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Photodisintegration of Helium†

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The integrated electric dipole absorption cross section σ_{int} and bremsstrahlung-weighted cross section σ_b are calculated for He^4. By using a two-body interaction operator having a Serber exchange character with a repulsive core and tensor component, it is found that $\sigma_{\text{int}}=107$ MeV-mb and $\sigma_b=2.73$ mb, in reasonable agreement with the current experimental values: $\sigma_{\text{int}}=95\pm7$ MeV-mb and $\sigma_b=2.4\pm0.15$ mb.

RECENT experimental studies[1] on the photodisintegration of He^4 have indicated the need for a theoretical re-evaluation of the integrated electric dipole absorption cross section,

$$\sigma_{\text{int}}=\int_0^\infty \sigma(W)dW, \tag{1}$$

and bremsstrahlung-weighted cross section,

$$\sigma_{\text{b}}=\int_0^\infty \sigma(W)W^{-1}dW. \tag{2}$$

Here $\sigma(W)$ represents the total electric dipole absorption cross section at incident energy W. Since σ_{int} and σ_{b} can be expressed quite generally through sum rules in terms of the ground-state wave function for the system,[2] it is expected that any appreciable discrepancy with experimental values must be attributed to defects in the selected ground state function or interaction operator. It is the intent of the present note to show that by an appropriate choice of the interaction operator and wave function, discrepancy between experiment and theory can be resolved.

In the sum rule calculations of Levinger and Bethe,[2] and Rustgi and Levinger,[3] it is shown that Eq. (1) can be written in the form

$$\sigma_{\text{int}}=[2\pi^2e^2\hbar/(Mc)]\Big\{NZ/A\,(Mx/3\hbar^2)]\times\int\Psi_0^*\sum_{i,j}V(r_{ij})r_{ij}^2P_{ij}^M\Psi_0d\tau-[My/(3\hbar^2)]\times\int\Psi_0^*\sum_{i,j}V(r_{ij})r_{ij}^2P_{ij}^H\Psi_0d\tau\Big\}, \tag{3}$$

where the summations extend over i protons and j neutrons, and x and y are the fractions of Majorana and Heisenberg exchange forces present in the two-body interaction operator $V(r_{ij})$. The other notation follows that of reference 3. Similarly, Foldy[4] has shown that σ_{b} is directly related to the mean square radius R^2 of the nucleus,

$$\sigma_{\text{b}}=(4\pi^2/3)[e^2/(\hbar c)][zN/(A-1)]R^2, \tag{4}$$

insofar as the ground-state wave function ψ_0 is symmetric in the space coordinates of all the nucleons.

The current experimental values for σ_{int} and σ_{b} as derived from photonuclear[1,3] and electron scattering[4] data are listed in Table I along with the previous theoretical estimates of Rustgi and Levinger.

Although the Rusti-Levinger theoretical results for σ_{int} compare not unfavorably with the latest value of Gorbunov and Spiridonov, the corresponding theoretical bremsstrahlung-weighted cross section yields an rms radius for He^4 that is much smaller than that required by the electron scattering data[5] ($R=1.44\times10^{-13}$ cm). That this discrepancy is due to the unrealistic nature[6] of the assumed wave function has already been suggested by Rustgi and Levinger. In view of the recent results of Clark,[7] the neglect of repulsive core effects,[8] in particular, appears unwarranted.

In the present calculation, we employ a two-body interaction operator having a Serber exchange character

TABLE I. Values of σ_{int} and σ_{b} for He^4.

	σ_{int} (MeV-mb)	σ_{b} (mb)
Experiment[a]	95±7	2.4 ±0.15
Experiment[b]	124	2.7
Theory[c]	86	1.23
Theory[d]	102	1.23

[a] Reference 1.
[b] Reference 3. σ_{int} is based on a compilation of photonuclear data, while σ_{b} is on the electron scattering data of reference 5 ($\sigma_{\text{b}}=2.66$ mb).
[c] Reference 3. Based on Irving's variational wave function with Serber mixture.
[d] Reference 3. Based on Irving's variational wave function with Rosenfeld or Inglis mixture.

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[1] A. Gorbunov and V. Spiridonov, J. Exptl. Theoret. Phys. (U. S. S. R.) **33**, 21 (1957) [Soviet Phys.—JETP **6**, 16 (1958)]; **34**, 862, 866 (1958) [Soviet Phys.—JETP **7**, 596, 600 (1958)].
[2] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 116 (1950), and J. S. Levinger, *Nuclear Photodisintegration* (Oxford University Press, New York, 1960).
[3] M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957).
[4] L. L. Foldy, Phys. Rev. **107**, 1303 (1957).
[5] R. W. McAllister and R. Hofstadter, Phys. Rev. **102**, 851 (1956).
[6] In both the work of reference 3 and that of R. H.Bransden, A. C. Douglas, and H. H. Robertson [Phil. Mag. **2**, 1211 (1957)], the effect of a repulsive core was not considered.
[7] J. W. Clark, Can. J. Phys. **39**, 385 (1961).
[8] The question of whether the repulsive core effect might not actually be due to a velocity dependence has not been touched upon. It appears, however, that the two effects may be essentially indistinguishable as far a σ_{int} is concerned; see O. Roj and J. S. Levinger, Phys. Rev. **123**, 2177 (1961).

TABLE II. Contributions to σ_{int} (in MeV-mb) for He^4.

Type of contribution	Reference 3[a]	Eqs. (3), (5), and (6)
Nonexchange	60	60
1st order central	...	28
2nd order central	...	−0.3
Total central	20.4	27.7
Total tensor	5.5	19.7
Total	86	107

[a] For Serber mixture.

($x=0.5$, $y=0$) with a repulsive core and tensor component,

$$\begin{aligned} V(r_{ij}) = {} & J_R \exp[-r_{ij}^2/r^2] \\ & + J_C[(1-\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(3+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)/16 \\ & + (3+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(1-\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)/16] \exp(-r_{ij}^2/r_0^2) \\ & + J_S[(1-\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)/4](r_{ij}^2/r_0^2) \\ & \times \exp(-r_{ij}^2/r_0^2)(\boldsymbol{\sigma}_i\cdot\mathbf{n}_{ij}\boldsymbol{\sigma}_j\cdot\mathbf{n}_{ij} - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j/3)], \end{aligned} \tag{5a}$$

where $\mathbf{r}_{ij}=\mathbf{n}_{ij}r_{ij}$ is the separation vector for particles i and j. The parameters,

$$\begin{aligned} & J_R = +189.75 \text{ MeV}, \quad J_C = -58.65 \text{ MeV}, \\ & J_S = -107.29 \text{ MeV}, \quad r_0 = 1.54\times10^{-13}\text{cm}, \\ & r = r_0/\sqrt{8}, \end{aligned} \tag{5b}$$

have those values previously determined[9] to give a reasonable fit to the stationary properties of H^2, H^3, He^3, He^4 within the accuracy of the Bolsterli–Feenberg perturbation procedure.[10] For He^4, the second-order perturbation calculation yields a binding energy of 28.4 MeV and an rms radius $R=1.46\times10^{-13}$ cm to be compared with the experimental values of 28.2 MeV and 1.44×10^{-13} cm.

In both the previous[9] and present calculation, the interaction operator (5a)–(5b) is used to generate a perturbed wave function,

$$\Psi_0 = N^{-1}\{\psi_0 + \sum_{n\neq0}(E-E_n)^{-1} \times [\sum_{i<j} V(r_{ij}) - \tfrac{1}{2}(M\omega^2/A)\sum_{i<j} r_{ij}^2]_{n0}\psi_n\} \tag{6}$$

where ψ_0 is taken to be a single determinant of s-state particle orbitals belonging to a sum of single-particle harmonic oscillator Hamiltonians with $\hbar\omega=24.15$ MeV, and $N^2=1.14$. The particular choice of ψ_0 in terms of oscillator functions permits the sum over n to be simply expressed in closed form.[10]

Substituting Eqs. (5) and (6) into (3), and carrying out the calculation through second order in $V(r_{ij})$, we arrive at a value of $\sigma_{\text{int}}=107$ MeV-mb. The individual contributions to the integrated cross section are shown in Table II. It can be seen that the principal contribution from the central force arises in first order, the cancellation being almost complete between the repulsive and attractive terms in second order. For comparison we also list the corresponding contributions found by Rustgi and Levinger using the Pease–Feshbach[11] potential with Irving's[12] wave function. For both the central and tensor part, we find a larger value. This is partly due to the fact that Irving's wave function contains only a 2.6% D state while Eq. (6) has a 10.6% D state. The presence of a large D state particularly enhances the tensor contribution.

Since the above value of $\sigma_{\text{int}}=107$ MeV-mb was derived using $R=1.46\times10^{-13}$ cm, the related bremsstrahlung-weighted cross section is found from Eq. (4) to be $\sigma_{\text{b}}=2.73$ mb. Reference to Table I shows that both of these values are in satisfactory agreement with the experimental results. Although the data of Gorbunov and Spiridonov are slightly lower, it should be noted that their results lead to a charge radius for He^4 of $(1.57\pm0.06)\times10^{-13}$ cm, rather than the $(1.61\pm0.08)\times10^{-13}$ cm found from electron scattering. Perhaps an interesting number for comparison here is the harmonic mean energy $W_H=\sigma_{\text{int}}/\sigma_{\text{b}}$ which gives a measure of the position of the resonance peak.[2] Using $\sigma_{\text{int}}=95\pm7$ MeV-mb and $\sigma_{\text{b}}=2.4\pm0.15$ we find $W_H=39.6$ MeV. The results of the present calculation give $W_H=39.2$ MeV which is in quite reasonable agreement, implying that the photodisintegration of the α particle may be consistently interpreted in terms of a simple interaction operator of the form displayed in Eq. (5a).

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[9] P. Goldhammer, Phys. Rev. **116**, 676 (1959).
[10] M. Bolsterli and E. Feenberg, Phys. Rev. **101**, 134 (1956).
[11] R. L. Pease and H. Feshbach, Phys. Rev. **88**, 945 (1952).
[12] J. Irving, Proc. Phys. Soc. (London) **A66**, 17 (1953).