Low-Frequency Magneto-Plasma Resonances in Sodium*

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A detailed study of low-frequency magneto-plasma resonances ("helicons" or "whistlers") has been carried out in a series of rectangular parallelepipeds of sodium at 4°K. Many harmonics have been observed and quantitative correlation has been made between the observed resonances and the specimen geometry. The investigation yields the following value for the Hall coefficient in sodium: $|R| = 24.8 \times 10^{-11}$ m³/C at 4°K corresponding to 0.94 electron per atom.

We have reported the observation of a very low frequency magneto-plasma resonance in sodium at 4°K.[1] This resonance was associated with the macroscopic electron gas excitation called the "helicon" by Aigrain.[2] The effect is a manifestation in a metal of the phenomenon known as a "whistler" in ionosphere physics.[3] This identification was based on the qualitative agreement between theory and experiment for the resonant frequency observed in a cylindrical specimen. It is the purpose of this note to present results of a detailed study of the resonances obtained in rectangular parallelepipeds; many harmonics have been seen and detailed quantitative correlation has been made between the observed frequencies and the sample geometry.

If electron scattering is neglected, the dispersion relation in mks units is

$$2\pi\nu = RH_0kk_z, \tag{1}$$

where R is the Hall coefficient, H_0 is the applied field taken to be the z direction, ν and k are the frequency and wave vector respectively.[1,2] The excitation consists of a circularly polarized wave, the sense of rotation being that of the cyclotron rotation. A finite specimen acts as a resonant cavity for this wave, the wave vector taking particular values determined by the geometry of the specimen.

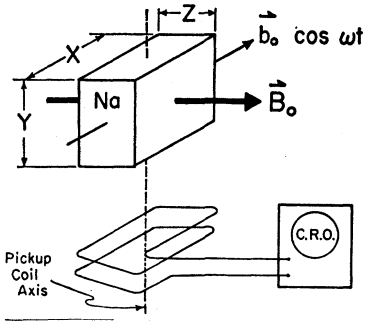

FIG. 1. Schematic diagram of experiment. Specimen and pickup coil (~1000 turns) are immersed in liquid helium at 4.2°K. The exciting coil is outside the cryostat.

Rectangular parallelepipeds were studied using the resonance version of the experiment described in reference 1. A sodium specimen (see Fig. 1) is placed inside a pickup coil whose axis is perpendicular to a large magnetic field H_0 (several thousand oersted). A small alternating field (~10 Oe) is applied perpendicular both to the pickup coil and to H_0. On varying the frequency of the exciting field, the pickup coil voltage exhibits maxima at certain frequencies thus indicating resonant modes of the cavity. Resonant frequencies were measured at several values of the field H_0 (maximum 7.4 kOe) in order to evaluate the ratio $\langle\nu/H_0\rangle$. The sodium used had a residual resistivity ratio $\rho_{300}/\rho_{4.2} \approx 7500$.[4] A rough estimate of $\omega_c\tau$ using the Hall coefficient and resistivity yields $\omega_c\tau \approx 30$ in 7.4 kOe.

In order to calculate the resonant frequencies of a finite cavity, we use the dispersion relation (1) and fix the wave vector by analogy with standard cavity theory. For parallelepipeds having sides of length X, Y, Z, we assume that the wave vector at resonance has the form:

$$\mathbf{k} = (\pi l/X, \pi m/Y, \pi n/Z),$$

where l, m, and n are integers. Then it follows that

$$\nu = (1/2\pi)RH_0kk_z = (\pi/2)RH_0(n/Z^2)G_{lmn}(X,Y,Z) \equiv \nu_{lmn}(X,Y,Z), \tag{2}$$

where

$$G_{lmn}(X,Y,Z) = [l^2(Z/X)^2 + m^2(Z/Y)^2 + n^2]^{1/2}.$$

G is a dimensionless parameter which we shall call the geometry factor.[5]

For a thin slab, X, $Y \gg Z$, Eq. (2) reduces to

$$\nu_{lmn}(X,Y,Z) \rightarrow \nu_n(Z) = (\pi/2)RH_0\, n^2/Z^2.$$

This relation has been derived rigorously by Jones and Chambers using an extension of skin-effect theory.[6] With our experimental arrangement, resonances corresponding to even values of n cannot be detected because such modes have no net flux linkage with the pickup coil. Similarly, we are not likely to excite these

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[2] P. Aigrain, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovak Academy of Sciences, Prague, 1961), p. 224.
[3] L. R. O. Storey, Phil. Trans. Roy. Soc. (London) **246A**, 113 (1953). H. Bremmer, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 16, p. 570.
[4] We are indebted to C. E. Taylor of the Lawrence Radiation Laboratory, Livermore, California, for giving us this very pure material.
[5] P. Cotti, P. Wyder, and A. Quattropani [Phys. Letters **1**, 50 (1962)] have discussed a similar relation for the particular case of the rectangular bar of infinite length ($X \gg Y$, Z).
[6] B. K. Jones and R. G. Chambers (to be published).

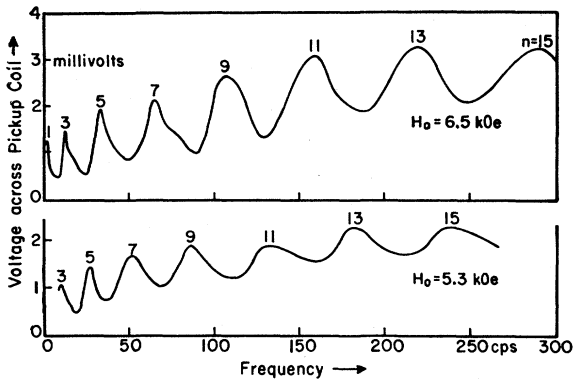

FIG. 2. Resonances observed in largest cube specimen (12.8-mm sides). Peak to peak voltage in pickup coil plotted against frequency of exciting field for two values of H_0. The number written above each peak is the value of the index n.

even harmonics because of the symmetry of our exciting field.

Our observations were made on 17 different parallelepipeds of widely different aspect ratios. The geometry factor, $G_{lmn}(X,Y,Z)$, for $l=m=n=1$ has been varied from 1.0 to 3.8. Measurements were made in the frequency range from 2 to 500 cps. For most specimens, about three resonant frequencies could be measured with adequate precision. In the largest cube specimen (12.8-mm sides) eight resonances were easily measured; these are shown in Fig. 2. The resonances observed had wavelengths varying from 0.6 to 15 mm.

All of the resonances observed can be fitted to Eq. (2) only by taking $l=m=1$ and assigning consecutive odd integers to n.[7] This is illustrated by the two graphs presented. Figure 3 refers only to the fundamental frequencies because these provide a more critical test of the geometry factor than do observations on higher harmonics; $G_{11n}(X,Y,Z)$ tends to n for large n. In Fig. 3 we plot $(2/\pi)\langle\nu_1/H_0\rangle Z^2$ vs the geometry factor $G_{111}(X,Y,Z)$.[8] The experimental results are well represented by a straight line through the origin; this validates the use of the geometry factor. Figure 4 shows a similar plot for both the fundamentals and the higher harmonics. Again we assume $l=m=1$. This is a plot of $(2/\pi)\langle\nu_n/H_0\rangle Z^2/n$ vs $G_{11n}(X,Y,Z)$ with $n=1, 3, 5, \cdots$ for each specimen. The straight line through the origin which fits the data best has a slope corresponding to $|R|=24.8\times10^{-11}$ m³/C±4% at 4°K. This corresponds to 0.94 electron per atom.[9] Studer and Williams[10]

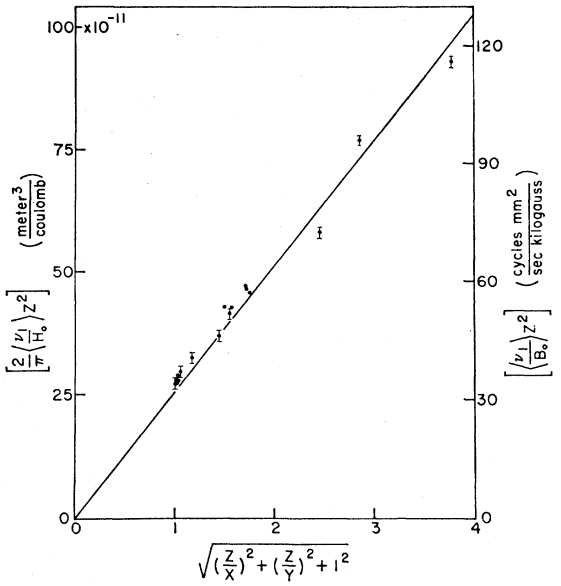

FIG. 3. Test of the appropriate geometry factor $[(Z/X)^2+(Z/Y)^2+1^2)]^{\frac{1}{2}}$ using only the fundamental resonant frequencies ν_1 of 17 rectangular parallelepipeds; Z is the specimen length parallel to the magnetic field.

report $R=-21\times10^{-11}$ m³/C at 25°C using conventional methods. *Note added in proof.* Deutsch, Paul, and Brooks[11] report $|R|=25.8\times10^{-11}$ m³/C ±7% at room temperature; $|R|$ decreased by 3% from room tem-

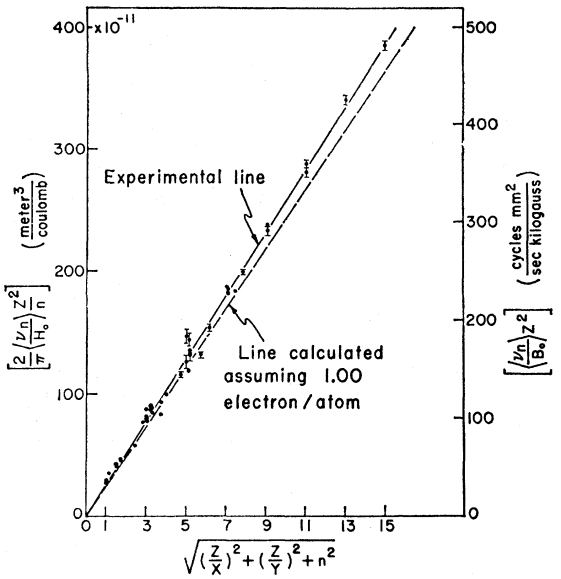

FIG. 4. Harmonic resonance data plotted against the appropriate geometry factor $[(Z/X)^2+(Z/Y)^2+n^2]^{\frac{1}{2}}$, where n is the harmonic number. A total of 53 distinct resonance peaks were measured in a series of 17 specimens.

[7] The choice of $l=m=1$ is based entirely on fitting the experimental points to Eq. (2). We have no firm theoretical reason to exclude other values of l and m.

[8] We have made no correction for the finite conductivity of the sodium; for the resistivity ratio of 7500 this correction is less than 1%.

[9] Thermal contraction has been taken into account by a correction of 1.5%.

[10] F. J. Studer and W. D. Williams, Phys. Rev. 47, 291 (1935).

[11] T. Deutsch, W. Paul, and H. Brooks, Phys. Rev. 124, 753 (1961).

perature to 77°K. Figures 3 and 4 provide quantitative experimental verification of Eq. (2)[7] and consequently verification of the dispersion relation (1).

Note added in proof. Jones and Chambers[6] have measured R for Li, Na, K, In, and Al using the resonance method in thin slabs. For sodium they find $R=-24.6\times10^{-11}$ m³/C $\pm1\%$ at 4°K.

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Nuclear Polarization in Polycrystalline Holmium-Indium Alloy*

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Holmium and indium nuclei have been polarized by nuclear hyperfine interaction in a polycrystalline ferromagnetic alloy, 92.2% holmium–7.8% indium. The nuclear polarization for each constituent was determined from the transmission of monochromatic, polarized neutron beams at neutron energies corresponding to resonances in the holmium and indium cross sections. A series of measurements at several temperatures from 0.071 to 4.225°K yield the following values for the hyperfine constant: for holmium, $A/k=0.58\pm0.03$°K (corresponding to an effective field at the nucleus, $H_{\text{eff}}=+8.4\times10^6$ Oe); and for In, $A/k=-0.013\pm0.002$°K (corresponding to $H_{\text{eff}}=-1.3\times10^5$ Oe). The effect on the nuclear polarization of the magnetic anisotropy of the polycrystalline sample is discussed.

I. INTRODUCTION

RECENT measurements[1] have demonstrated that large nuclear polarization (~0.4) occurs in holmium metal at relatively high temperatures (0.95°K). The polarization originates from an unusually large nuclear hyperfine interaction, which also occurs in various holmium compounds, e.g., holmium ethyl sulphate.[2] The existence of this large hyperfine interaction was independently confirmed by the specific heat measurements on holmium metal by Gordon, Dempesy, and Soller[3] who obtained the value $A/k=0.618$°K for the nuclear hyperfine interaction constant. Since our initial publication,[1] we have extended our measurements to both higher and lower temperatures. These new data yield a more precise value of A/k as well as providing additional facts about the magnetic nature of holmium metal at low temperatures.

The sample which was used for the new measurements was actually an alloy of 92.2% holmium–7.8% indium (at. %).[4] As will be seen, this alloy is ferromagnetic at low temperatures and, for the holmium atoms, appears to retain the value of A/k which is characteristic of pure holmium metal. There are several reasons for studying an Ho–In alloy: (1) The sign and magnitude of the hyperfine interaction at the diamagnetic indium in the strongly ferromagnetic rare-earth lattice are of interest, *per se*. (2) We are interested in finding suitable ferromagnetic "host" materials in which various impurity nuclei can be polarized, and which differ from iron with respect to alloying properties. (3) Another more technical reason of interest was to verify the fact that the sample comes to thermal equilibrium with the refrigerating salt at the lowest temperatures obtained during the reported measurements.

It appears that the "host" method is potentially useful for producing reasonably large nuclear polarization in many elements which normally can be polarized only by the "brute force" method. Existing data[5–7] indicate that the effective magnetic field at an impurity nucleus in a ferromagnetic host material can have a magnitude as large as 10^5 to 10^6 Oe. Since the fields which are practical and presently available for use in the brute

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[2] J. M. Baker, and B. Bleaney, Proc. Phys. Soc. (London) **A68**, 1090 (1955).

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