Pion Production in Neutrino-Nucleon Collisions*

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An evaluation of the scattering amplitude for pion production in neutrino-nucleon collisions is made assuming that the $\frac{3}{2}$-$\frac{3}{2}$ π-nucleon resonance dominates the dispersion integrals. No assumption is made concerning nucleon recoil. The vector part of the amplitude is related to the isovector part of the amplitude for electroproduction of pions by use of the Feynman–Gell-Mann conserved vector current theory. An independent calculation is given for the axial vector part of the amplitude.

I. INTRODUCTION

A SERIES of experiments are being undertaken to measure the cross sections for various processes involving the collision of high-energy neutrinos with protons or complex nuclei. Among the possible reactions that one would expect to observe when neutrinos have a laboratory energy of about 1 BeV, the following ones seem to be the most favored:

$$\nu+n\rightarrow p+e^- \text{ (or } \mu^-), \tag{1a}$$

$$\nu+n \begin{cases} \rightarrow e^-+p+\pi^0 \\ \rightarrow e^-+n+\pi^+, \end{cases} \tag{1b}$$

with possibly strange particle production in two-body final-state reactions.[1–3] A first estimate gives for the total cross sections:

$$\sigma_{(1a)}\approx 0.86\times 10^{-38} \text{ cm}^2,$$

$$\sigma_{(2a)}\approx 10^{-39} \text{ cm}^2,$$

for incident neutrino energies equivalent to a nucleon mass.

The single pion production reactions (1b) involve the strong as well as the weak interactions, and a relativistic evaluation of this amplitude is desirable, particularly since the nucleon recoil can be important.

To first order in the weak interaction, the leptons are assumed to interact with the strongly interacting particles via an intermediate vector boson of mass m_B (Fig. 1). The strong current consists of a vector part and an axial vector part,

$$J_\mu = J_\mu^{(V)} + J_\mu^{(A)}, \tag{2}$$

which are coupled to the leptons by their Møller potential multiplied by the boson propagator:

$$\epsilon_\mu = \epsilon_\mu^{(V)} + \epsilon_\mu^{(A)} = \frac{G_V}{\sqrt{2}} \frac{\bar{u}(e^-)[\gamma_\mu(1+\gamma_5)]u(\nu)}{k^2+m_B^2}, \tag{3}$$

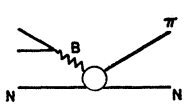

FIG. 1. Neutrino-pion production amplitude to first order in the weak interaction. The circle represents the effects of the strong interactions.

where k^2 is the square of the momentum transfer from the leptons, and G_V is the vector coupling constant. If one neglects the mass of the electron, then ϵ_μ obeys the supplementary condition:

$$k^\mu \epsilon_\mu = 0. \tag{4}$$

The vector part of the current (2) can be related to the isovector part of the electropion current by a rotation in isotopic spin space.[4] A relativistic calculation of electroproduction of pions has been given in a previous paper.[5] The axial vector contribution, however, must be calculated independently. We will assume that the $\frac{3}{2}$-$\frac{3}{2}$ π-nucleon resonance dominates the dispersion integrals; nevertheless, we will treat nucleon recoil exactly. We will adhere to the notations of (A) as closely as possible.

II. THE VECTOR CURRENT

The conserved vector hypothesis of Feynman and Gell-Mann[4] makes it possible to relate the vector part of the cross section for pion production in neutrino-nucleon collisions to the isovector part of the cross section for pion production in electron-nucleon collisions.[1,3]

The Feynman–Gell-Mann Theory postulates that the conserved vector current differs (aside from a factor of G_V/e) from the isovector part of the electromagnetic current by a rotation in isotopic spin space. The isovector part of the electromagnetic current is generated by the operator τ_3, whereas the weak vector current is generated by the isotopic boosting operator $\tau_+ = \tau_1 + i\tau_2$. In addition, if the weak interaction is intermediated by a vector boson of unit spin and mass m_B, the boson propagator

$$\frac{1}{m_B^2+k^2}\left[\delta_{\beta\alpha}+\frac{k_\alpha k_\beta}{m_B^2}\right]$$

will replace the photon propagator which appears in electroproduction. By neglecting the mass of the electron, one obtains the following ratio at given values of the total energy W, and of the square of the momentum transfer k^2:

$$\frac{d\sigma_\nu^{(V)}(k^2,W)}{d\sigma_e^{(V)}(k^2,W)} = 2\left(\frac{G_V k^2}{e^2}\right)^2 \left(\frac{m_B^2}{m_B^2+k^2}\right). \tag{5}$$

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[1] Y. Yamaguchi, CERN Report 61-2, 1961 (unpublished).
[2] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960).
[3] S. Berman, Proceedings of the International Conference on Theoretical Aspects of very High-Energy Phenomena, CERN Report 61-22, 1961 (unpublished).
[4] R. P. Feynmann and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[5] P. Dennery, Phys. Rev. **124**, 2000 (1961); quoted as (A).

The amplitude for the electroproduction of pions has been derived in (A). The intermediate boson must be at least as massive as the K meson, but experimentally, of course, there is as yet no upper limit on its mass. Since k^2 is spacelike, the ratio (5) will be greatest if $m_B=\infty$, corresponding to a point interaction between the leptons and the nucleons.

Berman[3] has extrapolated the electro-pion production data of Ohlsen[6] from 600 MeV to 1 BeV. He obtains

$$\sigma_\nu^{(V)}\leq 10^{-39}\text{ cm}^2.$$

III. AXIAL-VECTOR CURRENT

We must now calculate the matrix element

$$\langle N_1|J_\mu^{(A)}|N_2,\pi\rangle, \tag{6}$$

which involves only the strong interactions and where now the axial-vector current $J_\mu^{(A)}$ is no longer a divergenceless quantity. The fact that $J_\mu^{(A)}$ cannot be conserved follows from the observation that if it were, the decay rate of the charged pion would vanish.[7]

In calculating (6) we will consider the dispersion graphs of Fig. 2. At the vertices of these graphs, there will appear various form factors in terms of which, finally, the matrix element will be expressed. In particular, there will be the intermediate-boson–nucleon vertex associated with the nucleon pole term which, on general invariance grounds may be written[8]

$$\langle N_1|J_\mu^{(A)}|N_2\rangle=iG_A\bar{u}_N(p_2)\times[F_A(\lambda^2)\gamma_\mu\gamma_5-ibF_P(\lambda^2)k_\alpha\gamma_5]u_{N_1}(p_1), \tag{7}$$

where $k^2=\lambda^2$ is the square of the momentum transfer to the leptons. $F_A(\lambda^2)$ and $F_P(\lambda^2)$, the axial vector and pseudoscalar form factors are normalized to unity at $\lambda^2=0$. Experimentally $G_A\approx -1.25G_V$, where G_V is the vector coupling constant that appears in (5). The second term in (7), the induced pseudoscalar term, will be negligibly small, since $k_\mu\epsilon_\mu$ is proportional to the electron mass. In the following, we will not consider it; the meson pole term [Fig. 2(c)] will contain the bosonic form factor of the pion $F_\pi{}^A(\lambda^2)$. Similarly, we will consider transitions to the final $\frac{3}{2}$–$\frac{3}{2}$ π-nucleon resonant state and these will be characterized by three additional form factors. As in (A), we will show, by imposing the unitarity condition on the dispersion relations that these latter form factors can be expressed in terms of $F_A(\lambda^2)$ and $F_\pi{}^{(A)}(\lambda^2)$.

The matrix element $\epsilon_\mu\langle N_1|J_\mu^{(A)}|N_2,\pi\rangle$ may be decomposed into relativistic invariants as follows:

$$T^{(A)}=\bar{u}(p_2)\Big(\sum_{i=1}^{6}A_iN_i\Big)u(p_1),$$

$$\begin{aligned} &N_1=\gamma\cdot\epsilon, && N_4=i\gamma\cdot\epsilon\gamma\cdot k,\\ &N_2=iq\cdot\epsilon, && N_5=q\cdot\epsilon\gamma\cdot k,\\ &N_3=i(p_1+p_2)\cdot\epsilon, && N_6=(p_1+p_2)\cdot\epsilon\gamma\cdot k.\end{aligned} \tag{8}$$

[6] G. Ohlsen, Phys. Rev. **120**, 584 (1960).
[7] J. C. Taylor, Phys. Rev. **110**, 1216 (1958).
[8] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

p_1, p_2 are the initial and final 4-momenta of the nucleons; q, k are the momenta of the meson and of the intermediate boson. The amplitudes A_i will be functions of 2 of the 3 scalars:

$$s_1=-(p_1+k)^2,\quad s_2=-(p_1-k)^2,\quad t=-(q-k)^2,$$

where energy-momentum conservation implies that

$$s_1+s_2+t=2M^2+\mu^2-\lambda^2.$$

The dependence of the amplitude upon the momentum transfer to the nucleons t will be discussed later. For the moment, we shall consider it to be held fixed. The isotopic spin variables may be made explicit by writing

$$A_i=A_i{}^{+}\tfrac{1}{2}\{\tau_\alpha,\tau_+\}+A_i{}^{-}\tfrac{1}{2}[\tau_\alpha,\tau_+],$$

where α is the isotopic index of the meson.

The amplitudes $A_i{}^{\pm}$ will be a sum of terms representing the contributions from the graphs of Fig. 2. In the language of dispersion theory, $A_i{}^{\pm}$ will obey the representation

$$(A_i{}^{\pm})=(A_i{}^{\pm})_{\text{Born}}+(A_i{}^{\pm})_{\text{res}}, \tag{9}$$

where

$$(A_i{}^{\pm})_{\text{res}}=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\rho_i(s')\left(\frac{1}{s'-s_1}\pm\frac{1}{s'-s_2}\right)ds'.$$

$\rho_i(s)$ is the imaginary part of the amplitudes $A_i{}^{\pm}$ in the channel leading to the π-nucleon resonant state. $(A_i{}^{\pm})_{\text{res}}$ is the contribution from the direct and crossed resonant terms [Fig. 2(d), 2(e)]. The first term is the contribution from the Born term [Fig. 2(a)–2(c)], which may be evaluated directly. For these, one obtains:

$$(A^{\pm})_{\text{Born}}=R_s(A_i{}^{\pm})\left(\frac{1}{M^2-s_1}\pm\frac{1}{M^2-s_2}\right)+R_t(A_i{}^{\pm})\frac{1}{t-\mu^2}, \tag{10}$$

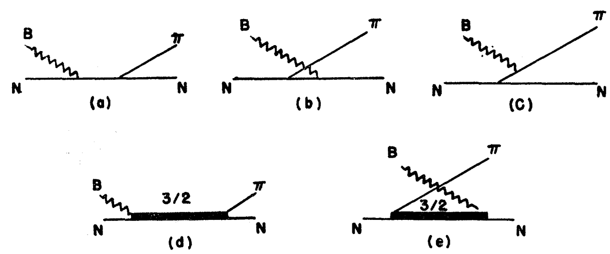

FIG. 2. Graphs considered in evaluating the neutrino-pion production amplitude.

where

$$R_s(A_1{}^{\pm})=2MgF_A(\lambda^2),$$
$$R_s(A_2{}^{\pm})=R_s(A_3{}^{\pm})=-R_s(A_4{}^{\pm})=gF_A(\lambda^2),$$
$$R_t(A_2{}^{-})=-2gF_\pi{}^A(\lambda^2).$$

The amplitudes $A_1{}^-$, $A_2{}^-$, $A_4{}^-$, and $A_3{}^+$ go with the positive sign in (10).

The problem of evaluating $(A_i{}^{\pm})_{\text{res}}$ may be divided into two parts. The first part consists in making a multipole expansion of the amplitudes $A_i{}^{\pm}$ (i.e., an expansion in terms of the orbital angular momentum of the intermediate boson) and keeping in this expansion only those amplitudes that can lead to a final $\frac{3}{2}$-$\frac{3}{2}$ π-nucleon resonant state. For these resonant amplitudes, one can write a dispersion relation; since their phase is simply the phase of the $\frac{3}{2}$-$\frac{3}{2}$ π-nucleon resonant state, one can obtain an explicit solution of the dispersion relations. The second part of the problem consists in evaluating the nonresonant waves. In order to evaluate these, we start from the dispersion relations for the relativistic amplitudes A_i [Eq. (9)] and keep in the multipole expansion of $\rho(s)$ only the resonant waves for which we now have a solution, since these will essentially exhaust the dispersion integral. The nonresonant waves can then be obtained by simple projections.

In the center-of-mass frame of the pion and nucleon, we may write (8) as

$$T^{(A)}=\chi_{N_2}{}^+\Big(\sum_{i=1}^{6} g_i\eta_i\Big)\chi_{N_1}, \tag{11}$$

where

$$\eta_1=i\sigma\cdot q\sigma\cdot\epsilon/q,\qquad \eta_4=iq\cdot\epsilon/q,$$
$$\eta_2=i\sigma\cdot k\times\epsilon/k,\qquad \eta_5=i\sigma\cdot q\sigma\cdot k\epsilon_0/qk,$$
$$\eta_3=i\sigma\cdot q\sigma\cdot kq\cdot\epsilon/q^2k,\quad \eta_6=i\epsilon_0.$$

The relations between the A_i and the g_i are given in the Appendix. The multipole expansion of the amplitudes g_i (axial vector case) may be obtained from those of the amplitudes F_i [vector case, see (A)] by effecting the correspondences:

$$M_l{}^+\rightarrow\mathfrak{M}_{l+1}{}^-, \quad (12a)\qquad E_{l-1}{}^+\rightarrow\mathcal{E}_l{}^-, \quad (12d)$$
$$M_l{}^-\rightarrow\mathfrak{M}_{l-1}{}^+, \quad (12b)\qquad L_{l+1}{}^-\rightarrow\mathcal{L}_l{}^+ \text{ (or } \mathcal{S}_l{}^+), \quad (12e)$$
$$E_{l+1}{}^-\rightarrow\mathcal{E}_l{}^+, \quad (12c)\qquad L_{l-1}{}^+\rightarrow\mathcal{L}_l{}^- \text{ (or } \mathcal{S}_l{}^-). \quad (12f)$$

The script letters refer to the axial-vector amplitudes. The subscripts are the values of the final orbital angular momentum l_f expressed in terms of the angular momentum of the multipole radiation. The superscripts ($\pm$) mean that the amplitudes lead to final π-nucleon states of total angular momentum $l_f\pm\frac{1}{2}$. Let us prove (12a).

For a vector boson inducing a magnetic transition of order l, the parity of the initial state is $-(-1)^l$, whereas it is $(-1)^l$ for an axial vector boson. Therefore, in the vector case, an initial state of total angular momentum $l+\frac{1}{2}$ will lead to a final state of orbital angular momentum $l_f=l$ whereas in the axial vector case such an initial state would lead to a final state of orbital angular momentum $l_f=l+1$ and therefore of total angular momentum $l_f-\frac{1}{2}$. The proof of the other relations follow along similar lines.

We will use the scalar amplitudes rather than the longitudinal ones [parentheses of (12e) and (12f)]. The reason is that, even though they are related by the condition (4) and are therefore similar, there appears in the longitudinal amplitudes an extra k_0 in the denominator which, for spacelike particles, gives rise to spurious singularities in these amplitudes at the center-of-mass energy $W=(M^2+\lambda^2)^{\frac{1}{2}}$ for each λ^2. These singularities are absent from the scalar amplitudes.

According to (12) and (A) we have for the multipole expansions[8a]

$$\begin{aligned}
g_1&=\sum_{l=0}^{\infty}\{[(l-1)\mathfrak{M}_l{}^-+\mathcal{E}_l{}^-]P_l'(x)+[(l+2)\mathfrak{M}_l{}^++\mathcal{E}_l{}^+]P_l'(x)\},\\
g_2&=\sum_{l=0}^{\infty}[l\mathfrak{M}_l{}^-P_{l-1}'(x)+(l+1)\mathfrak{M}_l{}^+P_{l+1}'(x)],\\
g_3&=\sum_{l=0}^{\infty}(\mathcal{E}_l{}^--\mathfrak{M}_l{}^-+\mathcal{E}_l{}^++\mathfrak{M}_l{}^+)P_l''(x),\\
g_4&=\sum_{l=1}^{\infty}[(\mathfrak{M}_l{}^--\mathcal{E}_l{}^-)P_{l-1}''(x)-(\mathfrak{M}_l{}^++\mathcal{E}_l{}^+)P_{l+1}''(x)],\\
g_5&=-\frac{k_0}{k}(g_1+xg_3)+\sum_{l=0}^{\infty}[l\mathcal{S}_l{}^--(l+1)\mathcal{S}_l{}^+]P_l'(x),\\
g_6&=-\frac{k_0}{k}g_4+\sum_{l=0}^{\infty}[(l+1)\mathcal{S}_l{}^+P_{l+1}'(x)-l\mathcal{S}_l{}^-P_{l-1}'(x)].
\end{aligned} \tag{13}$$

[8a] I thank Dr. A. Krass for a comment on Eq. (13).

The P_l's are the Legendre polynomials; they are functions of the cosine of the scattering angle in the center-of-mass system. The corresponding inversion formulas for the (+) amplitudes are:

$$\mathfrak{M}_l^+=\frac{1}{2(l+1)}\int_{-1}^{1}dx\left\{-g_1P_{l+1}(x)+g_2P_l(x)+g_3\frac{P_l(x)-P_{l+2}(x)}{2l+3}\right\},$$
$$\mathcal{E}_l^+=\frac{1}{2(l+1)}\int_{-1}^{1}dx\left\{g_1P_{l+1}(x)-g_2P_l(x)-g_3(l+2)\frac{P_l(x)-P_{l+2}(x)}{2l+3}-g_4(l+1)\frac{P_{l-1}(x)-P_{l+1}(x)}{2l+1}\right\},\tag{14}$$
$$\mathcal{S}_l^+=\frac{1}{2(l+1)}\int_{-1}^{1}dx\left\{\frac{k_0}{k}(g_1+\frac{k_0}{k}xg_3+g_5)P_{l+1}(x)+\frac{k_0}{k}(xg_4+g_6)P_l(x)\right\}.$$

From (14) and (10) one may obtain the multipole projections of the Born terms which will be grouped according to those proportional to $F_A(\lambda^2)$ and into those proportional to $F_\pi{}^A(\lambda^2)$. Only the electric dipole, the scalar dipole, and the magnetic quadrupole can lead to a final $\frac{3}{2}$-$\frac{3}{2}$ π-nucleon resonant state.

$$\begin{aligned}
&\mathfrak{M}_1^{+B}(F_A)|fF_A(\lambda^2)=-AI_2(a)+BI_1(a)-C[\tfrac{2}{3}-aI_3(a)],\\
&\mathfrak{M}_1^{+B}(F_\pi{}^A)fF_\pi{}^A(\lambda^2)=C[\tfrac{2}{3}-bI_3(b)],\\
&\mathcal{E}_1^{+B}(F_A)/fF_A(\lambda^2)=AI_2(a)-BI_1(a)+C[2-3aI_3(a)],\\
&\mathcal{E}_1^{+B}(F_\pi{}^A)/fF_\pi{}^A(\lambda^2)=-C[2-3bI_3(b)]+2DI_3(b),\\
&\mathcal{S}_1^{+B}(F_A)/fF_A(\lambda^2)=[AI_2(a)-CaI_2(a)]k_0/k\\
&\qquad-\frac{(2q_0-W-M)}{2q}CI_2(a)-k_0/kDaI_1(a)+\frac{D}{E_1+M}[W^2-M^2-2q_0(E_1+M)]\frac{I_1(a)}{2q},\\
&\mathcal{S}_1^{+B}(F_\pi{}^A)/fF_\pi{}^A(\lambda^2)=k_0/kCbI_2(b)+q_0CI_2(b)/q+k_0/kbDI_1(b)+q_0DI_1(b)/q,
\end{aligned}\tag{15}$$

where

$$A=\frac{(E_1+M)^{\frac{1}{2}}(W+M)}{4k(E_2+M)^{\frac{1}{2}}},\qquad B=\frac{(E_2+M)^{\frac{1}{2}}(W-M)}{4k(E_1+M)^{\frac{1}{2}}},$$
$$C=\frac{q}{2(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}},\qquad D=\frac{(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}}{2k},$$
$$2WE_1=W^2+M^2+\lambda^2;\quad 2WE_2=W^2+M^2-\mu^2;\quad 2Wk_0=W^2-M^2-\lambda^2,\quad k_0{}^2=k^2-\lambda^2;\quad q_0{}^2=q^2+\mu^2,$$
$$I_1(y)=2-y\ln\frac{y+1}{y-1};\quad I_2(y)=3y+\frac{1-3y^2}{2}\ln\frac{y+1}{y-1};\quad I_3(y)=y+\frac{1-y^2}{2}\ln\frac{y+1}{y-1},$$
$$a=\frac{2k_0E_2+\lambda^2}{2qk};\quad b=-\left(\frac{2q_0k_0+\lambda^2}{2qk}\right);\quad f^2\approx0.08.$$

Let us now define three new functions

$$\mathfrak{M}_1^{+\prime}\equiv\frac{(E_1+M)^{\frac{1}{2}}\mathfrak{M}_1^+}{qk(E_2+M)^{\frac{1}{2}}},\quad \mathcal{E}_1^{+\prime}\equiv\frac{\mathcal{E}_1^+}{q(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}},\quad \mathcal{S}_1^{+\prime}\equiv\frac{\mathcal{S}_1^+}{2Wqk(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}}.\tag{16}$$

It can be shown, starting from (9), that these functions satisfy simple dispersion relations. Typically:

$$\mathcal{E}_1^{+\prime}=\mathcal{E}_1^{+B\prime}+\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\operatorname{Im}\mathcal{E}_1^{+\prime}(W')}{W'-W}dW',\tag{17}$$

with similar relations holding for $\mathfrak{M}_1^{+\prime}$ and $\mathcal{S}_1^{+\prime}$.

In the above, we have neglected the crossed terms. This will be justified in the following. A numerical evaluation of the Born terms as functions of W and λ^2 has been done on a Univac computer. The results show that the mag-

TABLE I. Numerical values of the functions appearing in Eq. (18).

$W \backslash \lambda^2$	0	2	4	8	10	15	20	25	30	40	50	80
						$\mathcal{P}_1(W,\lambda^2)$						
7.9	1.17	1.15	1.14	1.11	1.10	1.07	1.05	1.02	0.99	0.94	0.89	0.78
8.4	1.17	1.17	1.16	1.15	1.14	1.12	1.10	1.08	1.07	1.03	1.00	0.90
8.9	1.19	1.18	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.09	1.06	0.99
9.2	1.20	1.19	1.19	1.18	1.18	1.17	1.16	1.15	1.14	1.12	1.09	1.03
9.4	1.21	1.20	1.20	1.19	1.19	1.18	1.17	1.16	1.15	1.13	1.11	1.05
						$\mathcal{P}_2(W,\lambda^2)$						
7.9	0.33	0.20	0.14	0.88	0.74	0.53	0.41	0.34	0.29	0.22	0.18	0.11
8.4	0.35	0.26	0.21	0.14	0.12	0.092	0.074	0.061	0.052	0.040	0.033	0.021
8.9	0.34	0.28	0.24	0.18	0.16	0.12	0.10	0.086	0.074	0.059	0.048	0.032
9.2	0.33	0.29	0.25	0.19	0.17	0.14	0.12	0.10	0.086	0.069	0.044	0.038
9.4	0.33	0.29	0.25	0.20	0.18	0.15	0.12	0.11	0.094	0.075	0.063	0.042
						$\mathcal{P}_3(W,\lambda^2)\times 10^4$						
7.9	1.10	4.4	2.7	1.5	1.20	0.78	0.56	0.42	0.34	0.23	0.16	0.075
8.4	1.24	7.1	4.9	2.9	2.4	1.6	1.2	0.92	0.73	0.50	0.36	0.17
8.9	1.16	7.9	6.0	3.9	3.3	2.3	1.7	1.35	1.10	0.75	0.55	0.26
9.2	1.08	8.0	6.3	4.3	3.6	2.6	2.0	1.6	1.3	0.9	0.66	0.32
9.4	1.03	7.9	6.3	4.4	3.8	2.8	2.1	1.7	1.4	1.0	0.73	0.35

netic Born terms are negligibly small as is the Born terms $\mathcal{S}_1{}^{+B}(F_\pi{}^A)$. The remaining electric and scalar terms may be approximated by simple poles when $\lambda^2=0$ which lie outside of the physical range. For $\lambda^2\neq 0$, these poles are multiplied by a real polynomial in λ^2 with coefficients that are weakly dependent upon W[9]:

$$\mathcal{E}_1{}^{+\prime}(F_A)=-\frac{fG_AF_A(\lambda^2)\mathcal{P}_1(\lambda^2,W)}{W-6.6};\quad \mathcal{E}_1{}^{+\prime}(F_\pi{}^A)=-\frac{fF_\pi{}^A(\lambda^2)\mathcal{P}_2(\lambda^2,W)}{W-7.3};\quad \mathcal{S}_1{}^{+\prime}=\frac{fG_AF_A\mathcal{P}_3(\lambda^2,W)}{W-7.5}. \tag{18}$$

Some of the values of $\mathcal{P}_i(\lambda^2,W)$ are given in Table I. The fact that the Born terms have a pole-like behavior greatly facilitates the task of finding solutions to the dispersion relations. If the $\mathcal{P}_i(\lambda^2,W)$ were independent of W, then the method of solution of the dispersion relations described in (A) could also be applied here. This is very nearly the case for $\mathcal{P}_1(\lambda^2,W)$ for $0<\lambda^2<80$, as it is for $\mathcal{P}_2(\lambda^2,W)$ and $\mathcal{P}_3(\lambda^2,W)$ for low values of λ^2. For higher values of λ^2, we can evaluate $\mathcal{P}_1$ and $\mathcal{P}_3$ at resonance, and then the solution that one obtains for $\mathcal{E}_1{}^+(F_\pi{}^A)$ and $\mathcal{S}_1{}^+(F_A)$ will be valid in the neighborhood of the resonance. One then obtains, according to (A),

$$\mathcal{E}_1{}^+(F_A)=-fG_AF_A(\lambda^2)K_1q(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}\mathcal{P}_1(\lambda^2,W)\frac{W-6.7}{W-6.6}\frac{h(W)}{q^2(W)}\quad\text{for}\quad 7.8<W<9.4,$$

while

$$\mathcal{E}_1{}^+(F_\pi{}^A)=-fF_\pi{}^A(\lambda^2)K_2q(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}\mathcal{P}_2(\lambda^2,W)\frac{W-6.7}{W-7.3}\frac{h(W)}{q^2(W)},$$

$$\mathcal{S}_1{}^+(F_A)=fG_AF_A(\lambda^2)K_3 2Wqk(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}\mathcal{P}_3(\lambda^2,W)\frac{W-6.7}{W-7.5}\frac{h(W)}{q^2(W)}\quad\text{for}\quad W\approx W_{\text{res}},$$

where

$$h(W)=e^{i\delta_{33}(W)}\sin\delta_{33}(W)/q(W),$$
$$K_1=\frac{-q^2(6.6)}{0.1\ \mathrm{Re}h(6.6)},$$
$$K_2=\frac{q^2(7.3)}{0.6\ \mathrm{Re}h(7.3)}, \tag{19}$$
$$K_3=\frac{q^2(7.5)}{0.8\ \mathrm{Re}h(7.5)}.$$

Of course, an exact solution of the dispersion relations (17) has been given by Omnès.[10] But the numerical integrations that remain are rather intractable, even on a computer, and for this reason we have preferred to give the above approximate solutions. If these solutions are inserted into the (unwritten) crossed terms of Eqs. (17), a numerical evaluation shows that these crossed terms are indeed completely negligible near resonance. The nonresonant waves may be obtained by projection using (14) together with similar relations for the $(-)$ amplitudes, once $\rho_i(W)$ [Eq. (9)] is expressed in terms

[9] We have set $\mu=1$, $M=6.7$.

[10] R. Omnès, Nuovo cimento 8, 316 (1958).

of the resonant waves. This may easily be done and the result is

$$\rho_i(W,\lambda^2)=\Delta(W,\lambda^2)[\delta_i(\mathfrak{M})(W,\lambda^2)\,\mathrm{Im}\mathfrak{M}_1^+(W) + \delta_i(\mathcal{E})(W,\lambda^2)\,\mathrm{Im}\mathcal{E}_1^+(W) + \delta_i(\mathcal{S})(W,\lambda^2)\,\mathrm{Im}\mathcal{S}_1^+(W)] \quad (20)$$

where

$$\Delta(W,\lambda^2)=M(E_1+M)^{\frac{1}{2}}/Wqk(E_2+M)^{\frac{1}{2}}$$

and the δ_i are given in the Appendix.

DISCUSSION

The neutrino-pion production amplitude given by (9) and (20) still does not contain the effects of a pion-pion interaction. If, instead of starting with (9), we had started with the Mandelstam representation and had approximated the cut in the momentum transfer to the nucleons t by poles at the π-π resonances, we would have been led to an expression similar to (9) but with an additional term representing these resonances. Such a term has been considered in (A), for example, and it would be a simple matter to add it here. We believe, however, that the amplitude given in (9) is already quite accurate near the π-nucleon $\frac{3}{2}$-$\frac{3}{2}$ resonance and that it would be best to clarify first the role of the pion-pion interaction in other more accessible processes such as photo- or electroproduction, before adding here a term, inherent in which there is some ambiguity.

As has been noted in (A), the resonant nucleon term due to the vector current falls off rather rapidly at high momentum transfers λ^2, due to the decrease of the Hofstadter form factors.[11] It may be that if $F_A(\lambda^2)$ remains larger than the Hofstadter form factors at high momentum transfers, the essential contribution to the neutrino-pion amplitude at resonance would come from the axial vector current.

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Note added in proof. After completion of this work a paper on the same subject [N. Dombey, this issue, Phys. Rev. **126**, 653 (1962)] was brought to my attention. The ideas presented in this paper are similar, although the methods of solution are perhaps different.

APPENDIX

The relations between the relativistic amplitudes (8) and the center-of-mass amplitudes (11) are as follows:

$$G_1\equiv\frac{2M(E_2+M)^{\frac{1}{2}}}{q(E_1+M)^{\frac{1}{2}}}g_1=A_1-(W-M)A_4,$$

$$G_2\equiv\frac{2M(E_1+M)^{\frac{1}{2}}}{k(E_2+M)^{\frac{1}{2}}}g_2=-A_1-(W+M)A_4,$$

$$G_3\equiv\frac{2M(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}}{q^2k}g_3=-(A_2-A_3)+(W+M)(A_5-A_6),$$

$$G_4\equiv\frac{2M}{q(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}}g_4=(A_2-A_3)+(W-M)(A_5-A_6),$$

$$G_5\equiv\frac{2M(E_1+M)^{\frac{1}{2}}(E_2+M)^{\frac{1}{2}}}{qk}g_5=A_1+q_0(A_2-A_3)+(W+M)A_4+2WA_3-q_0(W+M)(A_5-A_6)-2W(W+M)A_6,$$

$$G_6\equiv\frac{2M(E_1+M)^{\frac{1}{2}}}{(E_2+M)^{\frac{1}{2}}}g_6=A_1(W+M)-q_0(E_1+M)(A_2-A_3)-2W(E_1+M)A_3 -q_0(E_1+M)(W-M)(A_5-A_6)-2W(E_1+M)(W-M)A_6-\lambda^2A_4.$$

The inverse relations are:

$$2WA_1=-(W-M)G_2+(W+M)G_1,$$

$$2W(A_2-A_3)=(W+M)G_4-(W-M)G_3,$$

$$4W^2A_3=(W+M)G_1+[\lambda^2/(E_1+M)]G_2+q_0(W-M)G_3 -q_0(W+M)G_4+(W-M)G_5-[(W+M)/(E_1+M)]G_6,$$

$$2WA_4=-(G_1+G_2),$$

$$2W(A_5-A_6)=G_3+G_4,$$

$$4W^2A_6=G_1+[(W+M)/(E_1+M)]G_2-q_0(G_3+G_4)-G_5-G_6/(E_1+M).$$

[11] R. Hofstadter, F. Bumiller, and M. Croissiaux, Phys. Rev. Letters **5**, 263 (1960).

The coefficients δ_i of (20) are the following:

$$\delta_1(\mathfrak{M})=-6q(W-M)X+3k(W+M)(E_2+M)/(E_1+M),$$

$$\delta_1(\mathcal{E})=k(W+M)(E_2+M)/(E_1+M),$$

$$\delta_2(\mathfrak{M})=-3k(W+M)/(E_1+M)+\delta_3(\mathfrak{M}),$$

$$\delta_2(\mathcal{E})=-3k(W+M)/(E_1+M)+\delta_3(\mathcal{E}),$$

$$\delta_2(\mathcal{S})=\delta_3(\mathcal{S}),$$

$$\delta_3(\mathfrak{M})=\left[\frac{6q\lambda^2}{2W(E_1+M)}-\frac{3qk_0(W+M)}{2W(E_1+M)}\right]X+\frac{3k(W+M)(E_2+M)}{2W(E_1+M)}+\frac{3q_0k(W+M)}{2W(E_1+M)}-\frac{3(W-M)(E_2+M)k_0}{2Wk},$$

$$\delta_3(\mathcal{E})=-\frac{3qk_0(W+M)}{2W(E_1+M)}X+\frac{k(W+M)(E_2+M)}{2W(E_1+M)}+\frac{3q_0k(W+M)}{2W(E_1+M)}-(W-M)(E_2+M)k_0/2Wk,$$

$$\delta_3(\mathcal{S})=-\frac{3qk(W+M)}{W(E_1+M)}X-\frac{2(W-M)(E_2+M)}{2W},$$

$$\delta_4(\mathfrak{M})=-6qX-3k(E_2+M)/(E_1+M),$$

$$\delta_4(\mathcal{E})=-k(E_2+M)/(E_1+M),$$

$$\delta_5(\mathfrak{M})=-3k/(E_1+M)+\delta_6(\mathfrak{M}),$$

$$\delta_5(\mathcal{E})=-3k/(E_1+M)+\delta_6(\mathcal{E}),$$

$$\delta_5(\mathcal{S})=\delta_6(\mathcal{S}),$$

$$\delta_6(\mathfrak{M})=-\left[\frac{3qk_0}{2W(E_1+M)}+\frac{3q(W+M)}{W(E_1+M)}\right]X+\frac{3q_0k}{2W(E_1+M)}+\frac{3(E_2+M)k_0}{2Wk}+\frac{3k(E_2+M)}{2W(E_1+M)},$$

$$\delta_6(\mathcal{E})=-\frac{3qk_0}{2W(E_1+M)}X+\frac{3q_0k}{2W(E_1+M)}+\frac{(E_2+M)k_0}{2Wk}+\frac{k(E_2+M)}{2W(E_1+M)},$$

$$\delta_6(\mathcal{S})=(E_2+M)/W-3qk/W(E_1+M)X,$$

$$X=(t-\mu^2+\lambda^2+2q_0k_0)/2qk.$$