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Amplification of Acoustic Waves through Interaction with Conduction Electrons

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It is shown, from a Boltzmann equation treatment, that in the presence of a dc electric field it is possible for an acoustic wave to gain energy from the conduction electrons in a material. The criterion for such an amplification of an acoustic wave to take place is that the drift velocity given to the conduction electrons in the direction of propagation by the dc field must exceed the velocity of sound. In metals, dc fields of such a magnitude cannot be maintained, but in semiconductors the necessary conditions can be satisfied and an amplification of the acoustic wave can take place.

I. INTRODUCTION

In the past few years, much work, both theoretical[1–5] and experimental[6–8] has been done on the absorption of ultrasonic waves via an interaction with the conduction electrons in metals, semimetals, and semiconductors. However, only recently was it discovered by Hutson, McFee, and White[9] that amplification of ultrasonic waves occurred in CdS via the same interaction in the presence of a dc field. Weinreich has shown, using a phenomenological treatment,[10] that when there is a dc electric field which gives the conduction electrons a drift velocity the direction of propagation greater than in the velocity of sound, the wave is amplified instead of absorbed. It has been pointed out, however, that the phenomenological approach is only valid when the sound wavelength is longer than the mean free path, i.e., $ql<1$. A more general approach must be made through the use of the Boltzmann equation.[4] Since the electronic contribution to the absorption of ultrasound in materials at low temperatures can be quite large when $ql>1$, it is, therefore, of interest to examine the whole problem of the electron-acoustic wave interaction in the presence of a dc electric field using the Boltzmann equation treatment.

In Sec. II, we will use the model of a free electron gas developed by Cohen, Harrison, and Harrison for the conduction electrons in a metal[2] and, in general, adopt the formalism developed by them. This model has also been used for semimetals[3] and semiconductors.[5] In Secs. III and IV we shall consider the cases of the dc field parallel and transverse to the direction of propagation, respectively. In Sec. V we give a discussion of the results of our calculations.

II. CONSTITUTIVE EQUATION

The conduction electrons are replaced by the model of a free electron gas of density N_0. The sound wave of wave number q and frequency ω manifests itself as a velocity field $\mathbf{u}(\mathbf{r},t)\propto\exp[i(qz-\omega t)]$ in the positive background which has the same density as the electron gas. The interaction between the acoustic wave and the electrons can be represented partly through the means of a self-consistent internal electromagnetic field and partly by means of a deformation potential. The self-consistent electromagnetic field induced by the passage of the sound wave is derived from Maxwell's equations. In our case, the latter can be written in the form

$$\mathbf{J}_1+N_0e\mathbf{u}=-\sigma_0\mathbf{B}\cdot\boldsymbol{\mathcal{E}}, \tag{2.1}$$

* National Science Foundation Postdoctoral Fellow.
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where $\boldsymbol{\mathcal{E}}$ and $\mathbf{J}_1$ are the electric field and electronic currents accompanying the sound wave and $\mathbf{B}$ is a diagonal tensor with components $B_{xx}=B_{yy}=i\beta$, $B_{zz}=-i\gamma$. Here, σ_0 is the dc conductivity, $\gamma=\omega/\omega_p{}^2\tau$, $\beta=(c/v_s)^2\gamma$, and ω_p is the plasma frequency of the electrons.

The electronic current can be obtained from the distribution function in the usual manner:

$$\mathbf{J}=-e\int d\mathbf{v}\,\mathbf{v}f, \tag{2.2}$$

where $\mathbf{J}$ is the total electronic current. The Boltzmann equation from which the distribution function is determined has previously been derived for the case of the sound wave in the absence of an external dc electric field.[2,5] In the presence of an external dc electric field, the Boltzmann equation becomes

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}-\frac{e}{m}\left(\boldsymbol{\mathcal{E}}+\frac{\mathbf{qq}}{ei\omega}\cdot\mathbf{C}\cdot\mathbf{u}+\mathbf{E}\right)\cdot\frac{\partial f}{\partial \mathbf{v}}$$
$$=-(1/\tau)[f-f_0+(\partial f_0/\partial E)(m\mathbf{u}\cdot\mathbf{v}+\tfrac{2}{3}(N_1/N_0)E_{F_0})], \tag{2.3}$$

where $\mathbf{E}$ is the external dc field and $\mathbf{C}$ the deformation potential tensor. The extra terms on the right-hand side of (2.3) arise because the impurities on which the electrons scatter are moving with velocity $\mathbf{u}$ and because the electrons are nonuniformly distributed in the presence of the acoustic wave. Expanding f to first order in $\mathbf{E}$, $\mathbf{u}$ and quantities proportional to $\mathbf{u}$, and keeping terms that are first order in both $\mathbf{E}$ and $\mathbf{u}$, we have

$$f=f_0+f_1{}^0+f_1{}^1, \tag{2.4}$$

where f_0 is the unperturbed distribution function, $f_1{}^0$ is that part of the perturbed distribution function which is constant in space and time, and $f_1{}^1$ is that part of the perturbed distribution function which varies as $\exp[i(qz-\omega t)]$. Thus, we obtain

$$f_1{}^0=-m\mathbf{v}_d\cdot\mathbf{v}\partial f_0/\partial E,\quad \mathbf{v}_d=-(e\tau/m)\mathbf{E}, \tag{2.5a}$$

$$\begin{aligned}
f_1{}^1=\frac{\partial f_0/\partial E}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\Bigg[&e\tau\left(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\frac{\mathbf{Cu}}{ei\omega}-\frac{m\mathbf{u}}{e\tau}\right)\cdot\mathbf{v}-\frac{2}{3}\frac{N_1}{N_0}E_{F_0}-e\tau\left(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\frac{\mathbf{Cu}}{ei\omega}\right)\mathbf{v}_d-\frac{e\tau(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\mathbf{C}\cdot\mathbf{u}/eiv_s-m\mathbf{u}/e\tau)\cdot\mathbf{v}_d}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\\
&+\frac{i\mathbf{q}\cdot\mathbf{v}_d\tau[e\tau(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\mathbf{C}\cdot\mathbf{u}/ei\omega-m\mathbf{u}/e\tau)\mathbf{v}-\frac{2}{3}(N_1/N_0)E_{F_0}]}{(1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau)^2}\Bigg]-\frac{m\mathbf{v}_d\cdot\mathbf{v}\partial^2 f_0/\partial E^2}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\\
&\times\left[e\tau\left(\boldsymbol{\mathcal{E}}+\frac{\mathbf{qq}\cdot\mathbf{C}\cdot\mathbf{u}}{ei\omega}\right)\cdot\mathbf{v}+e\tau\frac{(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\mathbf{C}\cdot\mathbf{u}/ei\omega-m\mathbf{u}/e\tau)\cdot\mathbf{v}-\frac{2}{3}(N_1/N_0)E_{F_0}}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\right],
\end{aligned} \tag{2.5b}$$

where $\mathbf{v}_d$ is the drift velocity of the conduction electrons in the external field $\mathbf{E}$. From (2.2), we see that (2.5a) gives rise to a current that is constant in space and time and is just that current which would arise from the external field alone. It is (2.5b) that gives rise to the electronic current induced by the sound wave. Therefore, in considering the electron-sound wave interaction, we can neglect the constant current. From (2.2) and (2.5b), we obtain for the desired constitutive equation,

$$\mathbf{J}_1=\boldsymbol{\sigma}\cdot(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\mathbf{C}\cdot\mathbf{u}/ei\omega-m\mathbf{u}/e\tau)-\mathbf{R}N_1ev_s+\boldsymbol{\Sigma}\cdot(\boldsymbol{\mathcal{E}}+\mathbf{qq}\cdot\mathbf{C}\cdot\mathbf{u}/ei\omega), \tag{2.6}$$

where

$$\boldsymbol{\sigma}=e^2\tau\int\frac{d\mathbf{v}(-\partial f_0/\partial E)}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\left\{\mathbf{vv}\left[1-\frac{i\mathbf{q}\cdot\mathbf{v}_d\tau}{(1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau)^2}\right]+\frac{\mathbf{vv}_d}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\right\}, \tag{2.7a}$$

$$\mathbf{R}=\frac{2}{3}\frac{E_{F_0}}{N_0v_s}\int\frac{d\mathbf{v}(-\partial f_0/\partial E)}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\left\{\mathbf{v}\left[1-\frac{i\mathbf{q}\cdot\mathbf{v}_d\tau}{(1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau)^2}\right]+\frac{\mathbf{v}_d}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\right\}, \tag{2.7b}$$

$$\boldsymbol{\Sigma}=e^2\tau\int\frac{d\mathbf{v}(-\partial f_0/\partial E)\mathbf{v}}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\left[\mathbf{v}_d-\frac{i\mathbf{q}\cdot\mathbf{v}_d\tau\mathbf{v}}{1-i\omega\tau+i\mathbf{q}\cdot\mathbf{v}\tau}\right]. \tag{2.7c}$$

In (2.6), $\boldsymbol{\sigma}$ is the conductivity tensor which plays a part in all field-dependent transport phenomena. The second term arises from the diffusion of the nonuniformly distributed carriers and the third term arises because of an additional electron pressure due to the presence of the dc field.

The equation of continuity relates the nonuniform part of the electron density N_1 to J_{1z}; i.e., $J_{1z}=-N_1ev_s$.

Defining a tensor R by means of the relation

$$\mathsf{R}\cdot\mathbf{J}_1=\mathsf{R}J_{1z}, \tag{2.8}$$

we can rewrite (2.6) in the form

$$\mathbf{J}_1=\sigma_0\boldsymbol{\sigma}'\left(\boldsymbol{\mathcal{E}}+\frac{\mathbf{qq}\cdot\mathsf{C}\cdot\mathbf{u}}{ei\omega}-\frac{m\mathbf{u}}{e\tau}\right)+\sigma_0\boldsymbol{\Sigma}'\left(\boldsymbol{\mathcal{E}}+\frac{\mathbf{qq}\cdot\mathsf{C}\cdot\mathbf{u}}{ei\omega}\right), \tag{2.9}$$

where

$$\boldsymbol{\sigma}'=[\mathsf{I}-\mathsf{R}]^{-1}\boldsymbol{\sigma}/\sigma_0,\quad \boldsymbol{\Sigma}'=[\mathsf{I}-\mathsf{R}]^{-1}\boldsymbol{\Sigma}/\sigma_0. \tag{2.10}$$

Using (2.1), and (2.9), we find an expression for the self-consistent electric field induced by the sound wave:

$$\boldsymbol{\mathcal{E}}=-[\boldsymbol{\sigma}'+\boldsymbol{\Sigma}+\mathbf{B}]^{-1}\times\left[\mathsf{I}-\boldsymbol{\sigma}'+(\boldsymbol{\sigma}'+\boldsymbol{\Sigma}')\tau\frac{\mathbf{qq}\cdot\mathsf{C}}{im\omega}\right]\frac{m\mathbf{u}}{e\tau}. \tag{2.11}$$

The average power transferred between the electrons and the sound wave per unit volume is given by

$$Q=\tfrac{1}{2}\,\mathrm{Re}\left[\mathbf{j}_1{}^*\cdot\left(\boldsymbol{\mathcal{E}}+\frac{\mathbf{qq}\cdot\mathsf{C}\cdot\mathbf{u}}{ei\omega}\right)+\frac{m\mathbf{u}^*}{e\tau}\cdot(\mathbf{j}_1+N_0e\mathbf{u})\right]. \tag{2.12}$$

Using the expressions (2.9) and (2.11) for the electronic current and electric field, respectively, the power transferred per unit volume is found to be

$$Q=\tfrac{1}{2}N_0(m/\tau)|\mathbf{u}|^2\boldsymbol{\mu}\cdot\mathsf{S}\cdot\boldsymbol{\mu}, \tag{2.13a}$$

$$\mathsf{S}=\mathrm{Re}\left[\mathsf{I}+\mathbf{B}-\tau\frac{\mathsf{C}\cdot\mathbf{qq}\cdot\mathbf{B}}{im\omega}\right][\boldsymbol{\sigma}'+\boldsymbol{\Sigma}'+\mathbf{B}]^{-1}\times\left[\mathsf{I}-\boldsymbol{\sigma}'+(\boldsymbol{\sigma}'+\boldsymbol{\Sigma}')\frac{\tau\mathbf{qq}\mathsf{C}}{im\omega}\right], \tag{2.13b}$$

where $\boldsymbol{\mu}$ is a unit vector in the direction of polarization of the sound wave. The absorption coefficient is the average power transferred between the acoustic wave and the electrons per unit volume divided by the incident energy flux,

$$\alpha=Q/\tfrac{1}{2}\rho|u|^2v_s, \tag{2.14}$$

where ρ is the density of the material. Using (2.7a–c) and (2.13a–b) together with (2.14) we can determine the absorption coefficient for various orientations of $\mathbf{v}_d$, $\mathbf{u}$, and $\mathbf{q}$. In the next section we shall consider particular cases of interest.

III. LONGITUDINAL dc FIELD $v_d \| q$

When the dc electric field is in the direction of propagation of the sound wave, we find, by using Fermi-Dirac statistics in (2.7a–c), that the components of the effective conductivity tensor $\boldsymbol{\sigma}'$ and diffusion tensor $\boldsymbol{\Sigma}'$ are

$$\sigma_{xx}'=\left[\frac{(1-i\omega\tau)^2+(ql)^2-i\omega\tau\mu}{1+(ql)^2}\right]\left[g+\frac{3i\omega\tau}{(ql)^2}\right],$$
$$\sigma_{zz}'=(1-i\omega\tau)\left[\frac{3p}{(ql)^2}+\frac{i\omega\tau}{1+(ql)^2}\left(2g-\frac{3\mu}{1+(ql)^2}\right)\right]\Big/\left[p+\frac{1-\mu}{1+(ql)^2}+\frac{ip}{\omega\tau}\right], \tag{3.1a}$$

and

$$\Sigma_{xx}'=[3i\omega\tau/(ql)^2]\mu p,\quad \Sigma_{zz}'=-\frac{2i\omega\tau}{1+(ql)^2}\mu(1-i\omega\tau)g\Big/\left[p+\frac{1-\mu}{1+(ql)^2}+\frac{ip}{\omega\tau}\right]. \tag{3.1b}$$

Here $\mu=v_d/v_s$ and the functions p and g are

$$p=1-\frac{\arctan ql}{ql},\quad g=\frac{3}{2(ql)^2}\left(\frac{[1+(ql)^2]}{ql}\arctan ql-1\right). \tag{3.2}$$

All the other components of these tensors vanish. In this case we obtain the following expressions for the absorption coefficient for longitudinal and transverse polarized waves:

$$\alpha_{\mathrm{long}}=\frac{N_0m}{\rho v_s\tau}\,\mathrm{Re}\left[\frac{1-i\gamma+(v_F/v_s)^2(\omega/\omega_p)^2C_{zz}/mv_F{}^2)}{\sigma_{zz}'+\Sigma_{zz}'-i\gamma}\right]\left[1-\sigma_{zz}'-iql\frac{v_F}{v_s}\frac{C_{zz}}{mv_F{}^2}(\sigma_{zz}'+\Sigma_{zz}')\right], \tag{3.3a}$$

$$\alpha_{\mathrm{trans}}=\frac{N_0m}{\rho v_s\tau}\,\mathrm{Re}\left[\frac{(1+i\beta)(1-\sigma_{xx}')}{\sigma_{xx}'+\sum_{xx}'+i\beta}-\frac{iql(v_F/v_s)^2(\sigma_{zz}'+\sum_{zz}')}{\sigma_{zz}'+\sum_{zz}'-i\gamma}\left(\frac{C_{xz}}{mv_F{}^2}\right)^2\left(\frac{\omega}{\omega_p}\right)^2\right], \tag{3.3b}$$

where C_{zz} is the longitudinal deformation potential and C_{xz} is the shear deformation potential. We need only substitute (3.1a–b) into (3.3a–b) to obtain the absorption coefficient for longitudinal and transverse waves. We will

use the following limiting forms for p and g:

$$\begin{aligned} &p=\tfrac{1}{3}(ql)^2-\tfrac{1}{5}(ql)^2, \quad g=1-\tfrac{1}{5}(ql)^2, \quad ql<1 \\ &p=1-\pi/2ql, \qquad g=3\pi/4ql, \qquad ql>1. \end{aligned} \tag{3.4}$$

Using the above relations, we get the following expressions for the absorption coefficient in the different frequency ranges:

$$\begin{aligned} &ql<1, \quad (v_F/v_s)^2(\omega/\omega_p)^2(C_{zz}/mv_F^2)<1, \\ &\alpha=N_0m/\rho v_s\tau[-\mu+(4/15)(ql)^2]; \end{aligned} \tag{3.5a}$$

$$\begin{aligned} &ql<1, \quad (v_F/v_s)^2(\omega/\omega_p)^2(C_{zz}/mv_F^2)>1, \\ &\alpha=\frac{N_0m}{3\rho}\frac{v_F^2}{v_s}\frac{q^2\tau(1-\mu)(v_F/v_s)^2(\omega/\omega_p)^4(C_{zz}/mv_F^2)^2}{(1-\mu)^2(\omega/\omega_p)^4+(\omega\tau)^2[1+\frac{1}{3}(v_F^2/v_s)^2(\omega/\omega_p)^2]^2}; \end{aligned} \tag{3.5b}$$

$$\begin{aligned} &ql>1, \\ &\alpha=(\pi N_0m/6\rho)(v_F/v_s)q(1-\mu)\left[\frac{1+(v_F/v_s)^2(\omega/\omega_p)^2C_{zz}/mv_F^2}{1+\frac{1}{3}(v_F/v_s)^2(\omega/\omega_p)^2}\right]^2, \end{aligned} \tag{3.5c}$$

for longitudinal waves; and

$$\begin{aligned} &(v_F/v_s)^2(\omega/\omega_p)^2(C_{xz}/mv_F^2)\ll 1, \\ &\alpha=(N_0m/\rho v_s\tau)(1-g)(g+\beta^2)/g^2+\beta^2; \end{aligned} \tag{3.6a}$$

$$\begin{aligned} &ql<1, \quad (v_F/v_s)^2(\omega/\omega_p)^2(C_{xz}/mv_F^2)>1, \\ &\alpha=\frac{N_0m}{\rho v_s}\frac{v_F^2q^2\tau(1-\mu)(v_F/v_s)^2(\omega/\omega_p)^4(C_{xz}/mv_F^2)^2}{(1-\mu)^2(\omega/\omega_p)^4+(\omega\tau)^2[1+\frac{1}{3}(v_F/v_s)^2(\omega/\omega_p)^2]^2}; \end{aligned} \tag{3.6b}$$

$$\begin{aligned} &ql>1, \quad (v_F/v_s)^2(\omega/\omega_p)^2(C_{xz}/mv_F^2)>1, \\ &\alpha=\frac{\pi N_0m}{6\rho}\left(\frac{v_F}{v_s}\right)^5\frac{(1-\mu)q(\omega/\omega_p)^4(C_{xz}/mv_F^2)^2}{[1+\frac{1}{3}(v_F/v_s)^2(\omega/\omega_p)^2]^2}, \end{aligned} \tag{3.6c}$$

for transverse waves.

The expressions for the absorption coefficient (3.5a–c) and (3.6a–c) reduce to those derived previously[1,4] in the limit of zero dc electric field, $\mu=0$. For longitudinal waves there is a crossover from absorption to gain when $v_d=v_s$, and we get an amplification of the acoustic wave instead of its absorption in all the frequency ranges except (3.5a), when $v_d>v_s$. When $ql<1$ and $(v_F/v_s)^2(\omega/\omega_p)^2C_{zz}/mv_F^2\gg 1$, our result (3.5b) agrees with the result obtained by Weinreich using a phenomenological approach.[10] In this region we have either

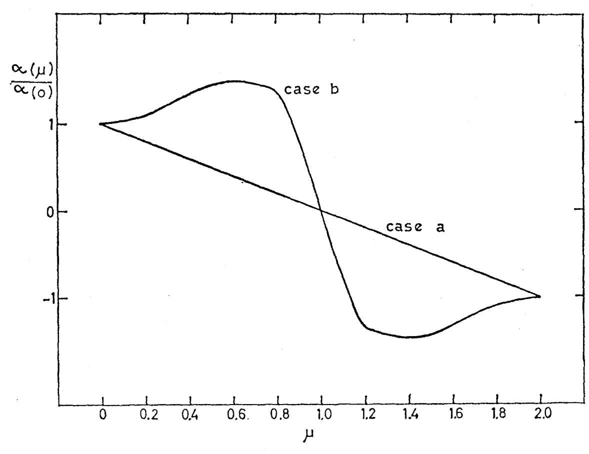

FIG. 1. The ratio of the absorption coefficient at finite dc field to that at zero field is shown as a function of v_d/v_s when $ql<1$, for (a) $\omega/\omega_p=10^{-2}$ and (b) $\omega/\omega_p=10$. In both cases, there is a crossover from absorption to gain when $v_d/v_s=1$.

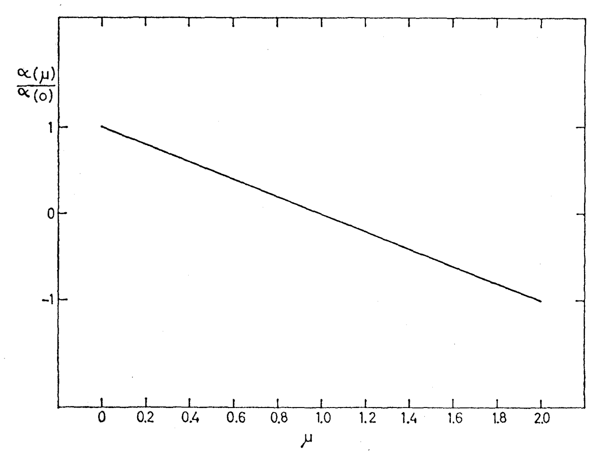

FIG. 2. The ratio $\alpha(\mu)/\alpha(0)$ is shown as a function of v_d/v_s for $ql>1$. The ratio is a linear function of the dc field.

maximum absorption or maximum amplification at a dc field, such that

$$\mu = v_d/v_s = 1 \pm (\omega_p{}^2\tau/\omega)[1+\tfrac{1}{3}(v_F/v_s)^2(\omega/\omega_p)^2], \quad (3.7)$$

the + sign corresponding to the amplification and the − sign to the absorption. When $ql > 1$, the amplification increases linearly with dc field and there is no maximum in the amplification. The absorption coefficient is shown as a function of dc field in Fig. 1 for condition (3.5b) and in Fig. 2 for $ql > 1$.

For transverse waves, there is no amplification of the acoustic wave in the presence of the dc electric field unless the shear deformation potential is large enough to satisfy the condition $(v_F/v_s)^2(\omega/\omega_p)^2 C_{xz}/mv_F{}^2 > 1$. In this case, the criterion for the amplification and the behavior of the absorption coefficient are similar to those for longitudinal waves.

IV. TRANSVERSE dc FIELD $v_d \perp q$

When the dc electric field is transverse to the direction of propagation of the sound wave, we find for the components of the effective conductivity tensor $\boldsymbol{\sigma}'$ and diffusion tensor $\boldsymbol{\Sigma}'$:

$$\sigma_{xx}' = \left\{ g - \frac{2\mu^2 i\omega\tau}{[1+(ql)^2]}\left[g + \frac{3i\omega\tau}{(ql)^2}\right]\right\} \Big/ \left[p + \frac{1}{1+(ql)^2} + \frac{ip}{\omega\tau}\right], \quad \sigma_{yy}' = g$$

$$\sigma_{zz}' = (1-i\omega\tau)\left[\frac{3p}{(ql)^2} + \frac{2i\omega\tau}{1+(ql)^2} g\right] \Big/ \left[p + \frac{1}{1+(ql)^2} + \frac{ip}{\omega\tau}\right],$$

$$\sigma_{xz}' = \frac{\mu(1-i\omega\tau)}{1+(ql)^2}\left[\frac{3p}{(ql)^2} + \frac{2i\omega\tau}{1+(ql)^2} g\right] \Big/ \left[p + \frac{1}{1+(ql)^2} + \frac{ip}{\omega\tau}\right], \quad (4.1a)$$

$$\sigma_{zx}' = -\frac{2i\omega\tau\mu}{1+(ql)^2}\left[g + \frac{3i\omega\tau}{(ql)^2}\right] \Big/ \left[p + \frac{1}{1+(ql)^2} + \frac{ip}{\omega\tau}\right],$$

and

$$\Sigma_{xx}' = \frac{-i\omega\tau\mu^2}{1+(ql)^2}\left[\frac{3p}{(ql)^2} + \frac{2i\omega\tau g}{1+(ql)^2}\right] \Big/ \left[p + \frac{1}{1+(ql)^2} + \frac{ip}{\omega\tau}\right],$$

$$\Sigma_{zx}' = -i\omega\tau\mu\left[\frac{3p}{(ql)^2} + \frac{2i\omega\tau g}{1+(ql)^2}\right] \Big/ \left[p + \frac{1}{1+(ql)^2} + \frac{ip}{\omega\tau}\right], \quad (4.1b)$$

where the dc field is in the x direction and the remaining components of these tensors vanish.

Using (4.1a–b) in the expressions for the absorption coefficient (2.13–14), we find that for both longitudinal and transverse waves, the values of the absorption coefficient are unchanged from those in the absence of the dc field as long as the drift velocity of the electrons is less than their Fermi velocity, i.e., $v_d < v_F$. However, this is the same condition needed for our linear Boltzmann equation treatment to be valid. Therefore, as long as our linear Boltzmann equation treatment is valid, there is no amplification of acoustic waves in the presence of a dc field transverse to the direction of propagation.

V. DISCUSSION

In our calculations, we have found that in the presence of a dc electric field in the direction of propagation of an acoustic wave, an amplification of the wave can occur. This happens when the drift velocity imparted to the conduction electrons by the field is greater than the sound velocity. In this section, we shall present a physical picture of this effect and suggest under what conditions it can best be observed.

When a particle is traveling in the direction of propagation of a wave with a velocity less than the wave velocity, the wave will overtake the particle and give up kinetic energy to it. On the other hand, when the particle is traveling in the direction of propagation with a velocity greater than the wave velocity, the particle will overtake the wave and lose kinetic energy to it. The dc electric field serves the purpose of giving all the conduction electrons a net drift velocity in the direction of propagation of the sound wave. When this drift velocity exceeds the sound wave velocity, there is a net loss of energy from the conduction electrons to the sound wave. Or in other words, the sound wave is amplified. This effect is similar to that discussed by Pines and Schrieffer[11] in connection with growing acoustic waves in an electron-hole plasma in semiconductors.

When the conduction electrons are moving transverse to the direction of propagation of the wave, they can no longer give up their kinetic energy to the wave. Therefore, the amplification of an acoustic wave can only take place if the drift velocity has a component

[11] D. Pines and J. R. Schrieffer, Phys. Rev. **124**, 1387 (1961).

along the direction of propagation which is greater than the sound velocity. Since the effect occurs only when the dc electric field is along the direction of propagation, transverse polarized waves are not affected unless there is a mechanism which couples shear waves to longitudinal currents. One such mechanism is the shear deformation potential. Thus, for an amplification to occur for transverse waves, the shear deformation potential forces must be stronger than the electrostatic forces, i.e., $(v_F/v_s)^2(\omega/\omega_p)^2C_{xz}/mv_F^2>1$.

Another case where amplification of transverse waves can occur is in a piezoelectric crystal where a longitudinal electric field of piezoelectric origin accompanies a transverse wave. This indeed is the case in the experiments of Hutson *et al.* on CdS,[9] which happens to be a strong piezoelectric. The field of piezoelectric origin which accompanies the transverse wave is $E_{pz}=-d_{xz}u_x/v_s$, where d_{xz} is the appropriate piezoelectric constant. We can take account of the piezoelectricity of CdS in our theory by adding the piezoelectric field produced to the term containing the other electric fields in (2.3). This is equivalent to replacing $\pm iqC_{xz}$ by $\pm iqC_{xz}-ed_{xz}$ in (2.5b), (2.6), (2.9), (2.11–13), and (3.3b). When $ed_{xz}>qC_{xz}$, we obtain in place of (3.6 b–c)

$$ql<1,\quad (\omega/\omega_p^2)(ed_{xz}/mv_s)>1,$$

$$\alpha=\frac{q^2\tau(1-\mu)(\omega/\omega_p)^2d_{xz}^2}{12\pi\rho v_s(1-\mu)^2(\omega/\omega_p)^4+(\omega\tau)^2[1+\frac{1}{3}(v_F/v_s)^2(\omega/\omega_p)^2]^2};\tag{5.1a}$$

$$ql>1,\quad (\omega/\omega_p^2)(ed_{xz}/mv_s)>1,$$

$$\alpha=\frac{qv_F(\omega/\omega_p)^2(1-\mu)^2d_{xz}^2}{24\rho v_s^3[1+\frac{1}{3}(v_F/v_s)^2(\omega/\omega_p)^2]^2}.\tag{5.1b}$$

Expression (5.1a) is just that derived by Hutson *et al.*, in order to explain the results of their experiments. It is only valid in the long-wavelength limit in which the experiment was performed. In the short-wavelength limit (5.1b) is the correct expression for the absorption coefficient. The neglect of the deformation potential forces compared to the forces of piezoelectric origin is valid since for CdS the piezoelectric constant $d_{xz}=10^7$ dyn/esu[12] and a typical value of the deformation potential for a semiconductor is $C_{xz}=10$ eV.[11,13] Therefore, the inequality $ed_{xz}>qC_{xz}$ holds for frequencies up to $\omega=10^{13}$ sec^{-1}, i.e., up to all attainable sound frequencies.

To obtain an estimate of the dc electric field strengths needed to give the conduction electrons a drift velocity that is equal to the sound velocity, we need to know the electron mobilities in various materials and for various temperature ranges. The critical dc field is given by the velocity of sound divided by the electron mobility. The electron mobility ranges in value from 300 cm^2/V sec for CdS,[9] 1200 cm^2/V sec for Si, and 3600 cm^2/V sec for Ge at room temperatures[14] to 10^4 cm^2/V sec for pure metals and semimetals at low temperatures[15] and 7×10^5 cm^2/V sec for InSb at 60°K.[16] Since the velocity of sound in most materials is of the order of 10^5 cm/sec, this means that for different materials the critical dc field at which we get a crossover from acoustic absorption to amplification can vary from less than 1 V/cm to 10^3 V/cm. In metals, dc electric fields of this order of magnitude cannot be obtained because of their high conductivity. However, in semimetals and semiconductors fields of the necessary order of magnitude can be obtained.

Related to the problem of maintaining large dc fields in metals is the problem of the dissipation of energy from the dc field to the conduction electrons. In Sec. II we neglected the current arising from the dc field alone in considering the interaction between the acoustic wave and the conduction electrons. However, this current is important in considering the power transferred from the dc field to the electrons. The average power transferred per unit volume from the dc field to the electrons is $\mathbf{J}_0\cdot\mathbf{E}$, where $\mathbf{J}_0=-N_0e\mathbf{v}_d$ is the current arising from the dc field alone. This can be rewritten in the form $Q_E=N_0(m/\tau)v_d^2$, where Q_E is the power transferred from the dc field to the electrons. When $v_d>v_s$, part of this power is transferred to the acoustic wave. The remainder is dissipated by collisions of the electrons with impurities. The ratio of the average power transferred from the electrons to the acoustic wave to the power transferred from the dc field to the electrons is

$$r=-\tfrac{1}{2}(\rho v_s\tau/N_0m)|u/v_d|^2\alpha.\tag{5.2}$$

The minus sign arises because when α is positive, power is transferred from the acoustic wave to the electrons. Using the various expressions derived for α (i.e., 3.5–6, 5.1), the ratio of the powers transferred can be calculated. We have found that this ratio is always much less than unity for all the cases we have considered. Since most of the power transferred from the field to the electrons is dissipated through collisions with impurities or by other mechanisms, we must make sure that the dissipation of this power will not cause

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drastic changes in the properties of the material used. At the critical field, $v_d = v_s$, and taking v_s to be 10^5 cm/sec, τ to be 10^{-11} sec, and $m = 10^{-28}$ g, we find the power dissipated by the dc field to be $QE = N_0 \times 10^{-7}$ erg/sec cm^3. For a rise in the temperature of the material of more than a few degrees to be avoided, the power dissipated must be less than a few hundred watts. Therefore, we are limited to materials with conduction electron densities $N_0 < 10^{18}$ cm^{-3}. The materials in which an amplification of acoustic waves by the mechanism discussed in this paper can take place are thus limited to semimetals and semiconductors.

A material which would be well suited for amplifying sound waves by this mechanism is indium antimonide because of its large mobility and the value of its deformation potential (about 7 eV). Thus, we would have crossover from absorption to gain of the acoustic wave at small values of the dc electric field and a large gain when the necessary criteria are satisfied. For instance, with a conduction electron density of 10^{15} cm^{-3}, we would have a gain of 10^3 dB/cm at a frequency of 10 kMc/sec. Piezoelectric semiconductors should also be well suited for amplification of sound waves via this mechanism as they would have a large gain with a relatively small number of conduction electrons. In the experiments performed on CdS,[9] gains as large as 150 dB/cm were measured at room temperatures and at frequencies in the megacycle range.

The theory presented here is only valid when the use of the linearized Boltzmann equation is justified. When the conduction electrons gain as much energy from the dc field as their original energy in thermal equilibrium, the dc field can no longer be treated as a perturbation and the use of the linear Boltzmann equation is no longer valid. Thus, the condition for the validity of the linear Boltzmann equation approach is that $v_d < v_F$, i.e., that the velocity gained by the electrons from the field be less than their velocity in thermal equilibrium. The use of the linear Boltzmann equation would also break down if the acoustic wave is so greatly amplified that the energy transferred between the wave and the electrons is greater than the original energy of the electrons.

The calculations here have been done using degenerate statistics for the conduction electrons. However, other treatments of the sound wave-conduction electron interaction have shown that using classical statistics for the electrons only change the expressions for α by a numerical constant of order unity.[4] Therefore, the general features of our treatment should hold for conduction electrons obeying either degenerate or classical statistics.

The mechanism discussed here is particularly well suited for amplifying microwave sound waves where the sound wave intensities generated by present methods are very low.[17] In semimetals and nonpiezoelectric semiconductors, the amplification increases linearly with wave number q in the high-frequency, short-wavelength limit (see 3.5c). So precisely in the region where we can only generate low-intensity sound waves by other methods, we can get a very large amplification by means of our mechanism.

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