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Nuclear Energy Levels in P^{29}†

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Excited levels in P^{29} have been observed as resonances in the differential elastic and inelastic scattering cross sections for protons incident upon natural silicon (92.2% Si^{28}) at proton energies from 4.8 to 7.0 MeV. Resonances corresponding to twenty-three levels in P^{29} from 7.4- to 9.5-MeV excitation were observed. The orbital angular momenta of the incident protons were determined for most of the resonances, using single-level dispersion theory.

I. INTRODUCTION

EXCITED levels in P^{29} resulting from the bombardment of Si^{28} by protons of less than 5-MeV bombarding energy have been investigated by several authors.[1–6] Among the latest work is that of Belote, Kashy, and Risser,[4] who measured the differential elastic scattering cross section from 2 to 5 MeV and inelastic cross section from 3 to 5.3 MeV; of Vorona, Olness, Hoeberli, and Lewis,[5] who investigated the energy region from 1.3 to 3.8 MeV; and of Cohen and Cookson,[6] who investigated the region between 10.0 and 12.3 MeV. However, levels excited by protons of energy from 5.0 to 8 MeV have not been previously resolved. The presence of resonances in the energy range of 4.9 to 5.5 MeV was well demonstrated by Oda, Takeda, Hu, and Kato,[7] who measured angular distributions of both $Si^{28}(p,p)$ and $Si^{28}(p,p')$ which varied greatly as a function of bombarding energy. The present work[8] covers the energy region from 4.8 to 7.0 MeV, thus giving a slight overlap with the work reported in reference 4. Since the separation energy of the last proton in P^{29} is only 2.724 MeV, relatively low-lying excited levels are reached by $Si^{28}+p$. This explains why several well-defined resonances are observed in the energy region investigated. One would expect a number of single-particle levels in P^{29} from the addition of a proton to the Si^{28} core and an energy-level scheme very similar to that of Si^{29}.

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II. EXPERIMENTAL PROCEDURE

The protons were accelerated by the MIT-ONR electrostatic accelerator. The protons scattered from the silicon target were deflected in the broad-range magnetic spectrograph into a scintillation counter consisting of a thin thallium-activated CsI crystal attached to an RCA 6199 photomultiplier tube. A 1-cm entrance slit situated in front of the crystal defined the window. The counter was at the focal surface of the spectrograph and observed protons with a trajectory radius from 50 to 51.8 cm. This region was about four times the half-width of the proton group. The bombarding energy was determined by means of an analyzing magnet of 60.51-cm radius, with slits having a width equivalent to 3-keV beam spread for 6-MeV protons. When measuring the cross sections, the magnetic field of the analyzing magnet was changed in steps corresponding to 10-, 5-, or 2.5-keV energy intervals. The scattered protons were maintained in the window by varying the field of the spectrograph.

The targets used were made by evaporating natural silicon dioxide onto thin Formvar film backings. The yields from Si^{29} and Si^{30} were assumed to be low because of their small abundance.

In order to obtain the absolute differential cross section, the excitation curves were normalized, using data by Belote *et al.*[4] in the region where the experiments overlap. The resulting cross sections are for natural silicon in the case of elastic scattering and for Si^{28} in the case of inelastic scattering. The probable error in the absolute cross section was estimated to be $\pm 16\%$. This figure does not include the statistical error of counting which, in general, was less than 2%.

III. RESULTS

We have measured the $Si(p,p)$ cross section at center-of-mass angles of 92.1 and 125.3 deg and the cross section for the $Si^{28}(p,p')Si^{28}$ reaction, which leaves the Si^{28} nucleus in its first excited level, at a laboratory angle of 90 deg. These curves are shown in Fig. 1. We have also measured the elastic scattering cross section at a center-of-mass angle of 54.7 deg, and this measurement helped considerably to facilitate the analysis. The energies and widths of the resonances observed, together with the excitation energies of the corresponding P^{29} levels, are

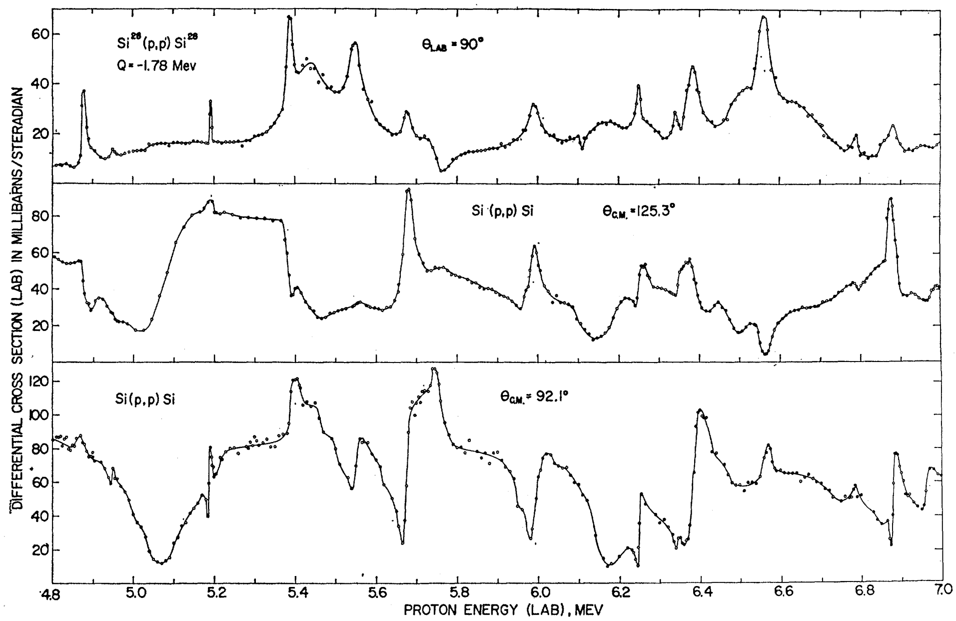

FIG. 1. Differential cross section for the $Si(p,p)Si$ and $Si^{28}(p,p')Si^{28}$ reactions.

listed in Table I. The uncertainties of excitation energies in P^{29} are of approximately 15 keV. Clearly, for some of the very broad levels, the uncertainty is considerably greater.

In order to interpret the resonance phenomena observed, a number of theoretical curves expected for the elastic scattering cross section were calculated for various possible values of total angular momentum and parity J, of the excited levels using single-level dispersion theory. The expression used for the differential cross sections was[9]

$$\sigma = \lambda\!\!\!^{-2}[|A|^2+|B|^2],$$

where

$$A = -(\eta/2)\csc^2(\theta/2)\exp[i\eta\ln\csc^2(\theta/2)] + \sum_{l=0}^{\infty}(l+1)P_l(\cos\theta)\exp(i\alpha_l+i\delta_l^+)\sin\delta_l^+ + \sum_{l=1}^{\infty} lP_l(\cos\theta)\exp(i\alpha_l+i\delta_l^-)\sin\delta_l^-,$$

and

$$B = \sin\theta\sum_{l=1}^{\infty}P_l'(\cos\theta)e^{i\alpha_l}[\sin\delta_l^-\exp(i\delta_l^-) - \sin\delta_l^+\exp(i\delta_l^+)].$$

Furthermore,

$$\eta = ZZ'/\hbar v,\quad \lambda\!\!\!^{-} = \hbar/mv,$$

and

$$\alpha_0 = 0,\quad \alpha_l = 2\sum_{s=1}^{l}\tan^{-1}(\eta/s)\quad \text{for}\quad l>0,$$

with v being the relative velocity and m the reduced mass of the system. The phase shifts $\delta_l^{\pm}$ were calculated as $\delta_l^{\pm} = \beta_l^{\pm} + \phi_l$. For β_l^+ and β_l^- the expression $\tan^{-1}[\frac{1}{2}\Gamma/(E_R-E)]$ was used. Here Γ = total width of the resonance and E_R is the resonant energy.

Theoretical curves for the differential elastic scattering cross section were calculated for an incident proton energy of 6.0 MeV. Two sets of values of the hard-sphere phases were assumed; these were: $\phi_0 = -43°$, $\phi_1 = -22.1°$, $\phi_2 = -7.2°$, $\phi_3 = -2.5°$, and $\phi_0 = -69.3°$, $\phi_1 = -42.4°$, $\phi_2 = -17.2°$, $\phi_3 = -4.5°$. The results of

TABLE I. Summary of results for the energy levels of P^{29}.

Excitation energy (MeV)	E_p(lab) (MeV)	Γ(lab) (keV)	l_p	J^π	$(2mR/3\hbar^2)\gamma_p^2$ [a]
7.43	4.88	12±3	3	$\frac{5}{2}^-$ [b]	0.013
7.50	4.95	7±3	2	$\frac{5}{2}^+$, $(\frac{3}{2}^+)$	0.0024
7.62	5.07	165±25	0	$\frac{1}{2}^+$	0.025
7.74	5.19	∼2	2	$\frac{5}{2}^+$, $(\frac{3}{2}^+)$	0.0007
7.92	5.39	14±4	3	$\frac{5}{2}^-$, $(\frac{7}{2}^-)$	0.013
7.97	5.44	125±25	1	$\frac{1}{2}^-$, $\frac{3}{2}^-$	0.022
8.08	5.55	36±10	2	$\frac{5}{2}^+$, $(\frac{3}{2}^+)$	0.011
8.20	5.67	20±4	2	$\frac{5}{2}^+$, $(\frac{3}{2}^+)$	0.006
8.26	5.75	∼40	1	$\frac{1}{2}^-$, $\frac{3}{2}^-$	0.007
8.49	5.97	36±10	(0)	$(\frac{1}{2}^+)$	0.005
8.51	5.99	25±7	2	$\frac{5}{2}^+$, $(\frac{3}{2}^+)$	0.007
8.62	6.11	∼10	(1)	$(\frac{1}{2}^-, \frac{3}{2}^-)$	0.0016
8.67	6.16	120±30	0	$\frac{1}{2}^+$	0.016
8.76	6.25	14±3	0	$\frac{1}{2}^+$	0.002
8.85	6.34	9±3	0	$\frac{1}{2}^+$	0.001
8.89	6.39	33±6	2	$\frac{5}{2}^+$, $(\frac{3}{2}^+)$	0.0083
8.97	6.48	∼50	...	...	...
9.06	6.56	23±5	1	$\frac{1}{2}^-$, $\frac{3}{2}^-$	0.0035
9.10	6.60	∼150	...	...	...
9.28	6.79	7±3	(1,3)	...	0.0012
9.34	6.86	...	...	...	...
9.37	6.88	13±5	(2,0)	...	...
9.44	6.95	20±5	0	$\frac{1}{2}^+$	0.0023

[a] These represent upper values calculated assuming $\Gamma_p = \Gamma$.
[b] See reference 4.

[9] R. A. Laubenstein and M. J. W. Laubenstein, Phys. Rev. 84, 18 (1951).

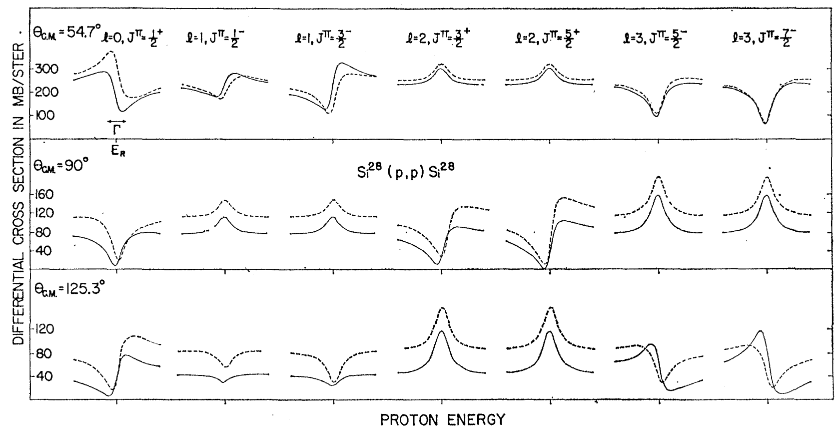

FIG. 2. Theoretical curves for the differential scattering cross section of protons by Si^{28}. The solid curves represent calculations with $\phi_0=-43°$, $\phi_1=-22.1°$, $\phi_2=-7.2°$, and $\phi_3=-2.5°$; while the dashed curves are for $\phi_0=-69.3°$, $\phi_1=-42.4°$, $\phi_2=-17.2°$, and $\phi_3=-4.5°$.

the calculations, which were carried out graphically, are shown in Fig. 2 and show clearly that the general characteristics of the cross section are not changed by such a variation in hard-sphere phases. These calculations assume that the elastic-scattering channel is the only one open; that is, the total width Γ and elastic partial width Γ_p are the same. This is not so in the experiment, since the (p,p') cross section is large, and the partial width for inelastic proton scattering $\Gamma_{p'}$ must make up a considerable fraction of the total width for most levels. It would be necessary to know the angular distribution of the inelastically scattered protons in order to determine $\Gamma_{p'}$. We, therefore, were able to determine only the l value for the incident protons and thus limit the J^π of the levels to two values, except in the case $l_p=0$ where then $J^\pi=\frac{1}{2}^+$.

Because of the overlapping of resonances, we were not able to make definite assignments for all the levels. A list of the assignments is given in Table I. Our assignment for the 7.43-MeV level of $J^\pi=(\frac{5}{2}^-,\frac{7}{2}^-)$ agrees with that of reference 4 where J^π is given as $\frac{5}{2}^-$, while the $\frac{1}{2}^+$ level at about 7.9 MeV and with width $\Gamma\sim300$ keV, which Belote, Kashy, and Risser[4] hypothesized in order to improve the theoretical elastic scattering fit is in fact the 7.62-MeV, $J^\pi=\frac{1}{2}^+$, $\Gamma=165$-keV level of Table I. Upper limits of the reduced widths for the elastically scattered protons have also been calculated and are listed in Table I as fractions of the Wigner limit $3\hbar^2/2mR$, with $R=5.85\times10^{-13}$ cm. The small values of the reduced widths are not surprising and are in agreement with a plot by Vogt[10] of reduced widths as functions of atomic weight.

[10] E. Vogt, in *Nuclear Reactions I*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), p. 215.

It is interesting to consider the levels that have been assigned $J^\pi=\frac{1}{2}^+$. Those with large reduced widths (7.62 and 8.67) probably belong to the $3s_{\frac{1}{2}}$ shell-model configuration. By comparison of the level scheme of P^{29} and of some other neighboring nuclei, where a number of shell-model levels have been identified,[11] levels belonging to other shell-model configurations can be identified. The $l=2$ levels at about 8.1 MeV would belong to the $2d_{\frac{5}{2}}$ configuration, while the $l=3$ level at 7.92 and the levels at 7.43 and 5.71 MeV probably make up the $1f_{\frac{5}{2}}$ configuration.

From recent $Si^{28}(p,p'\gamma)$ angular correlation experiments performed by Bowsher *et al.*,[12] it has been concluded that the direct interaction mechanism plays a dominant role in the proton energy region from 5.8 to 7 MeV. However, our results show strong resonance phenomena and a relatively weak nonresonant contribution. It would be interesting to investigate this point further by performing on- and off-resonance angular correlation experiments in this energy region.

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