Wave Propagation in a Gyromagnetic Solid Conductor: Helicon Waves

A. LIBCHABER AND R. VEILEX
Laboratoire de Physique, Ecole Normale Superieure, Paris, France
(Received February 6, 1962; revised manuscript received April 18, 1962)

The propagation of circularly polarized electromagnetic waves in a solid under a high dc magnetic field is examined theoretically. An experiment is performed, in which there is transmission through a thin slab of InSb of a 10 000-Mc/sec wave, at liquid nitrogen and room temperatures. Results are in good agreement with the "helicon" theory, which implies that $\omega \ll \omega_c$, $\omega_c\tau \gg 1$.

A. INTRODUCTION

It has been shown by Aigrain[1] that in a solid conductor, under a dc magnetic field, circularly polarized waves with frequencies lower than the cyclotron resonance frequency and wave vectors parallel to the dc magnetic field, can propagate with small losses if $\omega_c\tau \gg 1$; ω_c is the cyclotron angular rotation frequency and τ the relaxation time of the free charge carriers.

This can be proved by the direct application of the Appleton and Hartree equations, giving the effective electrical permittivity. We derive these equations for one type of carrier in order to make the hypothesis more precise, and present a transmission experiment at 10 000 Mc/sec.

B. ELECTROMAGNETIC WAVE PROPAGATION IN A GYROMAGNETIC SOLID CONDUCTOR

The classical Drude-Zener model is used. A calculation starting from the Boltzmann transport equation leads to identical results.

Consider the propagation of a plane electromagnetic wave in a medium of electrical permittivity ϵ and with N free electrons per unit volume with a single effective mass (constant energy surfaces being spheres). Let Oz be the direction of both the dc magnetic field B_0 and the wave vector K.

If $\mathbf{v}$ is the drift velocity and $\mathbf{E}$ the radio-frequency electric field inside the sample, the equation of motion of the free charge carriers is

$$m^* d\mathbf{v}/dt + (m^*/\tau)\mathbf{v} = e(\mathbf{E} + \mathbf{v}\times\mathbf{B}_0),$$

where we neglect the effect of the radio-frequency magnetic field.

The motion of the free charge carriers being confined in the xy plane, the stationary solutions are[2]

$$v_\pm = (eE_\pm)/[m^*/\tau + i(m^*\omega \pm eB_0)],$$

where

$$v_\pm = v_x \pm iv_y, \quad E_\pm = E_x \pm iE_y,$$

thus introducing two circularly polarized waves. Let us define $\sigma_\pm$ as the complex conductivity

$$\sigma_\pm = Nev_\pm/E_\pm, \quad \sigma_\pm = \epsilon_0\omega_p{}^2\tau/[1+i(\omega\pm\omega_c)\tau],$$

where $\omega_p{}^2 = Ne^2/m^*\epsilon_0$ and $\omega_c = eB_0/m^*$. Applying the Appleton and Hartree equation,

$$\epsilon_\pm = \epsilon/\epsilon_0 - i\omega_p{}^2\tau/\omega[1+i(\omega\pm\omega_c)\tau],$$

the dispersion equation is

$$K_\pm{}^2c^2 = \epsilon\omega^2/\epsilon_0 - i\omega_p{}^2\omega\tau/[1+i(\omega\pm\omega_c)\tau].$$

If we make the following hypothesis:

$$\omega \ll \omega_c, \quad \omega_c\tau \gg 1,$$

the formulas are greatly simplified and one finds that the propagation is strongly dependent on the type of circularly polarized wave—ordinary or extraordinary.

As a practical case, one can neglect the effect of the static electrical permittivity. The dispersion equation becomes

$$K_\pm{}^2 \simeq \mp\omega_p{}^2\omega/\omega_c c^2 - i\omega_p{}^2\omega/\omega_c{}^2\tau c^2,$$

where we write

$$K_\pm = \alpha_\pm + i\beta_\pm,$$
$$\alpha_- \simeq \omega_p\omega^{\frac{1}{2}}/c\omega_c{}^{\frac{1}{2}}, \quad \beta_- \simeq [\omega_p\omega^{\frac{1}{2}}/c\omega_c{}^{\frac{1}{2}}][1/2\omega_c\tau],$$
$$\alpha_+ \simeq [\omega_p\omega^{\frac{1}{2}}/c\omega_c{}^{\frac{1}{2}}][1/2\omega_c\tau], \quad \beta_+ \simeq \omega_p\omega^{\frac{1}{2}}/c\omega_c{}^{\frac{1}{2}};$$

here $1/\beta$ is the attenuation length and α is the propagation vector. α_- and β_-, which represent the extraordinary wave, can be large; for example, in InSb with $N = 10^{15}$ electron/cm^3, $\mu_e = 2\times10^5$ cm^2/V sec, and $B_0 = 10\,000$ G, we have

$$\alpha_- \simeq 3600 \text{ m}^{-1},$$

which leads to a wavelength in the sample of 1.75 mm, and

$$\beta_- \simeq 90 \text{ m}^{-1},$$

i.e., an attenuation length of about 1 cm. The normal skin effect is of the order of 10^{-1} mm.

We have thus shown that in a solid conductor with one type of free charge carrier, under a dc magnetic field, and under the following conditions,

$$\omega \ll \omega_c, \quad \omega_c\tau \gg 1,$$

the extraordinary wave can propagate with a small attenuation and with a slow phase velocity. This wave is called a "helicon" wave.

When two types of free carriers are present, the calculation remains valid; the two currents are just to

[1] P. Aigrain, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961).
[2] See for instance: R. Rau and M. Caspari, Phys. Rev. **100**, 632 (1955).

be added. This supposes that the electron-hole interaction is unimportant.

If $\omega_{c1}\tau_1 \gg 1$ and $\omega_{c2}\tau_2 \ll 1$, the effect of the low-mobility carriers is not important; it increases slightly the attenuation of the helicon wave. This is the case of InSb at 300°K.

When $\omega_{c1}\tau_1 \gg 1$, $\omega_{c2}\tau_2 \gg 1$, and $N=P$, where P is the density of holes, helicon waves become the well-known Alfvén waves. Experiments on Alfvén waves have been performed in bismuth.[3]

C. DIMENSIONAL RESONANCE

To show this effect in a simple way, we study the transmission of guided waves through a thin slab of a conducting solid. In the sample itself, the wavelength is that of nonguided waves, for the effective electrical permittivity is large.

Let λ_0 be the wavelength in the air, λ the wavelength in our sample, corresponding to the working frequency, and l the thickness of the sample. Maxima of transmission occur for

$$l=n\lambda/2=n\lambda_0/2\epsilon^{\frac{1}{2}}, \quad l=\pi c n \omega_c^{\frac{1}{2}}/\omega_p \omega^{\frac{1}{2}}.$$

The Q of the resonance depends on both the loss angle $\omega_c\tau$ and the Q of the slab which is a "Perot-Fabry" type of structure. For low B, losses are important. For high B, the Q of the sample is poor, the reflection coefficient being small.

D. EXPERIMENTAL SETUP

InSb is a semiconductor noted for its good relaxation time and a number of free carriers such that dimensional resonance can occur in a reasonable thickness of the specimen.

We work in the X band, with a circularly polarized wave. The magnet is an iron-core solenoid, of the type described by Bitter, which gives a dc field continuously variable from 0 to 16 000 G.

In Fig. 1 is shown the experimental arrangement for circular polarization study. The Dewar is not shown. The magnetic field is parallel to the axis of the waveguide. Circular polarization is obtained by a quarter-wavelength dielectric plate. Attenuators are placed in order to prevent reflection of power not accepted by the rectangular guides.

The sample is inserted in a molybdenum ring; it is important to avoid any high-frequency leakage which can interfere with the signal transmitted through the sample.

The signal is detected by a crystal, working in the quadratic range of its characteristic, followed by a lock-in detection.

We worked with monocrystals and polycrystals of InSb and found no detectable difference, as could have been predicted (spherical energy surfaces).

[3] S. J. Buchsbaum and J. K. Galt, Phys. Fluids 4, 1514 (1961).

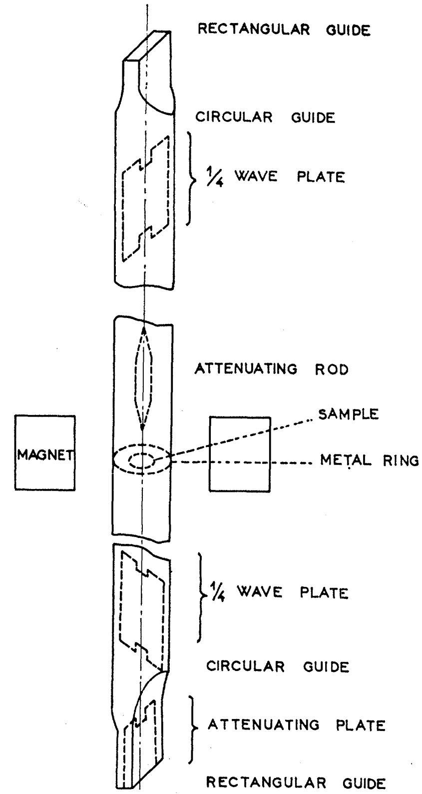

FIG. 1. Experimental arrangement for circular polarization studies of helicon wave propagation.

E. RESULTS

Figure 2 shows a transmission experiment through a sample 2 mm thick, 7 mm in diameter, with the following characteristics given by the usual conductivity and

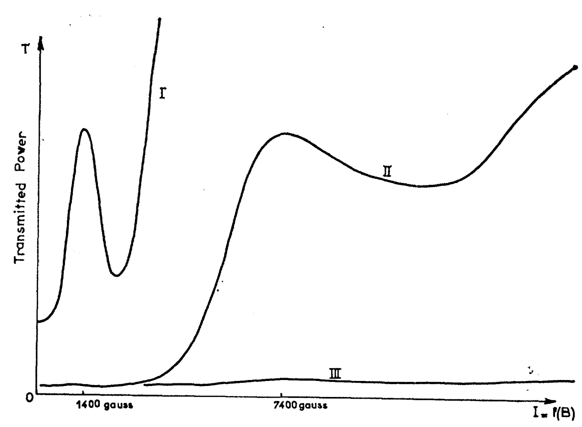

FIG. 2. Plot of transmitted power vs magnetic field at liquid nitrogen temperature. Curve II is the transmitted power for the direction of the magnetic field which allows helicon wave propagation. Curve III for the reversed direction of the magnetic field. Curve I is a part of curve II magnified by 12 dB, showing a resonance at small field.

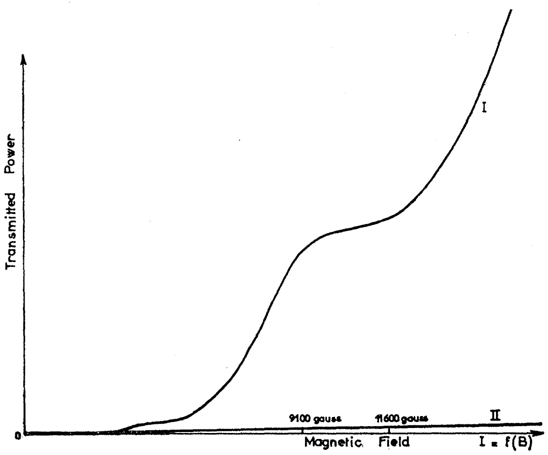

FIG. 3. Plot of transmitted power vs magnetic field at room temperature. Curve I: helicon wave propagation; Curve II: reversed direction for the magnetic field.

Hall effect experiments:

$$N=1.2\times10^{14}\ \text{electron/cm}^3,\quad N\gg P,$$
$$\mu_e=3\times10^5\ \text{cm}^2/\text{V-sec},\quad \text{temperature}=78°\text{K},$$
$$\omega=9000\ \text{Mc/sec},\quad \omega_c\tau\gg1\quad \text{for}\quad B_0\gg400\ \text{G}.$$

Two resonances are observed for a magnetic field of 1400 and 7400 G corresponding to $n=2$ and $n=1$. The calculated value for the number of free charge carriers is $N\simeq1.3\times10^{14}$ electrons/cm³. In the calculation we have to take into account the effect of the static dielectric permittivity.

Figure 3 shows an experiment at room temperature through a sample 0.65 mm thick, 9 mm in diameter, with the characteristics

$$N=1.35\times10^{16}\ \text{electron/cm}^3,\quad N=P,$$
$$\mu_e=5\times10^4\ \text{cm}^2/\text{V-sec},\quad \mu_h=800\ \text{cm}^2/\text{V-sec},$$
$$(\omega_c\tau)_e\gg1\quad \text{for}\quad B_0\gg2000\ \text{G}.$$

The attenuation distance being of the order of 1.5 mm at 10 000 G, dimensional resonances cannot be clearly observed. If we suppose that the two points indicated on the curve correspond to $n=3$ and the related minimum, we find $N\simeq1.6\times10^{16}$ electron/cm³. With the magnetic field reversed, no signal is detected.

We have also observed the shifts of the resonances with temperature, frequency, and thickness of the samples.

F. CONCLUSION

We have investigated the propagation of "helicon" waves in a semiconductor. These waves have also been studied in sodium[4] and indium[5] at a much lower frequency, the temperature being that of liquid helium.

These experiments in solids have a double interest, in the first place as a measuring tool and secondly as a technique for the study of plasmas in solids.

One can indeed measure the number of free charge carriers in a solid and their mobility (the Hall effect represents a limiting case of the helicon wave propagation when the frequency goes to zero). By choosing a suitable working frequency, the static dielectric permittivity can also be measured.

For the study of plasmas in solids, helicon wave propagation represents an experiment with a small number of well-defined parameters. Another step will be to apply a dc electric field parallel to the dc magnetic field, which can, under some conditions, amplify the helicon waves.[1,6]

ACKNOWLEDGMENTS

We thank Professor P. Aigrain for giving us the idea of the experiment and for his constant help during the work; Professor J. Bok and Professor P. Nozières for very enlightening discussions.

[4] R. Bowers, C. Legendy, and F. Rose, Phys. Rev. Letters **7**, 339 (1961).
[5] P. Cotti, A. Quattropani, and P. Wyder (private communication).
[6] J. Bok and P. Nozières (to be published).