ij}u^iu^j)^{\frac{1}{2}}]ds. \quad (9)$$

Here, $m(x)$ and $\bar{m}(x)$ are masses which characterize the structure of the particle. $\bar{m}$ is a measure of the mass of the particle associated with its electromagnetic character.

Here, the path of the particle depends on its structure. However, to make the theory consistent with the Eötvös experiment, it is only necessary to require that the acceleration should be sensibly independent of structure for a particle at rest on the earth. It is important that the earth is moving slowly relative to the comoving coordinate frame of the universe, the relative velocity being between 30 km/sec (the speed of the earth relative to the sun) and 300 km/sec (the upper limit determined from the approximately isotropic distribution of galactic red shifts).

In regions of space well removed from local inhomogeneities the universe may be reasonably assumed to appear very nearly isotropic, so that with appropriate coordinates, the comoving coordinate frame, the metric tensor g_{ij}, and the gravitational field h_{ij} are diagonal, with $g_{11}=g_{22}=g_{33}$ and $h_{11}=h_{22}=h_{33}$. Since the time scale for expansion of the universe is large with respect to processes we want to consider, the components of h_{ij} and g_{ij} are substantially constant. The effect on the components of h_{ij} and g_{ij} from the presence of the sun is small and will be neglected, but not the effect on the gradients of these tensors.

By appropriately redefining the fields g_{ij} and h_{ij}, and $m(x)$ and $\bar{m}(x)$, it is possible (if $m\neq 0$) to write the action (9) so that m is constant, and in the special frame mentioned above, $g_{00}=h_{00}$. Then it may be verified that if $\bar{m}(x)$ satisfied the condition

$$\frac{1}{\bar{m}}\frac{\partial\bar{m}}{\partial x^\alpha}=\frac{1}{2}\frac{1}{g_{00}}\frac{\partial g_{00}}{\partial x^\alpha}\left(\frac{h_{\alpha\alpha}}{g_{\alpha\alpha}}-1\right), \quad (10)$$

(no sum on α; $\alpha=1, 2, 3$) where the field components in this equation are supposed to be evaluated in the special coordinate frame, the gravitational acceleration of a particle at rest in this frame would be independent of m and $\bar{m}$, that is, independent of the structure of the particle.

For a particle moving with speed v with respect to this frame, accelerations of different particles may differ by the following amount, taken relative to the gravitational acceleration in the frame

$$|\delta g/g|\sim\bar{m}/m(v/c)^2(h_{\alpha\alpha}/g_{\alpha\alpha}-1). \quad (11)$$

We have assumed here that $\bar{m}/m\ll 1$.

As discussed above, $\bar{m}$ is the contribution to the mass of the particle associated with its electromagnetic structure, and $\bar{m}/m=E_e/(E-E_e)$, $\bar{m}/m\lesssim 10^{-2}$. For the conditions under which the Eötvös experiment was performed $(v/c)^2\lesssim 10^{-6}$, and by Eqs. (3) and (10) $|1-h_{\alpha\alpha}/g_{\alpha\alpha}|\sim 10^{-2}$, so the anomalous acceleration (11) here is less than one part in 10^{10}. This is consistent with Eötvös's limit of five parts in 10^9 on possible departures from exact structure independence of gravitational acceleration.[7]

The next step would be to use a more reasonable model for the test particle. We have considered a test particle consisting of a distribution of interacting classical fields.[14] To include Eqs. (1) and (2) in this model, Maxwell's equations for the electromagnetic field in the particle would be suitably modified. Then, following the above example [Eq. (9)] we would suppose that all the fields in the particle, save the electromagnetic field, obey the usual field equations with metric tensor g_{ij}, while in the action for the electromagnetic field g_{ij} is replaced everywhere with a symmetric tensor gravitational field h_{ij}. This theory can be consistent with the results of the Eötvös experiment.

There are several observational tests of covariant field theories of gravity in which particles have variable energies. Inertial mass is not equivalent to energy in such a theory,[15] and the momentum of a particle with high laboratory energy is not related to the energy of the particle in the usual way. A most significant test is provided by the condition of spatial isotropy.[2–4]

Consider a nucleus moving relative to the special coordinate frame defined above with speed v. With coordinates chosen such that the nucleus is at rest, and the comoving coordinate frame of the universe moving in the x^3 direction, and such that the metric tensor g_{ij} has the Minkowski form, the components h_{11} and h_{33} of the tensor field are not equal,

$$|(h_{11}-h_{33})/h_{11}|\sim\alpha(v/c)^2. \tag{12}$$

We have used the conditions (3) and (10) on the components of h_{ij}.

The nucleons in the nucleus have some electromagnetic structure, so the inertial mass of a nucleon, in the point particle picture [Eq. (9)], is expected to have an anisotropic part in this coordinate frame. A more important effect is that the electromagnetic field equations have an anisotropic form. To estimate how this perturbs the energy levels of the nucleus, we can go to a rest coordinate frame chosen so that locally h_{ij} has the Minkowski form. By (12) the shape of the nucleus in this frame is distorted along the direction x^3, and the electrostatic energy of a nucleus may depend on orientation relative to the x^3 axis. The relative shift in energy levels belonging to different magnetic quantum numbers is approximately[14]

$$\alpha(v/c)^2(Ze^2q/R^3). \tag{13}$$

Here, q is the quadrupole moment, Z the atomic number, and R the radius of the nucleus. e is the charge on a proton.

The recent experiments have used Li^7, with $q\sim2\times10^{-26}$ cm^2.[16] The speed of the earth relative to the special coordinate frame is expected to be at least 30 km/sec, (the speed of the earth relative to the sun), and by Eq. (13) the relative shift in energy levels is at least 10^{10} cps. This is larger than the experimental limit[2–4] by a factor of about 10^{10}.

It is apparent that the assumption of general covariance places a very important restriction on possible gravity theories. We have shown that it is possible to find a generally covariant theory for the motion of point particles which is consistent with the Eötvös experiment, and in which the ratios of masses of particles may vary appreciably with position. It was mentioned that this theory can be adapted to a more realistic model for physical particles. We have found from the discussion of point particles that a generally covariant theory in which particles have variable energies seems necessarily to involve at least two tensor fields. In this case, it is very difficult to make the theory consistent with the accurate observations of spatial isotropy.

It was earlier suggested by one of us (R. H. D.)[15] that the systematic discrepancy between the inertial and Q-value mass scale may be related to a variable fine structure constant. Assuming the validity of the above interpretation of the significance of the space isotropy experiments, this possibility now seems quite unlikely.

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