Influence of Interference Scattering on the Scattering of Slow Neutrons by Gaseous Methane*

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Recent computations on the partial differential cross section for the scattering of slow neutrons from gaseous methane have been extended to momentum transfers large enough to investigate the full range of the available experimental results. It is found that the interference scattering from carbon-proton and proton-proton pairs has an effect on the scattering that can be observed. The experimental results appear to show such an effect and are in agreement with the results of the computations.

I. INTRODUCTION

IN an article (hereafter referred to as I) on the scattering of slow neutrons by methane,[1] methods were developed to treat the quantum nature of the rotational levels. It was found that the quantum effects have a marked effect on the scattering of neutrons which was also observed in the experiments.[2] The computational program used in I was designed to investigate scattering angles and neutron energies such that the argument of the spherical Bessel functions appearing in the theory would be less than two. This limitation was removed by writing a computational program that made it possible to investigate the full range of the experiments.

The present paper presents the results of the computations on the partial differential cross section for larger scattering angles and incident neutron energies than in I. In addition the calculations show that proton-proton and carbon-proton interference scattering in methane has an effect which can be observed and indeed is observed in the experiments.

II. GENERAL REMARKS

For subsequent presentation of the results it will be necessary to discuss aspects of the theory in I. The partial differential cross section of methane for an incident neutron energy E_0, energy transfer ϵ, and a scattering angle θ can be written as

$$\sigma(E_0,\epsilon,\theta)=4\sigma_{pp}(E_0,\epsilon,\theta)+12\sigma_{pp'}(E_0,\epsilon,\theta)+8\sigma_{Cp}(E_0,\epsilon,\theta)+\sigma_{CC}(E_0,\epsilon,\theta). \quad (1)$$

In (1), p denotes a proton, p' a different proton, and C the carbon nucleus. The dependence of the cross section on the temperature T of the gas is suppressed. Analytical expressions for the quantities appearing on the right-hand side of (1) are given in I. If the momentum transfer $\boldsymbol{\kappa}$ and the energy transfer ϵ are considered as independent variables[3] then multiplying (1) by $(k_0/k)\exp(\epsilon/2T)$ defines a function

$$S(\kappa^2,\epsilon)=S_{pp}(\kappa^2,\epsilon)+S_{pp'}(\kappa^2,\epsilon)+S_{pC}(\kappa^2,\epsilon)+S_{CC}(\kappa^2,\epsilon), \quad (2)$$

where

$$S_{pp}(\kappa^2,\epsilon)=4(k_0/k)\exp(\epsilon/2T)\sigma_{pp}(E_0,\epsilon,\theta).$$

The remaining terms are similarily defined with $\mathbf{k}_0$ and $\mathbf{k}$ being the initial and final neutron momentum. It is easily verified that $S(\kappa^2,\epsilon)$ is an even function of ϵ for

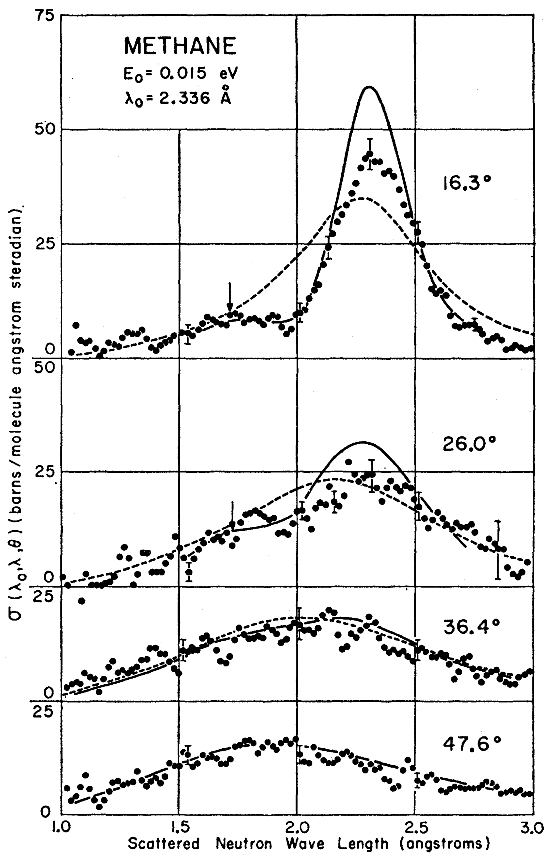

FIG. 1. The experimental and computed partial differential cross section shown as a function of the outgoing neutron wavelength. The solid-line curve presents the results when the quantum nature of the rotational levels is considered. The dashed curve presents the results when the rotational levels are treated classically. The arrow indicates the feature due to rotations. The temperature of the gas is 21°C.

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[1] G. W. Griffing, Phys. Rev. 124, 1489 (1961).

[2] P. D. Randolph, R. M. Brugger, K. A. Strong, and R. E. Schmunk, Phys. Rev. 124, 460 (1961).

[3] L. Van Hove, Phys. Rev. 95, 249 (1954).

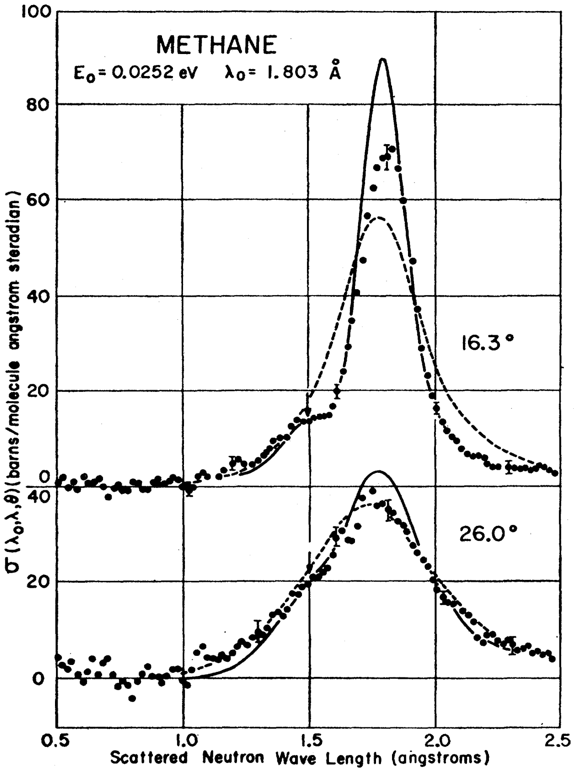

FIG. 2. The experimental and computed partial differential cross section shown as a function of the outgoing neutron wavelength. The solid-line curve presents the results when the quantum nature of the rotational levels is considered. The dashed curve presents the results when the rotational levels are treated classically. The arrow indicates the feature due to rotations. The temperature of the gas is 21°C.

a fixed value of κ^2. This expresses the condition of detailed balance of the cross section.[4] This function, besides being of intrinsic interest in itself, is of particular convenience for an internal check on the consistency of the experimental as well as the numerical results.

The partial differential cross section of molecular systems in contrast to atomic systems cannot be separated into two factors,[3] one depending only on the dynamical and nuclear properties of the scattered particle and the other depending only on the dynamical properties of the scattering system. This is because the latter will always contain nuclear properties through the scattering lengths of the individual nuclei. To be sure, such a separation would be possible for practical purposes if the cross section was determined essentially by the first term on the right-hand side of (1). However, results to be presented later show that in a certain range of κ^2 the second and third term representing carbon-proton and proton-proton interference scattering give contributions of approximately ten percent to the total and produces an effect which can be observed with sufficiently accurate experimental results. Thus these terms must be included. In addition the last term of (1) will give contributions amounting to about 5%.

III. RESULTS

Figures 1, 2, and 3 present a synopsis of the computations[5] on the partial differential cross section. Plots of other cases are not presented since an over-all representation of the results can more conveniently be given by plotting $S(\kappa^2,\epsilon)$ as given by (2). In addition, if differences of the order of 10% or less are disregarded the results which have not been presented do not differ in any essential manner from the computations that are plotted in reference 2.

It is to be noted that as the scattering angle increases the feature that is due to the discrete nature of the rotational levels disappears. The same statement is relevant regarding the incident neutron energy. Although the results suggest that if the incident energy or

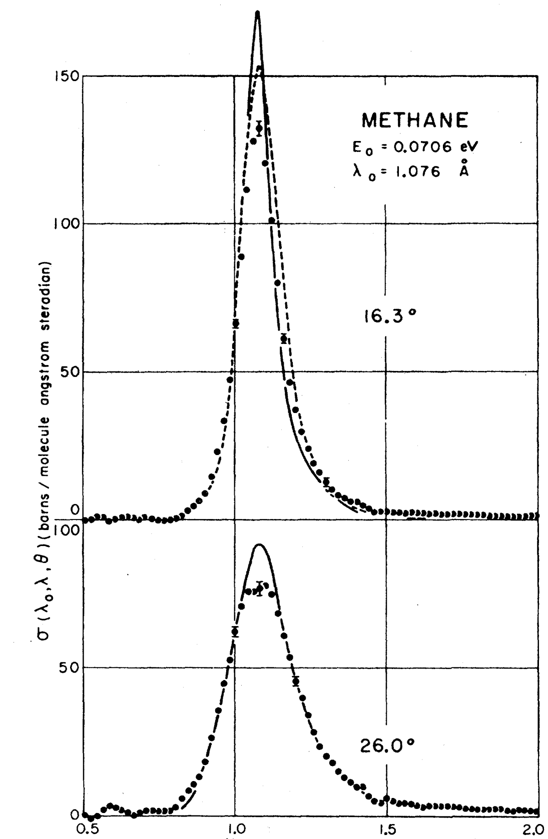

FIG. 3. The experimental and computed partial differential cross section shown as a function of the outgoing neutron wavelength. The solid-line curve presents the results when the quantum nature of the rotational levels is considered. The dashed curve presents the results when the rotational levels are treated classically. The temperature of the gas is 21°C.

[4] P. Schofield, Phys. Rev. Letters 4, 239 (1960).

[5] Two of the plots in Figs. 1 and 2 differ from those reported in I. This was due to an error in the computational program used in I. The plots of I may be corrected by multiplying the partial differential cross section $\sigma(\lambda_0,\lambda,\theta)$ at each scattered wavelength by $(\lambda/\lambda_0)^3$. No conclusions or statements in I need be revised.

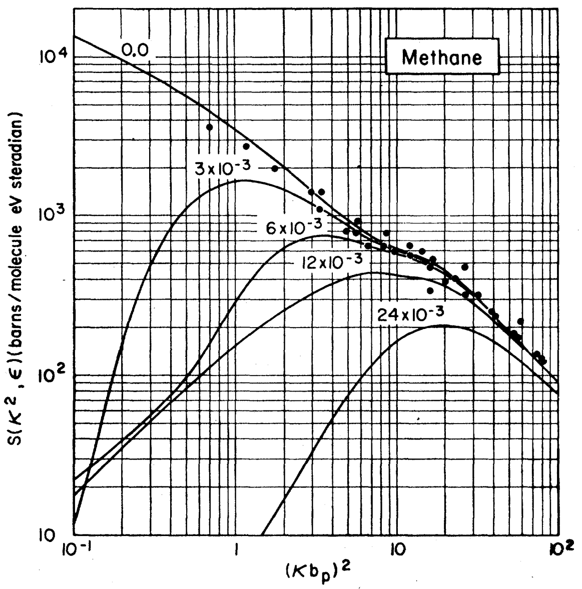

FIG. 4. $S(\kappa^2, \epsilon)$ shown as a function of $(\kappa b_p)^2$ for selected energy transfers. The experimental points are for $\epsilon=0.0$. The temperature of the gas is 21°C.

the scattering angle or both become large enough, it would be sufficient to treat the rotational levels as a continuum, the author was not able to make any analytical proofs.

There are certain features in the experimental data in reference 2 which are tempting to ascribe as being due to the discrete rotational structure of the molecule. An example of this is for an incident neutron energy of 0.0706 eV and scattering angles of 47.6° and greater. The computations did not predict a double maxima and consequently must be of experimental origin as concluded in reference 2.

In Fig. 4, $S(\kappa^2,\epsilon)$ is shown as a function[6] of $(\kappa b_p)^2$ for a few selected values of the energy transfer ϵ. The distance from the carbon nucleus to the proton is denoted by b_p. The dimensionless parameter (κb_p) occurs as the argument of the spherical Bessel functions in the theory and is particularly appropriate for discussion of the results. The values of the energy transfer were chosen so that $S(\kappa^2,\epsilon)$ is illustrated at various significant sections of the partial differential cross section curve. Thus considering Fig. 1, the case $\epsilon=0.0$ corresponds to $\lambda=2.336$ Å which locates a position near the main peak of the cross section, $\epsilon=3\times10^{-3}$ eV corresponds to $\lambda=2.13$ Å which locates a position where the cross section is changing quite rapidly, $\epsilon=6\times10^{-3}$ eV corresponds to $\lambda=1.97$ Å which locates a position where the influence of the rotational levels are beginning to be noticeable, $\epsilon=12\times10^{-3}$ eV corresponds to $\lambda=1.76$ Å which locates a position near the peak of the feature due to rotations, while $\epsilon=24\times10^{-3}$ eV corresponds to $\lambda=1.45$ Å which locates a position well beyond a position where the influence of the rotations can be noted. One feature readily apparent is that there is a marked change in the slope of the $S(\kappa^2,\epsilon)$ curve at small $(\kappa b_p)^2$ as the influence of the rotational levels begin contributing.

A feature in Fig. 4 of particular interest concerns the influence of the interference scattering. This interference scattering is mainly responsible for the dip in the curves for values of $(\kappa b_p)^2$ lying between 5 and 15 and the subsequent hump between 15 and 30. An examination of the terms contributing to $S(\kappa^2,\epsilon)$ show that for $(\kappa b_p)^2<5$ the interference terms as given by the second and third term of (2) tend to cancel each other. For $5<(\kappa b_p)^2<15$ both of these terms give negative contributions amounting to about 10% of the total. For $15<(\kappa b_p)^2<30$ both terms give positive contributions of about 10% of the total. For $(\kappa b_p)^2>30$ the contributions of these terms is more complicated and is usually less than about 5% of the total. Inspection of the analytical expression of (2) shows that this behavior is due to the spherical Bessel functions occurring in the theory.

The experimental data for $\epsilon=0.0$ has been plotted on Fig. 4. There is agreement between the experimental and theoretical results. Since there is considerable scatter in the experimental data it is not possible to conclude conclusively that this effect due to interference scattering has been observed. However, data for energy transfers of 3×10^{-3} eV and 6×10^{-3} eV support the conclusion that such an effect has been observed. In addition, it would be expected that scattering results on gaseous propane[7] should also show this effect. An examination indicates that this is the case. Experimental data for the remaining curves were not plotted since the figure would become too confusing. In reference 2 it was mentioned that there were serious difficulties in attempting to compare the experimental data for small momentum transfers with the theory that was used. No such difficulty was experienced in the present work.

ACKNOWLEDGMENTS

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[6] There are no advantages in introducing the dimensionless variables $\alpha=[\kappa^2/(2MT)]$ and $\beta=\epsilon/(2T)$ as used in reference 2. One reason for so doing would be to eliminate the temperature T so that results for the gas at other temperatures could be predicted. However, the introduction of such variables will not eliminate the xplicit dependence of $S(\kappa^2,\epsilon)$ on the temperature for a molecular gas.

[7] K. A. Strong, G. D. Marshall, R. M. Brugger, and P. D. Randolph, Phys. Rev. **125**, 933 (1962).