Alpha Decay of Nonaxial Nuclei*†

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A theory is developed, within the context of the Davydov-Filippov model of the nucleus, to describe the even-parity alpha decay of nonaxial nuclei. The wave equation of the system consisting of a daughter nucleus and an alpha particle is solved under the boundary conditions that on the nuclear surface the wave function shall be equal to a given function and that the function shall represent only an outgoing wave for large values of r. It is found that if the given function is assumed to be equal to a constant, the observed variations of the alpha transition probabilities with respect to Z in even–even nuclei cannot be explained. But if one assumes that this given function can be represented as an expansion in spherical harmonics, keeping only quadrupole terms, the expansion coefficients being determined by the empirical data (that is, setting the boundary conditions by the empirical data), the observed variations can be theoretically reproduced. Even-parity alpha transitions in odd-A nuclei are also considered. The comparison between theory and the empirical data is quite satisfactory.

I. INTRODUCTION

ALPHA decay of axially symmetric nuclei have been studied by various authors.[1–4] They have shown that if the alpha wave function on the deformed nuclear surface is assumed to be constant then the empirical transition probabilities of various alpha groups can be explained by including multipole moments of the nuclear surface deformation. This introduces unknown parameters which are the coefficients in the expansion of the surface in terms of the Legendre polynomials P_l, and one needs as many free parameters as there are alpha groups to be explained. Fröman[4] determined these parameters by the empirical data; in other words, the shape of the daughter nucleus was prescribed by the empirical data of the alpha groups emitted from the parent nucleus.

On the other hand, the Davydov-Filippov[5] model of the nucleus has had a considerable amount of success in predicting the energies and spins of the rotational states of the daughter nucleus, which are found to be related to the alpha fine structure. This model, according to which the nucleus is assumed to be represented, in first approximation, by a triaxial ellipsoid, has one free parameter γ which measures the deviation of the nuclear shape from axial symmetry and which is determined from the experimental energy ratio of two of the rotational states of the daughter nucleus. In contrast to Fröman's theory, the shape of the nucleus is now given and need not be determined by the alpha groups emitted from the parent nucleus. From this consideration and the fact that the nonaxiality might have an effect on the barrier penetration of the alpha particles it seems advisable to develop a theory of alpha decay within the context of the Davydov-Filippov model.

Recently, Rostovskii[6] has developed a theory for the relative probabilities of alpha decay to rotational states of nonaxial even–even nuclei. The wave function of the system consisting of the daughter nucleus and the alpha particle has been derived under the assumption that it is constant on the surface of the nonaxial nucleus. The theoretical values fail to show the variational character of the relative transition probabilities with respect to Z, as one should expect from the variation of the empirical values with Z. Further, the relative transition probabilities for different alpha groups for a given nucleus are not reproduced theoretically. The reason for this is due to the fact that the alpha wave function on the nonaxial nucleus is strongly coupled with the nuclear deformation and as such it is distorted about some average value. Therefore, in order to understand the empirical data one should remove from the theory the restriction that the alpha wave function on the nuclear surface is constant.

The aim of this work is to remove the above-mentioned restriction, to develop a theory which is different from Rostovskii's theory in analytical approach, and to see to what extent one can understand the empirical data on the relative transition probabilities of the various alpha groups.

In Sec. II the theory of the penetration of an alpha particle through an anisotropic potential barrier is developed by following Fröman's method.[4] In Sec. III the theory is compared with the empirical data, first for the even parity transitions in even–even nuclei and then for similar transitions in odd-A nuclei. Finally, certain conclusions are presented in Sec. IV.

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II. THEORY

The wave function for the description of the penetration of an alpha particle through an anisotropic potential barrier is, in the laboratory coordinate system, of the form

$$|I_iM_i\rangle = e^{-iEt/\hbar}\sum_{I_fl}\frac{1}{r}f_{lI_f}{}^{I_i}(r)\Phi_{M_i}{}^{I_i}(lI_f;\omega,\theta,\varphi), \quad \text{(II 1)}$$

where the index i attached to the quantum numbers refers to the parent nucleus and index f refers to the daughter nucleus. E is the total energy of the alpha particle and the daughter nucleus; (θ,φ) are the polar angles of the alpha particle with respect to the laboratory coordinate system; r is the distance between the center of the daughter nucleus and the alpha particle; ω represents, symbolically, the three Eulerian angles specifying the orientation of the daughter nucleus.

The Φ functions in (II 1) are defined by

$$\Phi_{M_i}{}^{I_i}(lI_f;\omega,\theta,\varphi) = \sum_{M_fm} C(I_flI_i;M_fmM_i)|I_fM_f,\tau\rangle Y_l{}^m(\theta,\varphi), \quad \text{(II 2)}$$

where

$$|I_fM_f,\tau\rangle = \left(\frac{2I_f+1}{16\pi^2}\right)^{\frac{1}{2}}\sum_{K_fj_fk_f}A_{K_f}{}^{I_f\tau}B_{k_f}{}^{j_f} \times\{\chi_{k_f}{}^{j_f}D_{M_f,K_f}{}^{I_f}(\omega) +(-1)^{I_f-j_f}\chi_{-k_f}{}^{j_f}D_{M_f,-K_f}{}^{I_f}(\omega)\}, \quad \text{(II 3)}$$

are the eigenfunctions of rotational and particle Hamiltonians. The quantum numbers I, M, K, with appropriate index, refer to the total angular momentum, its components along the z axis (lab system) and along the 3 axis (body fixed), respectively, whereas the total angular momentum of the odd particle and its component along the 3 axis are denoted by j and k, respectively. K_f and k_f are assumed to be non-negative unless otherwise noted. The quantity τ characterizes the rotational band of a particular I_f. The angular momentum of the alpha particle and its projections on the Z axis and the 3 axis are denoted, respectively, by l, m, and Ω.

The wave function (II 1) must satisfy the wave equation and the boundary conditions that the function is assumed to be equal to a given function on the nuclear surface (Dirichlet condition) and that it shall represent only an outgoing wave for large values of r.

In the center-of-mass coordinate system of the daughter nucleus and the alpha particle, the Hamiltonian of this two-body system is

$$H = -(\hbar^2/2M)\nabla_r{}^2 + V(\mathbf{r}') + H_{\text{part}} + H_{\text{rot}}, \quad \text{(II 4)}$$

where M is the reduced mass of the alpha particle; the interaction energy between the daughter nucleus and the alpha particle, which is assumed to be purely electrostatic, is denoted by $V(\mathbf{r}')$; the vector $\mathbf{r}'=(r,\theta',\varphi')$ is the position vector relative to a c.m. system fixed in the body of the daughter nucleus; the operator for the excitation energy of the daughter nucleus is denoted by $H_{\text{part}}+H_{\text{rot}}$. In the operator for the excitation energy of the daughter nucleus we have not included the term arising from the nuclear vibrations, since the large excitation energy for such vibrations ($\gtrsim 1$ MeV) makes them relatively unimportant for the even-parity alpha decay.

The surface of the daughter nucleus is represented by[5]

$$R(\theta',\varphi') = R_0[1+\sum_{\mu=-2}^{2}\alpha_\mu Y_2{}^\mu(\theta',\varphi')], \quad \text{(II 5)}$$

where the quantities α_μ are related to the nuclear surface-deformation parameter β and shape parameter γ by[7]

$$\begin{aligned}\alpha_0 &= \beta\cos\gamma,\\ \alpha_{\pm1} &= 0, \qquad\qquad \text{(II 5a)}\\ \alpha_{\pm2} &= 2^{-\frac{1}{2}}\beta\sin\gamma.\end{aligned}$$

The nuclear surface (II 5) differs from that of Fröman's theory,[4] by the fact that he considered only the axially symmetric case ($\gamma=0$) and included higher deformation parameters β_4, β_6 etc.

Corresponding to the nuclear surface (II 5) the anisotropic part of $V(r')$ can be expanded as

$$V(\mathbf{r}') - \frac{2(Z-2)e^2}{r} = \sum_{\mu=-2}^{2} V_\mu(\beta,\gamma;r)Y_2{}^\mu(\theta',\varphi'), \quad \text{(II 6)}$$

where

$$V_\mu(\beta,\gamma;r) = \frac{8\pi e}{5r^3}Q_\mu{}^2 = \frac{8\pi e}{5r^3}\int_{\text{nuclear volume}}(r'')^2\rho(r'') \times Y_2{}^\mu(\theta'',\varphi'')d\mathbf{r}'', \quad \text{(II 6a)}$$

and $(Z-2)e$ is the charge of the daughter nucleus.

The wave equation is solved by following the procedure of Fröman,[4] which employs a WKB approximation method and the ray concept of optics first introduced by Christy.[1] This involves an extremal problem

$$\int_P^{P'} Kds = \text{minimum}, \quad \text{(II 7)}$$

where P and P' are two points inside the anisotropic potential barrier, and the arc length connecting these two points is measured in the direction from P to P'. K

[7] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat-fys. Medd. **26**, No. 14 (1952).

is defined by

$$K=K(\mathbf{r}')=k[(V(\mathbf{r}')-\mathcal{E})/\mathcal{E}]^{\frac{1}{2}}$$
$$=k\left(\frac{2\eta}{kr}-1\right)^{\frac{1}{2}}+\frac{4\pi\eta}{5(Z-2)e}\left(\frac{2\eta}{kr}-1\right)^{-\frac{1}{2}}$$
$$\times\sum_{\mu=-2}^{2}\frac{1}{r^3}Q_\mu{}^2Y_2{}^\mu(\theta',\varphi')$$
$$=K_0(r)+\Delta K(r,\theta',\varphi'), \qquad \text{(II 8)}$$

where k is the wave number corresponding to the energy $\mathcal{E}$. The quantity η is defined by

$$\eta=2(Z-2)e^2/\hbar v, \qquad \text{(II 8a)}$$

with v the velocity of the alpha particle after penetration of the barrier.

In the general case of an anisotropic barrier, the extremal problem (II 7) involves solving a set of coupled differential equations defining the path which minimizes the integral $\int_P{}^{P'} Kds$. Assuming the purely anisotropic part, $V(\mathbf{r}')-2(Z-2)e^2/r$, to be small compared with the spherically symmetric part $2(Z-2)e^2/r$ we can use the same paths as for the spherically symmetric potential. Although this assumption is not very good, it is adequate. The result of these considerations is an approximate solution of the wave equation, from which, following Fröman's[4] procedure, we obtain the formulas for the reduced transition probabilities. We discuss these transition probabilities in the following section.

III. COMPARISON BETWEEN THEORY AND EMPIRICAL DATA

A. Even-Parity Alpha Transitions in Even–Even Nuclei

Most of the alpha groups which have been observed in even–even nuclei are of the even-parity type (l= even). The nuclear transitions are $(I_i=0)\rightarrow(I_f=l;\tau)$, the alpha groups corresponding to these transitions being denoted by $\alpha_{l\tau}$.

The reduced transition probability[4] (reciprocal of the hindrance factor) $c_{l\tau}$ is

$$c_{l\tau}=\left|\frac{\sum_{K_f}A_{K_f}{}^{l\tau}\sum_{l'\Omega'}h_{l,-K_f;\,l'\Omega'}\epsilon_{l'\Omega'}}{\sum_{l'\Omega'}h_{00;\,l'\Omega'}\epsilon_{l'\Omega'}}\right|^2$$
$$\times\exp\left[\frac{-l(l+1)}{\eta}\left(\frac{2\eta}{kR_0}-1\right)^{\frac{1}{2}}\right]. \qquad \text{(III 1)}$$

The l dependence of $c_{l\tau}$, which is due to the ordinary centrifugal barrier, can be separated out by introducing a quantity $b_{l\tau}$ defined by[4]

$$b_{l\tau}=(c_{l\tau})^{\frac{1}{2}}\exp\left[\frac{l(l+1)}{2\eta}\left(\frac{2\eta}{kR_0}-1\right)^{\frac{1}{2}}\right], \qquad \text{(III 2)}$$

so that

$$b_{l\tau}=\left|\frac{\sum_{K_f}A_{K_f}{}^{l\tau}\sum_{l'\Omega'}h_{l,-K_f;\,l'\Omega'}\epsilon_{l'\Omega'}}{\sum_{l'\Omega'}h_{00;\,l'\Omega'}\epsilon_{l'\Omega'}}\right|. \qquad \text{(III 3)}$$

The formulas (III 1) to (III 3) are obtained by assuming that the wave function on the nuclear surface $\Psi_0(\theta',\varphi')$ is expressed by

$$\Psi_0(\theta',\varphi')=\psi_0\sum_{l'\Omega'}\epsilon_{l'\Omega'}Y_{l'}{}^{\Omega'}(\theta',\varphi'). \qquad \text{(III 4)}$$

It may be stressed at this point that Fröman obtained his formula for $b_{l\ \tau}$ by assuming that the alpha wave function on the deformed nuclear surface, $\Psi_0(\theta',\varphi')$ is constant and independent of θ' and φ'. The matrix elements $h_{l\Omega;\,l'\Omega'}$, which are functions of β and γ, are defined as

$$h_{l\Omega;\,l'\Omega'}$$
$$=\langle Y_l{}^\Omega(\theta',\varphi')|\sum_{nm}C_{nm}Y_n{}^m(\theta',\varphi')|Y_{l'}{}^{\Omega'}(\theta',\varphi')\rangle, \qquad \text{(III 5)}$$

where C_{nm} are the expansion coefficients defined by

$$\sum_{nm}C_{nm}Y_n{}^m(\theta',\varphi')=\exp[\sum_\mu B_\mu Y_2{}^\mu(\theta',\varphi')], \qquad \text{(III 5a)}$$

with

$$B_\mu=\tfrac{2}{5}\alpha_\mu\left[\frac{kR_0}{2\eta}\left(1-\frac{kR_0}{2\eta}\right)\right]^{\frac{1}{2}}(4\eta-kR_0). \qquad \text{(III 5b)}$$

The quantities $\epsilon_{l'\Omega'}$ depend on B_μ and Ψ_0 is some average value. In the Appendix we give the approximate formulas for $h_{l\Omega;\,l'\Omega'}$ for $\Omega=-K_f$ and $l'=2$, which we have used for this discussion. The other elements ($\Omega\neq-K_f$; $l'\neq2$) can be obtained from formulas (III 5).

From the formula (III 3) it is seen that the $b_{l\tau}$-values for the even-parity alpha groups depend on the coefficients $A_{K_f}{}^{l\tau}$, $\epsilon_{l'\Omega'}$ and the matrix elements $h_{l,-K_f;\,l'\Omega'}$ corresponding to the even values of both l and l'. The quantities $A_{K_f}{}^{l\tau}$ are functions of γ, which are obtained by solving the Schrödinger equation for the rotational states of the daughter nucleus. In the following discussion we shall confine ourselves to transitions to the main rotational band ($\tau=1$).[8] In Table I we give the empirical values of the reduced transition probabilities c_{l1} and the quantities b_{l1} for various heavy nuclei. These values were taken from Fröman's paper.[4]

We shall discuss these empirical data in relation to the theoretical formula (III 3). It seems most reasonable to proceed by making simple assumptions about the alpha wave function $\Psi_0(\theta',\varphi')$ on the nuclear surface. If we assume that $\Psi_0(\theta',\varphi')$ is constant, as Fröman[4] did, then the quantities $\epsilon_{l'\Omega'}$ are different from zero only for

[8] In the very heavy region $Z\geq88$, anomalous transitions ($\tau\geq2$) have not been observed except for Cm^{242} in which the observed alpha intensity is found to be very weak [B. S. Dzhelepov and L. K. Peker, Acad. Sci. U. S. S. R. "Decay Schemes of Radioactive Nuclei" (1958)].

TABLE I. Empirical hindrance factors, reduced transition probabilities c_{l1}, and the quantities b_{l1} calculated from (III 2) of some even–even nuclei ($Z \geq 88$).

Parent nucleus	l	Hind. factor	c_{l1}	b_{l1}	Parent nucleus	l	Hind. factor	c_{l1}	b_{l1}
$_{88}Ra^{222}$	2	1.20	0.83	1.18	$_{94}Pu^{236}$	2	1.5	0.67	1.05
Ra^{224}	2	1.20	0.83	1.19	Pu^{238}	2	1.6	0.62	1.02
Ra^{226}	2	0.90	1.11	1.38		4	115.0	0.0087	0.22
$_{90}Th^{226}$	2	1.50	0.67	1.06		6	385.0	0.0026	0.30
	4	7.80	0.13	0.84		8	15 000.0	0.000067	0.18
Th^{228}	2	0.90	1.11	1.37	Pu^{240}	2	1.7	0.59	0.99
	4	12.0	0.083	0.69		4	68.0	0.015	0.28
Th^{230}	2	1.0	1.00	1.30	$_{96}Cm^{242}$	2	1.65	0.61	1.00
	4	9.8	0.10	0.77		4	380.0	0.0026	0.12
	6	8500.0	0.00012	0.070		6	260.0	0.0038	0.36
Th^{232}	2	0.80	1.25	1.47		8	4850.0	0.00021	0.28
$_{92}U^{230}$	2	1.1	0.91	1.23	Cm^{244}	2	1.8	0.56	0.96
	4	9.6	0.10	0.76		4	1000.0	0.0010	0.073
U^{232}	2	1.1	0.91	1.24		6	340.0	0.0029	0.31
	4	16.0	0.062	0.60	$_{98}Cf^{246}$	2	2.7	0.37	0.77
	6	270.0	0.0037	0.37		4	115.0	0.0087	0.21
U^{234}	2	1.2	0.83	1.18		6	320.0	0.0031	0.31
	4	14.5	0.069	0.62	Cf^{250}	2	3.0	0.33	0.73
U^{236}	2	1.1	0.91	1.24	Cf^{252}	2	3.2	0.31	0.71
U^{238}	2	1.5	0.67	1.06		4	94.0	0.0011	0.23
					$_{100}Fm^{254}$	2	4.0	0.25	0.63
						4	54.0	0.018	0.30
						6	800.0	0.0012	0.19

$l'=\Omega'=0$ $[\epsilon_{00}=(4\pi)^{\frac{1}{2}}]$ and (III 3) reduces to (for $\tau=1$)

$$b_{l1}=\left|\frac{\sum_{K_f} A_{K_f}{}^{l1} h_{l,-K_f;00}}{h_{00;00}}\right|. \tag{III 6}$$

The formula (III 6) is different from that of Fröman,[4] although we have here assumed that $\Psi_0(\theta',\varphi')$ is constant. However, the simple formula (III 6) fails to explain the empirical data. The calculated b_{l1} values deviate from the corresponding empirical values, and the deviations become larger for higher values of l. Besides, the calculated values fail to show a Z dependence which is similar to the empirical b_{l1} values.[9] These discrepancies are also found in Fröman's theory.[4]

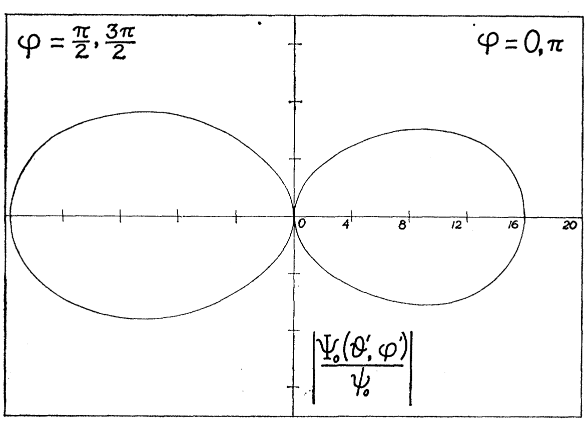

FIG. 1. A polar plot of the relative strength of the alpha-particle wave function on the nuclear surface. The coefficients ϵ_{20}, $\epsilon_{2\pm2}$ used in the expression $\Psi_0(\theta',\varphi')/\psi_0=1+\sum_\mu \epsilon_{2\mu}Y_2{}^\mu(\theta',\varphi')$ are those empirically determined for Cm^{242} (see Table II). Note that the right side of the figure is for $\varphi=0,\pi$ while the left side is for $\varphi=\pi/2, 3\pi/2$.

Thus, the assumption that $\Psi_0(\theta',\varphi')$ is constant is not a good one. The fact that the nuclear deformation causes a distortion of the alpha wave function on the surface should be taken into account in order to explain the empirical data for the intensities of various alpha groups. The alpha wave function $\Psi_0(\theta',\varphi')$ may contain higher angular momenta but for the sake of simplicity we make the assumption that it is of the form

$$\Psi_0(\theta',\varphi')=\psi_0[1+\sum_{\Omega'} \epsilon_{2\Omega'}Y_2{}^{\Omega'}(\theta',\varphi')], \tag{III 7}$$

so that

$$b_{l1}=\left|\frac{\sum_{K_f} A_{K_f}{}^{l1}(C_{lK_f}+\sum_{\Omega'}\epsilon_{2,-\Omega'}h_{lK_f;2,-\Omega'})}{C_{00}+\sum_{\Omega'}\epsilon_{2,-\Omega'}h_{00;2,-\Omega'}}\right|. \tag{III 8}$$

From (III 7) and (III 8) we see, as was pointed out before, that we shall need the matrix elements $h_{l-K_f;2,\Omega'}$ (which are given in the Appendix).

For nuclei having $\gamma \leq 15°$, the quantities $A_{K_f}{}^{l1}$ appearing in (III 8) can be approximated by

$$A_{K_f}{}^{l1}\simeq\delta_{0K_f}, \tag{III 9}$$

and thus (III 8) reduces to

$$b_{l1}=\left|\frac{C_{l0}+\sum_{\Omega'}\epsilon_{2,-\Omega'}h_{l0;2,-\Omega'}}{C_{00}+\sum_{\Omega'}\epsilon_{2,-\Omega'}h_{00;2,-\Omega'}}\right|. \tag{III 10}$$

Empirically, there is not much known about the varia-

[9] The theory of reference 6, which has been developed in a different analytical approach under the assumption that the alpha wave function on the nuclear surface is constant, also fails to show the dependence on Z of the relative alpha transition probabilities.

TABLE II. Theoretical b_{ll} values calculated from (III 8), the quantities β, ϵ_{20}, and $\epsilon_{2,\pm2}$ being determined by the empirical data. The quantities Q_0^2 and $Q_{\pm2}^2$ are calculated from (II 6a) by assuming $R_0=1.44\times10^{-13}(A-4)^{\frac{1}{3}}$ cm, where A is the mass number of the parent nucleus. The calculated values for b_{81} may be compared with the empirical values for Pu^{238} and Cm^{242} given in Table I.

Parent nucleus	Daughter nucleus	γ^0	β	b_{21}	b_{41}	b_{61}	ϵ_{20}	$\epsilon_{2,\pm2}$	b_{81}	Q_0^2 (barns)	$Q_{\pm2}^2$ (barns)
$_{90}Th^{230}$	$_{88}Ra^{226}$	20.4	0.21	1.30	0.77	0.10	4.73	−16.88	0.007	10.30	2.71
$_{92}U^{232}$	Th^{228}	9.7	0.24	1.24	0.60	0.31	−2.40	−72.81	0.11	12.66	1.53
$_{94}Pu^{238}$	U^{234}	13.1	0.24	1.02	0.22	0.30	−1.62	−23.58	0.10	12.65	2.08
$_{96}Cm^{242}$	Pu^{238}	8.13	0.25	1.00	0.12	0.36	−2.42	−23.08	0.10	14.20	1.44
$_{98}Cf^{246}$	Cm^{242}	14.4	0.25	0.77	0.21	0.32	−5.15	−219.1	0.09	14.02	2.55

tion of β with Z in the region of very heavy nuclei. Furthermore, the relative coefficients $\epsilon_{2\Omega}$ can only be determined from the empirical data, which is equivalent to setting the boundary conditions by the empirical data. However, a theory of alpha-particle formation within the nucleus may yield an analytical formula for the quantities $\epsilon_{2\Omega'}$. Since at the present moment no such theory exists, it seems most reasonable to consider the quantities $\epsilon_{2\Omega'}$ as adjustable parameters. We, therefore, adjust β and the quantities ϵ_{20} and $\epsilon_{2\pm2}$ such that the calculated values agree with the corresponding empirical values and obtain results that are shown in Table II. The parameters γ for the nuclei considered were determined from the ratio of observed energies of two of the rotational states of the daughter nucleus, as prescribed by the Davydov-Filippov model.[5] The calculated b_{ll} values can be checked only for two parent nuclei Pu^{238} and Cm^{242}, which are found to be in better agreement than those of Fröman.[4] For b_{81} values, a comparative study of the variation with Z cannot be made since except for Pu^{238} and Cm^{242} no such empirical data exist. It is seen that β increases slightly with increasing Z in the region $Z\geq88$[10], which differs significantly in character from that of Fröman's paper.[4] Table II also shows that the alpha wave function on the nuclear surface is strongly distorted by the deformation of the nuclear surface. The relative coefficients ϵ_{20} and $\epsilon_{2\pm2}$, which are the measure of such distortion, are given in columns eight and nine. It is seen that for all the nuclei considered the absolute values of $\epsilon_{2\pm2}$ are significantly larger than the corresponding values of ϵ_{20}. In Fig. 1 we give a plot of $|\Psi_0(\theta',\varphi')/\psi_0|$ against θ' for various values of φ' for the parent nucleus Cm^{242}, which is of much interest. One finds that the alpha wave function on the nuclear surface $\Psi_0(\theta',\varphi')$ is highly elongated at the equator and considerably lowered at the poles. At this point it is worth mentioning that in order to explain the $l=4$ alpha intensity for Cm^{242} Christy[1] suggested that one should either introduce some fourth-order deformation in the nuclear surface or decrease the alpha wave function on the nuclear surface near the poles. The result of Table II and Fig. 1 supports this suggestion. Finally, using the values of β and γ, we calculate Q_0^2 and $Q_{\pm2}^2$, according to formula (II 6a), for the daughter nuclei, by assuming $R_0=1.44\times10^{-13}(A-4)^{\frac{1}{3}}$ cm, where A is the mass number of the parent nucleus. These calculated values are shown in the last two columns of Table II. One may also use a smaller value for R_0, namely $R_0=1.21\times10^{-13}(A-4)^{\frac{1}{3}}$ cm, which changes only slightly the calculated values of Q_μ^2 shown in Table II.

B. Even-Parity Alpha Transitions in Odd-A Nuclei

The reduced transition probability $c_{l\tau}$ for the transition $(I_i)\rightarrow(I_f,\tau)$ is

$$c_{I_f,\tau}=\frac{\left\{\sum_l\left|\sum_{K_f\Omega}A_{K_f}{}^{I_f\tau}(-)^{\Omega}C(I_ilI_f;\,K_f+\Omega,\,-\Omega,\,K_f)\sum_{l'\Omega'}h_{l\Omega;\,l'\Omega'}\epsilon_{l'\Omega'}\right|^2\exp\left[-\frac{l(l+1)}{\eta}\left(\frac{2\eta}{kR_0}-1\right)^{\frac{1}{2}}\right]\right\}}{\left\{\sum_l\left|\sum_{K_f\Omega}A_{K_f}{}^{I_i\tau}(-)^{\Omega}C(I_ilI_i;\,K_f+\Omega,\,-\Omega,\,K_f)\sum_{l'\Omega'}h_{l,\Omega;\,l'\Omega'}\epsilon_{l'\Omega'}\right|^2\exp\left[-\frac{l(l+1)}{\eta}\left(\frac{2\eta}{kR_0}-1\right)^{\frac{1}{2}}\right]\right\}}. \quad \text{(III 11)}$$

For the case of even-parity alpha groups we must consider only even l values for the sum over l in both numerator and denominator of (III 11) and note that only terms for Ω even contribute to $c_{I_f\tau}$.

Again we discuss the transitions to the main rotational band of the daughter nucleus ($\tau=1$). Empirically there is nothing known about β and γ for the odd-A nuclei, and therefore the matrix elements $h_{l\Omega;\,l'\Omega'}$ cannot be determined. The quantities $A_{K_f}{}^{I_f\tau}$ can be determined by solving separately the Schrödinger equations for the rotational and intrinsic motion of the daughter nucleus,

[10] The method of reference 6 shows a decrease of β with increase of Z and A. However, by assuming that alpha wave function on the nuclear surface is constant, the determination of β from the empirical data on alpha decay can yield only the effective values but not the actual ones.

which requires elaborate computational work.[11] Thus, we make the following assumptions:

(i) The alpha wave function on the nuclear surface, $\Psi_0(\theta',\varphi')$, is given by (III 7).

(ii) For nuclei having $\gamma \leq 15°$, $\Psi_0(\theta',\varphi')$ are calculated by interpolating the corresponding function $\Psi_0(\theta',\varphi')$ for neighboring even–even nuclei to the Z value of the odd-A nucleus considered, and then multiplying the resulting function by an angular independent factor having an absolute value of 0.7, in order to account for the observed hindrance of the alpha transitions in odd-A nuclei compared to that in even–even nuclei.

(iii) For nuclei having $\gamma \leq 15°$, the quantities $A_{K_f}{}^{I_f1}$ are approximated by the formula

$$A_{K_f}{}^{I_f1} \simeq \delta_{I_iK_f}, \qquad \text{(III 12)}$$

which is a generalization of (III 11).

The assumption (III 12) can be justified only by calculating $A_{K_f}{}^{I_f1}$ as a function of γ and then comparing the values for $\gamma \leq 15°$ with (III 12). The assumption (III 12) implies that for odd-A nuclei having $\gamma \leq 15°$, like similar even–even nuclei, the alpha wave function contains only the angular momentum component $\Omega = 0$. The corresponding alpha transitions may then be looked upon as the usual "favored transitions."

Under the above assumptions (III 11) reduces to (for $\tau = 1$)

$$c_{I_f1} = \frac{\sum_l c_l' |C(I_ilI_f; I_i0I_i)|^2}{\sum_l c_l' |C(I_ilI_i; I_i0I_i)|^2}, \qquad \text{(III 13)}$$

where

$$c_l' = |C_{lI_i} + \sum_{\Omega'} h_{lI_i;2,-\Omega'}\epsilon_{2,-\Omega'}|^2 \times \exp\left[-\frac{l(l+1)}{\eta}\left(\frac{2\eta}{kR_0} - 1\right)^{\frac{1}{2}}\right]. \qquad \text{(III 13a)}$$

The formula (III 13a) is similar to the formula derived by Fröman[4] for nuclei having $\gamma = 0°$. However, the quantities c_l' appearing in (III 13) and defined by (III 13a) are different from the similarly defined quantities in Fröman's formula by the fact that the former quantities depend on both β and γ whereas the latter quantities depend on β and other higher deformation parameters, β_4, β_6 etc.

Even under the assumptions (i)–(iii), one faces difficulty in employing Eq. (III 13). For the quantities c_l', being dependent on both β and γ, cannot be determined without knowing empirically the latter quantities, and, as pointed out before, these latter quantities are not known for odd-A nuclei.

We therefore follow the procedure first suggested by Bohr *et al.*[3] and later employed by Fröman[4] to try to understand the so-called favored transitions of alpha groups in odd-A nuclei in terms of the alpha transitions in neighboring even–even nuclei. Under the assumptions (i)–(iii), one may expect that the approximate favored transition probabilities in odd-A nuclei ($\gamma \leq 15°$) will, in some respects, be similar to that in similar neighboring even–even nuclei. The quantities c_l' for an odd-A nucleus (III 13a) are then proportional to the reduced transition probabilities c_{l1} for even–even nuclei, interpolated to the Z value of the odd-A nucleus considered. Hence in Eq. (III 13), we may simply replace c_l' values by these interpolated values. This then makes it possible to calculate the reduced transition probabilities for the approximate "favored" alpha groups of odd-A nuclei ($\gamma \leq 15°$) on the basis of the empirically determined reduced transition probabilities for the alpha groups of even–even nuclei ($\gamma \leq 15°$). The formula thus modified becomes exactly the same as the corresponding formula used by Fröman.[4] The results are to be found in Table III.

When the empirical and theoretical values are compared one finds very good agreement, considering the approximations involved in the theoretical formula and the uncertainties in the empirical data. From this comparison, one concludes that if the odd-A nuclei considered are axially asymmetric, then the parameters γ associated with the asymmetry of these nuclei must not exceed 15°. This fact also supports our assumptions (i)–(iii) for these odd-A nuclei, which should fairly well represent the odd-A nuclei in the very heavy region ($Z \geq 91$). The results of Table III, however, do not necessarily prove the existence of nonaxial nuclei, for the empirical data could also be accounted for by assuming $\gamma = 0$.

IV. CONCLUSIONS

From the work of Sec. III, one can conclude two things of importance regarding the alpha decay of nonaxial nuclei. The first is that for deformed triaxial ellipsoids the alpha wave function on the nuclear surface cannot be represented by a constant function, if one wants to account for the variational character of the empirical reduced transition probabilities with respect to both l and Z. On the other hand, if the wave function on the nuclear surface is assumed to be equal to a constant function then the multipole moments higher than the quadrupole ones should also be considered in the effective nuclear deformation. This, however, requires as many free parameters, which are the coefficients in the expansion of the surface in terms of the spherical harmonics $Y_\lambda{}^\mu(\theta',\varphi')$, as there are alpha groups to be explained. But, the present theory is based on the quadrupole-type deformation of the surface, and one should expect that the wave function on the nuclear

[11] Chi and Davidson have obtained the energy levels of odd-A nuclei in the region of $A \simeq 25$, by diagonalizing the collective Hamiltonian, and have calculated these quantities [B. E. Chi and J. P. Davidson, Bull. Am. Phys. Soc. 6, 233 (1961); and J. P. Davidson and B. E. Chi, in *Proceedings of the Conference of Electromagnetic Lifetimes and Properties of Nuclear States, Gatlinburg, 1961* (Publication No. 974, National Research Council—National Academy of Sciences, Washington, D. C., 1962)]. It is expected that their calculations will extend to the region of $A \geq 220$, when it will be possible to apply the formula (III 11).

TABLE III. Theoretical reduced transition probabilities $c_{I_f 1}$, calculated from (III 13), by replacing the c_l' values by the reduced transition probabilities of the neighboring even–even nuclei, interpolated to the Z value of the odd-A nucleus considered.

Parent nucleus	I_i	I_f	$c_{I_f 1}$ (emp.)	c_{l1} (interpolated) $l=0$	$l=2$	$l=4$	$l=6$	$c_{I_f 1}$ (theor.)
$_{91}Pa^{231}$	$3/2^-$	$3/2^-$	1	1	0.87	0.091		1
		$5/2^-$	0.32					0.38
		$7/2^-$						0.23
$_{92}U^{233}$	$5/2^+$	$5/2^+$	1	1	0.83	0.971		1
		$7/2^+$	0.34					0.31
		$9/2^+$	0.093					0.13
		$11/2^+$						0.018
$_{94}Pu^{239}$	$1/2^+$	$1/2^+$	1	1	0.63	0.011	0.0026	1
		$3/2^+$	0.35					0.26
		$5/2^+$	0.35					0.39
		$7/2^+$						0.0049
$_{95}Am^{241}$	$5/2^-$	$5/2^-$	1	1	0.59	0.004	0.0031	1
		$7/2^-$	0.31					0.24
		$9/2^-$	0.083					0.083
		$11/2^-$	0.0018					0.0016
		$13/2^-$	0.00076					0.0013
Am^{243}	$5/2^-$	$5/2^-$	1	1	0.59	0.004	0.0031	1
		$7/2^-$	0.25					0.24
		$9/2^-$	0.052					0.083
		$11/2^-$						0.0016
		$13/2^-$						0.0013
$_{96}Cm^{243}$	$5/2^+$	$5/2^+$	1	1	0.57	0.0017	0.0033	1
		$7/2^+$	0.29					0.23
		$9/2^+$						0.080
$_{99}Es^{253}$	$(7/2^+)$	$(7/2^+)$	1	1	0.29	0.014	0.0022	1
		$(9/2^+)$	0.13					0.11
		$(11/2^+)$	0.050					0.033
		$(13/2^+)$						0.0034

surface, being coupled with the latter, is strongly distorted about some average value corresponding to a spherical nucleus. The empirical data can be explained if one assumes a quadrupole type distortion, the two expansion coefficients of which being determined by the empirical data, i.e., setting the boundary condition by the empirical data. (The possibility of determining these coefficients analytically may result from a quantitative theory of the alpha particle formation.) Unfortunately, there exists no experimental information about Q_μ^2 of the nuclei considered in the region $Z \geq 88$, and therefore the quantities β for these nuclei are also considered as adjustable parameters, which would, otherwise, have been obtained from the experimental values of Q_μ^2. Thus, there are actually three parameters β, ϵ_{20}, and $\epsilon_{2\pm 2}$ which are adjusted to account for the empirical data. The theory predicts the b_{81} values, which may be compared with the empirical values only for Pu^{238} and Cm^{242}. It is found that the present theory predicts b_{81} values for these two nuclei which are significantly better than those predicted by other theories.[4,6]

The second major conclusion is that both the deformation parameter β and the shape parameter γ strongly affect the alpha transition probabilities. For $\gamma \leq 15°$, the alpha transitions to the main rotational band ($\tau = 1$) of even–even nuclei are found to be approximately favored. The same is true for odd-A nuclei with $\gamma \leq 15°$ under the assumptions (i)–(iii). Furthermore, under these assumptions the reduced transition probabilities for odd-A nuclei can be obtained in terms of the empirically known reduced transition probabilities for the neighboring even–even nuclei. Although this approximate method is similar, in formalism, to that of Fröman[4] one cannot say definitely that the odd-A nuclei considered are nonaxial. For the empirical data for these nuclei are explained in terms of those for even–even nuclei which may not be axially symmetric, as is shown by the present theory. One may expect that the odd-A nuclei considered are also nonaxial. But the present approximate method does not prove the existence of the nonaxial odd-A nuclei in the very heavy region ($Z \geq 91$). All that can be said is that if nonaxial odd-A nuclei do exist in this region then γ for these nuclei should probably be less than 15°. However, the present theory provides necessary formulas for odd-A nuclei, which are free from the assumptions (i)–(iii). And therefore by solving the Schrödinger equation for the collective motion of the daughter nucleus one can determine the coefficients $A_{K_f}{}^{I_f}$ as a function of γ, which then can be used to calculate, from the theoretical formula, the reduced transition probabilities of various alpha groups. It is expected that the slight discrepancy between the theoretical and the empirical values of Table III may be removed in this way. The numerical calculations in this respect are in progress and will be reported later.[11]

The theory is developed for the even-parity alpha groups. However, it can be extended to include the odd-parity alpha groups by assuming asymmetric vibrations of the nuclear surface corresponding to odd order spherical harmonics in the deformation. The simplest

case would be to consider Y_3^μ terms in the deformation (octupole deformation).[12] However, the relatively few cases of these groups thus far observed do not, at the present moment, warrent such consideration.

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APPENDIX

The calculations of the matrix elements $h_{l\Omega;l'\Omega'}$ are facilitated by making use of the expansion

$$\exp[\sum_\mu B_\mu Y_2^\mu(\theta',\varphi')]=\sum_{n,m} C_{nm}Y_n^m(\theta',\varphi').$$

The coefficients C_{nm} are functions of β and γ through the parameters B_μ given in Eq. (III 5b). Only the values of $h_{l\Omega;l'\Omega'}$ are listed for $\Omega=K_f$ and $l'=2$.

$$h_{00;20}=0.2821C_{20},$$

$$h_{00;2,-2}=h_{00;22}=0.2821C_{22},$$

$$h_{20;20}=0.2821C_{00}+0.1803C_{20}+0.2418C_{40},$$

$$h_{20;2,-2}=h_{20,22}=-0.18C_{22}+0.0588C_{42},$$

$$h_{22;20}=-0.18C_{22}+0.0404C_{42},$$

$$h_{22;2,-2}=0.3375C_{44},$$

$$h_{22;2,2}=0.2821C_{00}-0.1803C_{20}+0.0404C_{40},$$

$$h_{40;20}=0.2417C_{20}+0.164C_{40}+0.2384C_{60},$$

$$h_{40;2,-2}=h_{40;22}=0.18C_{22}-0.1912C_{42}+0.1299C_{62},$$

$$h_{42;20}=0.15C_{22}+0.0662C_{42}+0.2067C_{62},$$

$$h_{42;2,-2}=-0.1063C_{44}+0.2307C_{64},$$

$$h_{42;22}=0.1563C_{20}-0.1904C_{40}+0.062C_{60},$$

$$h_{44;20}=-0.2292C_{44}+0.1067C_{64},$$

$$h_{44;2,-2}=0.3563C_{66},$$

$$h_{44;22}=0.3351C_{22}-0.1063C_{42}+0.0155C_{62},$$

$$h_{60;20}=0.2389C_{40}+0.1609C_{60}+0.2405C_{80},$$

$$h_{60;2,-2}=h_{60;22}=0.0625C_{42}-0.1948C_{62}+0.125C_{82},$$

$$h_{62;20}=0.2063C_{42}+0.115C_{62}+0.2222C_{82},$$

$$h_{62;2,-2}=0.0158C_{42}-0.1538C_{62}+0.1893C_{82},$$

$$h_{62;2,2}=0.133C_{40}-0.1804C_{60}+0.0706C_{80},$$

$$h_{64;20}=0.1083C_{44}-0.0227C_{64}+0.1686C_{84},$$

$$h_{64;2,-2}=-0.1437C_{66}+0.2625C_{86},$$

$$h_{64;22}=0.2305C_{42}-0.1538C_{62}+0.0329C_{82},$$

$$h_{66;20}=-0.25C_{66}+0.081C_{86},$$

$$h_{66;2,-2}=0.3599C_{88},$$

$$h_{66;22}=0.3542C_{44}-0.076C_{64}+0.0078C_{84},$$

$$h_{80;20}=0.2384C_{60}+0.1639C_{80}+0.2372C_{10\ 0},$$

$$h_{80;2,-2}=h_{80;22}=0.0714C_{62}-0.1934C_{82}+0.1173C_{10\ 2},$$

$$h_{82;20}=0.1938C_{62}+0.1317C_{82}+0.2267C_{10\ 2},$$

$$h_{82;2,-2}=0.0327C_{64}-0.1706C_{84}+0.1667C_{10\ 4},$$

$$h_{82;22}=0.1231C_{60}-0.1927C_{80}+0.0764C_{10\ 0},$$

$$h_{84;20}=0.5987C_{64}+0.0529C_{84}+0.1947C_{10\ 4},$$

$$h_{84;2,-2}=0.0088C_{66}-0.1263C_{86}+0.2249C_{10\ 6},$$

$$h_{84;22}=0.1889C_{62}-0.1708C_{82}+0.0441C_{10\ 2},$$

$$h_{86;20}=0.0813C_{66}-0.075C_{86}+0.1415C_{10\ 6},$$

$$h_{86;2,-2}=-0.060C_{88}+0.2916C_{10\ 8},$$

$$h_{86;22}=0.2686C_{64}-0.0627C_{84}+0.0204C_{10\ 4},$$

$$h_{88;20}=-0.30C_{88}+0.0652C_{10\ 8},$$

$$h_{88;2,-2}=0.3669C_{10\ 10},$$

$$h_{88;22}=0.3624C_{66}-0.0625C_{86}+0.0053C_{10\ 6},$$

where C_{nm} are given by

$$\begin{aligned}C_{00}=3.545&+0.141B_0^2+0.282B_2^2+0.0085B_0^3\\&-0.0533B_0B_2^2+0.002B_0^4+0.0088B_0^2B_2^2\\&-0.00079B_0^3B_2+0.00154B_0B_2^3+0.0065B_2^4\\&+0.00024B_0^5-0.000089B_0^4B_2-0.00036B_0^3B_2^2\\&-0.0004B_0^2B_2^3-0.00146B_0B_2^4+\cdots,\end{aligned}$$

$$\begin{aligned}C_{20}=B_0&+0.091B_0^2-0.181B_2^2+0.0285B_0^3\\&+0.057B_0B_2^2+0.0043B_0^4+0.0005B_0^3B_2\\&-0.004B_0^2B_2^2-0.001B_0B_2^3-0.00554B_2^4\\&+0.00043B_0^5+0.00003B_0^4B_2+0.00076B_0^3B_2^2\\&+0.00031B_0^2B_2^3+0.00179B_0B_2^4+\cdots,\end{aligned}$$

$$\begin{aligned}C_{22}=C_{2,-2}=B_2&-0.181B_0B_2-0.0055B_0^3+0.034B_0^2B_2\\&+0.011B_0B_2^2+0.046B_2^3-0.001B_0^4\\&-0.00117B_0^3B_2-0.0031B_0^2B_2^2-0.0102B_0B_2^3\\&-0.00012B_0^5+0.00032B_0^4B_2+0.00019B_0^3B_2^2\\&+0.00168B_0^2B_2^3+0.00036B_0B_2^4+0.00081B_2^5\\&+\cdots,\end{aligned}$$

$$\begin{aligned}C_{40}=0.121B_0^2&+0.04B_2^2+0.014B_0^3-0.037B_0B_2^2\\&+0.0033B_0^4-0.00011B_0^3B_2+0.0046B_0^2B_2^2\\&+0.0002B_0B_2^3+0.0011B_2^4+0.00042B_0^5\\&+0.000025B_0^4B_2-0.00037B_0^3B_2^2\\&-0.00013B_0^2B_2^3-0.0011B_0B_2^4+\cdots,\end{aligned}$$

$$\begin{aligned}C_{42}=C_{4,-2}=0.156B_0B_2&-0.009B_0^2B_2-0.018B_2^3\\&-0.00022B_0^4+0.0027B_0^3B_2+0.00042B_0^2B_2^2\\&+0.007B_0B_2^3-0.000033B_0^5-0.000017B_0^4B_2\\&-0.000058B_0^3B_2^2-0.00075B_0^2B_2^3\\&-0.000058B_0B_2^4-0.00036B_2^5+\cdots,\end{aligned}$$

[12] P. O. Lipas and J. P. Davidson, Nuclear Phys. **26**, 80 (1961) and P. O. Lipas, Doctor's thesis, Rensselaer Polytechnic Institute, 1961 (unpublished).

$$C_{44}=C_{4,-4}=0.169B_2^2-0.0387B_0B_2^2-0.00046B_0^3B_2+0.0062B_0^2B_2^2+0.001B_0B_2^3+0.006B_2^4-0.000045B_0^4B_2-0.00044B_0^3B_2^2-0.00026B_0^2B_2^3-0.00128B_0B_2^4+\cdots,$$

$$C_{60}=0.0097B_0^3+0.0095B_0B_2^2+0.0012B_0^4-0.00063B_0^2B_2^2-0.00029B_2^4+0.00023B_0^5-0.000011B_0^4B_2+0.00032B_0^3B_2^2+0.000021B_0^2B_2^3+0.00013B_0B_2^4+\cdots,$$

$$C_{62}=C_{6,-2}=0.016B_0^2B_2+0.0027B_2^3-0.00175B_0B_2^3-0.001B_0^3B_2-0.000009B_0^5+0.000235B_0^4B_2+0.000013B_0^3B_2^2+0.00057B_0^2B_2^3+0.000048B_2^5+\cdots,$$

$$C_{64}=C_{6,-4}=0.018B_0B_2^2-0.0008B_0^2B_2^2-0.00063B_2^4-0.00002B_0^4B_2+0.00022B_0^3B_2^2+0.000042B_0^2B_2^3+0.00026B_0B_2^4+0.000083B_2^5+\cdots,$$

$$C_{66}=C_{6,-6}=0.016B_2^3-0.0021B_0B_2^3-0.000037B_0^3B_2^2+0.0004B_0^2B_2^3+0.000074B_0B_2^4+0.0006B_2^5+\cdots,$$

$$C_{80}=0.00058B_0^4+0.00113B_0^2B_2^2+0.0001B_2^4+0.000075B_0^5-0.00011B_0^3B_2^2-0.00011B_0B_2^4+\cdots,$$

$$C_{82}=C_{8,-2}=0.00117B_0^3B_2+0.00058B_0B_2^3+0.000079B_0^4B_2-0.00018B_0^2B_2^3-0.000021B_2^5+\cdots,$$

$$C_{84}=C_{8,-4}=0.0015B_0^2B_2^2+0.00017B_2^4-0.000092B_0^3B_2^2-0.00015B_0B_2^4+\cdots,$$

$$C_{86}=C_{8,-6}=0.00154B_0B_2^3-0.00013B_0^2B_2^3-0.00005B_2^5+\cdots,$$

$$C_{88}=C_{8,-8}=0.00146B_2^4-0.00025B_0B_2^4+\cdots,$$

$$C_{10,0}=0.000028B_0^5+0.000071B_0^3B_2^2+0.000013B_0B_2^4+\cdots,$$

$$C_{10,2}=C_{10,-2}=0.000052B_0^4B_2+0.000013B_0^2B_2^3+0.000026B_0B_2^4+0.0000015B_2^5+\cdots,$$

$$C_{10,4}=C_{10,-4}=0.0001B_0^3B_2^2+0.000033B_0B_2^4+\cdots,$$

$$C_{10,6}=C_{10,-6}=0.00011B_0^2B_2^3+0.000033B_0B_2^4+\cdots,$$

$$C_{10,8}=C_{10,-8}=0.00011B_0B_2^4+\cdots,$$

$$C_{10,10}=C_{10,-10}=0.00011B_2^5+\cdots.$$

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Gamma Decay of the 7.57-MeV Level of N^{15}†*

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The $C^{12}(\alpha,p\gamma)N^{15}$ reaction was studied using particle-gamma coincidence techniques and the 22-MeV alpha-particle beam of the Indiana University cyclotron. The 7.57-MeV level of N^{15} is observed to decay by gamma emission to either the 5.28- or 5.30-MeV level. An upper limit of 10% is placed on the intensity of the ground-state branch. This information, together with the results of other experiments, suggests a spin and parity assignment of $\frac{5}{2}^+$ or $\frac{7}{2}^+$ for the 7.57-MeV level, which is consistent with shell-model predictions.

I. INTRODUCTION

The shell-model calculations of Halbert and French[1] have been successful in predicting many of the properties of the positive-parity states of N^{15}: a spin of $\frac{5}{2}^+$ or $\frac{7}{2}^+$ is predicted for the 7.57-MeV level. The $N^{14}(d,p)$ stripping reaction studies of Green and Middleton[2] and Warburton and McGruer[3] indicate that the 7.57-MeV level has positive parity and spin $\leqslant\frac{7}{2}$. The present experiment was undertaken to help determine the spin of the 7.57-MeV level.

II. APPARATUS AND PROCEDURES

Gamma radiation from N^{15} was studied employing the $C^{12}(\alpha,p\gamma)N^{15}$ reaction with the 22-MeV alpha-particle beam of the Indiana University cyclotron. Standard fast-slow coincidence circuitry was arranged so that gamma rays could be studied in time coincidence with charged reaction particles of a selected energy. In this way it was possible to study the decay modes of individual levels resolved by the particle detector. As a check on the interpretation of the coincidence gamma-ray spectra, measurements were also made of charged-

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