Fission Fragment Angular Distributions by Exact Power Series*

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A method is illustrated for computing exactly the various terms of the power series for fission fragment angular distributions. No semiclassical approximation to the wave functions is made. The results are therefore accurate even for small angular momenta. The angular distributions are calculated explicitly to second order, including perturbative effects of spin and of angular-momentum-dependent fissionability. Results are compared with exact numerical calculations to illustrate the adequacy of the first- and second-order approximations under various circumstances.

I. INTRODUCTION

THE anisotropies of fission fragments[1] emitted from compound nuclei was first considered in the semiclassical approximation,[2,3] with such small effects as target and projectile spin and the possible angular momentum dependence of fissionability omitted. More recently these effects have been incorporated into the semiclassical framework.[3] Exact calculations[4] of the full angular distributions, including these effects[5] and utilizing realistic neutron penetration probabilities, have also been made on digital computers.

We show here how the semiclassical restriction to large angular momenta can be obviated, with the result that calculations of the full angular distributions analogous to those of reference 3 become precise even for small angular momenta, while the numerical effort involved in using realistic penetration probabilities is reduced to the evaluation of a single sum.[6]

II. FISSION FRAGMENT ANGULAR DISTRIBUTIONS

Consider an energetic projectile of spin (S,σ) incident on an unpolarized target nucleus of spin (I_0,μ). These spins combine to various channel spins (j,m) which in turn add to the projectile's orbital angular momentum (L) to give the compound nuclear angular momenta (I,M). The total probability of forming a compound nucleus with angular momentum I and z component M is then given, for a fixed L and j, by

$$P(L,j;IM)=[\sum_{\mu\sigma}(2L+1)T_L|C_{0mM}{}^{LjI}|^2|C_{\mu\sigma m}{}^{I_0Sj}|^2]\times[(2I_0+1)(2S+1)\sum_L(2L+1)T_L]^{-1}. \quad (1)$$

We have chosen the z axis along the projectile beam, thereby guaranteeing $L_z\equiv 0$. $T_L(E)$ is the penetration coefficient for a projectile of angular momentum L and energy E.

The compound nucleus is assumed subsequently to fission through barrier states[1] with various projections, K, of angular momentum on the nuclear symmetry axis. The probability of fission with a given value of K is assumed proportional to the statistical (Gaussian) probability of barrier states with that value of K. The angular distribution of the fragments emitted from a compound nucleus (I,M), fissioning through a barrier state K, is assumed proportional to the square of the corresponding symmetric top wave function.[1] Then the normalized angular distribution from a given compound nuclear state (I,M) is given by

$$W(\vartheta,I,M)=\{\sum_{K=-I}^{I}\exp[-\beta K^2]|\bar{D}_{MK}{}^I(\psi,\vartheta,\psi)|^2\}\times\{\sum_{K=-I}^{I}\exp(-\beta K^2)\}^{-1}. \quad (2)$$

The functions, $\bar{D}_{MK}{}^I$, are normalized to unity for integration over the angles describing the nuclear orientation. β is a function of the internal excitation energy, E^*-E_f, at the fission barrier deformation.

In the case where the fission probability is independent of angular momentum, the full expression for the angular distribution from a compound nuclear process is obtained by combining (1) and (2):

$$W(\vartheta)=2\pi\sum_L\sum_j\sum_I\sum_M P(L,j;IM)W(\vartheta;I,M). \quad (3)$$

The coefficient 2π arises from integration over the irrelevant angle which specifies the degree of rotation of the nucleus about its symmetry axis.

One may also wish to include the dependence of fissionability on angular momentum. This dependence arises in the liquid drop picture from the modification of the fission threshold as a result of the centrifugal

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[1] A. Bohr, *Proceedings of the International Conference on the Peaceful Uses Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 2, p. 151.
[2] I. Halpern and V. Strutinskii, *Proceedings of the Second United Nations International Conference on the Peaceful Uses Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. 15, p. 408.
[3] R. Leachman and E. Sanman, Ann. Phys. (New York) (to be published).
[4] J. Griffin, Phys. Rev. **116**, 107 (1959).
[5] L. Blumberg, thesis, Columbia University, 1962 (unpublished.)
[6] A similar treatment has been developed by V. Strutinskii [J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 781 (1960); Nuclear Phys. **27**, 348 (1961)] from a somewhat different viewpoint. Comparison of our results with those of the latter reference is hampered by numerous typographical errors therein, but seems to indicate disagreement with the β^2 terms of Eqs. (7).

force of rotation,[7,8] or in a statistical picture from the difference in the moments of inertia for the nuclei formed in the competing processes of neutron emission and approach to the fission barrier.[9] It can be included easily by inserting a (normalized) weighting of angular momentum in the summand of (3). To first order in I the appropriate function is

$$\bar{\gamma}(I)=1+[\alpha/(1+\gamma_0)][I(I+1)-N(I_0,S)], \quad (4a)$$

where

$$N(I_0,S)=[\sum_j \sum_L (2L+1)T_L \sum_{IM}|C_{0mM}{}^{LjI}|^2 I(I+1)] \times[(2I_0+1)(2S+1)\sum_L(2L+1)T_L]^{-1}, \quad (4b)$$

and γ_0 is the ratio of the probabilities of fission and neutron emission for $I=0$. Here, α is related to the moments of inertia and temperatures of the fissioning nucleus and the nucleus left after neutron emission by

$$\alpha=(\hbar^2/2)[1/T_n\mathfrak{F}_n-1/T_f\mathfrak{F}_f]. \quad (5)$$

The function $\bar{\gamma}(I)$ is appropriately normalized to unity for summation over L, j, I, and M. For simplicity, this effect is not included in the discussion, although the result of including it is given in Eqs. (7).

Other more specialized effects such as possible polarization or orientation of projectile and/or target nuclide may be treated by the methods used here. In this paper, however, we make no attempt to catalog and compute the numerous possibilities they present.

III. ACCURATE CALCULATION OF LEADING TERMS OF POWER SERIES

When the anisotropy is small, one obtains a good approximation by expanding (2) in powers of β. The simplicity of the present calculations stems essentially from the fact that in spite of the complexity, as a function of angle, of the component $\bar{D}_{MK}{}^I$ functions, sums of the form

$$S_M{}^I(\vartheta,n)=\sum_{K=-I}^{I} K^n|\bar{D}_{MK}{}^I(\vartheta)|^2 \quad (6)$$

are quite simple for small n, involving at most the nth power of $\cos\vartheta$. Moreover, factorization of the differential equation for the symmetric top wave functions allows such sums to be calculated in a rather direct fashion by means of recursion relations. This calculation is outlined in the Appendix and the results required to evaluate (3) are summarized there.

One next carries out the trivial (in the absence of polarization or orientation of target or projectile) sums over μ and σ, and proceeds to the (I,M) sums of Eq. (3). These can be completed with the aid of certain identities involving the vector coupling coefficients, $C_{\alpha\beta\gamma}{}^{ABC}$. The Appendix lists the identities required and also indicates how they and similar identities can conveniently be proven.

At this stage there remain only the sums over j and L. If one assumes $T_L=1$ for $L\leq L_{\max}$ and $T_L=0$ for $L>L_{\max}$ as a crude representation of the neutron case, the summands involve only low powers of integers or half-integers. For more general projectile penetration probabilities, the final sum over L may require numerical evaluation.

In this way one obtains, in the former case, the final result

$$W(\vartheta)/W(90°)=1+(1/4)\beta\cos^2\vartheta\{L_m(L_m+2)\}-(1/144)\beta^2\cos^2\vartheta\{7L_m{}^2(L_m+2)^2 +8L_m(L_m+2)[I_0(I_0+1)+S(S+1)]-36L_m(L_m+2)\}+(1/16)\beta^2\cos^4\vartheta\{L_m{}^2(L_m+2)^2-3L_m(L_m+2)\} +[\alpha\beta\cos^2\vartheta/24(1+\gamma_0)]\{L_m{}^2(L_m+2)^2+8L_m(L_m+2)[I_0(I_0+1)+S(S+1)]\}+\cdots \quad (7a)$$

or, in the case of more general penetrabilities,

$$4\pi W(\vartheta)=[\sum(2L+1)T_L]^{-1}\times\sum_L(2L+1)T_L[1+(1/6)\beta L(L+1)[3\cos^2\vartheta-1]+(1/2160)\beta^2\{69L^2(L+1)^2 +80L(L+1)[I_0(I_0+1)+S(S+1)]-189L(L+1)\}-(1/72)\beta^2\cos^2\vartheta\{15L^2(L+1)^2 +8L(L+1)[I_0(I_0+1)+S(S+1)]-36L(L+1)\}+(\beta^2/16)\cos^4\vartheta[3L^2(L+1)^2-6L(L+1)] +[\alpha\beta/18(1+\gamma_0)]\{7L(L+1)[I_0(I_0+1)+S(S+1)]+3L^2(L+1)^2-3L(L+1)N(I_0,S)\} \times[3\cos^2\vartheta-1]]+\cdots. \quad (7b)$$

In both expressions the term in $\alpha\beta$ arises from the inclusion of the angular momentum dependent fissionability to lowest order by means of the expression (4).

Comparison with Exact Numerical Calculations

Exact numerical calculations have been performed and compared with Eq. (7a). In the simplest comparisons, target and projectile spins were set equal to zero and the exact anisotropy was compared with the power series to first and second order in β. These results are summarized in Fig. 1.

In other calculations the accuracy of the power series estimate of the spin effects was tested (without including any angular momentum dependence of fissionability). These results are tabulated in Table I.

The results plotted in Fig. 1 show that the expansion (7a) gives a very accurate description of the anisotropy in the region relevant to most presently available

[7] D. Sperber, thesis, Princeton University, May 1961 (unpublished); Princeton University Report NYO-2961 (unpublished).

[8] J. Hiskes, University of California Radiation Laboratory Report UCRL-9275 (unpublished).

[9] I. Halpern, Ann. Rev. Nuclear Sci. 9, 245 (1959).

TABLE I. This table compares exact and approximate calculations of the difference in anisotropy between situations with both target and projectile spin equal to zero and those with projectile spin, S, of 1/2 and target spin, I_0, as listed in column three. Columns four and five give the exact results used, and column six lists the ratio of the spin-dependent β^2 term in Eq. (7a) to the corresponding exact difference. The last column lists the analogous ratio resulting from the use of the heuristic factor, expression (8), to estimate the spin effect. In all of these calculations the fissionability was assumed independent of angular momentum.

L_m	β	I_0	$[W(0°)/W(90°)]_{I_0=S=0}$	Exact spin correction	Approximate/ Exact	Heuristic/ Exact
2	2×10^{-2}	1/2	1.040548	-2.92×10^{-4}	0.92	0.96
8	2×10^{-3}	1/2	1.040364	-2.81×10^{-5}	0.95	0.99
2	4×10^{-2}	1/2	1.082157	-1.27×10^{-3}	0.84	0.91
8	4×10^{-3}	1/2	1.081412	-1.22×10^{-4}	0.87	0.95
2	8×10^{-2}	1/2	1.168334	-5.81×10^{-3}	0.73	0.86
2	8×10^{-2}	7/2	1.168334	-6.09×10^{-2}	0.77	0.86
4	2.67×10^{-2}	1/2	1.166049	-1.87×10^{-3}	0.76	0.88
8	8×10^{-3}	1/2	1.165284	-5.51×10^{-4}	0.77	0.90
12	3.8×10^{-3}	1/2	1.165114	-2.60×10^{-3}	0.79	0.92
2	1.6×10^{-1}	1/2	1.350603	-2.82×10^{-2}	0.63	0.83
2	1.6×10^{-1}	7/2	1.350603	-2.18×10^{-1}	0.86	0.98
8	1.6×10^{-2}	1/2	1.338206	-2.53×10^{-3}	0.70	0.93
8	1.6×10^{-2}	7/2	1.338206	-3.43×10^{-2}	0.55	0.88

experimental data. It is interesting to notice that the term linear in β can give a fairly good rough approximation ($\sim$10%) to the exact anisotropy, $W(0°)/W(90°)$, even for anisotropies so large that the expression including β^2 is quite inaccurate. Of course, the angular dependence of (7) will not represent the exact results accurately under these circumstances.

Table I illustrates rather clearly the convergence of the power series result to the exact result as the anisotropy decreases (column 6). It also indicates that for large target spins ($\approx 7/2$) and large anisotropies (>1.30), this first approximation to the spin effect may be in error by a factor of 2 or more. The final column of Table I indicates the advantage for estimates of the anisotropy of the heuristic replacement of the target spin term by the following simple factor multiplying the zero-spin anisotropy:

$$\{1+(\beta^2/18)L_m(L_m+2)[I_0(I_0+1)+S(S+1)]\}^{-1}. \quad (8)$$

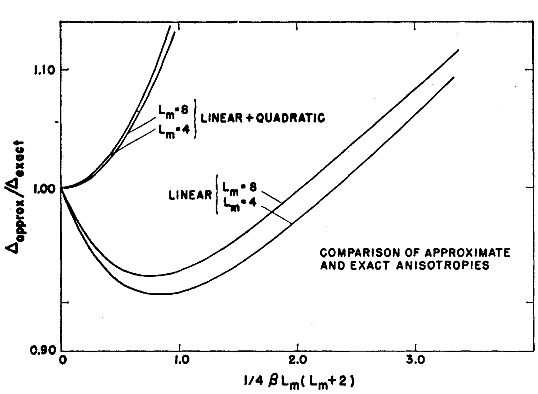

FIG. 1. This graph compares approximate and exact values of the quantity $\Delta=[W(0°)/W(90°)]-1$. The ratios of the linear (in β) and quadratic approximations to Δ to the exact value are shown as functions of the linear approximation for two values of L_m which span the range of primary interest in low-energy neutron-induced fission.

Note added in proof. The expression (8) was considered because it is the simplest non-negative algebraic factor which reduces to the correct power series limit for small β.
This replacement for anisotropy estimates is, of course, not extensible to the full angular distribution function on the basis of the results presented here.

Results

The anisotropy implied by Eq. (7a) differs from the large angular momentum approximation of earlier treatments primarily in the replacements

$$\begin{aligned} L_m^2 &\rightarrow L_m(L_m+2), \\ I_0^2 &\rightarrow [I_0(I_0+1)+S(S+1)]. \end{aligned} \quad (9)$$

Of these, the first is the most important for analysis of neutron fission anisotropies at low energies. For example, at a neutron energy of 2 MeV where $L_m\approx 3$, use of the large L_m approximation leads to a 70% overestimate of β.

This feature probably accounts for the tendency towards low inferred values of $K_0^2=(2\beta)^{-1}$ at low neutron bombarding energy in some of the simpler analyses of this type,[2,4,10,11] in contrast to the results of exact calculation[5,12] which indicate a linear increase from the threshold of K_0^2 with energy. It suggests, therefore, that the simplest pairing model viewpoint, without serious modification of the temperature-energy relationship it implies,[13] is consonant with the low-energy anisotropy data.

One also notes that the effect of target spin is predicted to be negative in the direction first suggested by Bohr,[1] but not born out empirically.[10,11] The exact

[10] J. Simmons and R. Henkel, Phys. Rev. **120**, 198 (1960).
[11] L. Blumberg and R. Leachman, Phys. Rev. **116**, 102 (1959).
[12] J. Simmons and J. Griffin (to be published).
[13] J. Griffin, *Proceedings of the International Conference on Nuclear Structure, Kingston* (University of Toronto Press, Toronto, 1960).

calculations[5] reinforce the present perturbative treatment in this respect, as do the results of the semiclassical analysis.[3] Nor can an angular-momentum-dependent fissionability explain the fact that U^{233} ($I_0=5/2$) has a larger anisotropy than Pu^{239} ($I_0=1/2$), except in conjunction with the assumption that the moments of inertia involved are much smaller than the rigid-body values. Finally, even apart from other difficulties which it involves,[3] it is difficult to give credence to the suggestion[6] that the distribution in K^2 deviates significantly from the statistical Gaussian form for values of K^2 so small as those involved in these low-energy neutron experiments.

One is therefore forced to consider seriously the possibility that $\beta=1/2K_0^2$ depends somewhat on the particular nucleus involved, and not alone on the excitation energy in excess of the fission barrier. A possible basis for such a dependence has been suggested by Chaudry *et al.*[14] from recent theoretical studies[15] of the uniformly charged liquid drop by Cohen and Swiatecki. The suggestion here is that different charge to mass ratios for different fissioning nuclei lead to more or less elongation at the fission barrier with consequent differences in K_0^2, even at the same value of E^*-E_f. Although this effect is not sufficient to explain the $U^{233}-Pu^{239}$ anisotropies,[10,11] it is probably comparable in magnitude to the spin and fissionability effects treated here.

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MATHEMATICAL APPENDIX

1. Sums of *D* Functions

In the expansion of Eq. (3) we have to deal with sums of the form

$$S_m{}^j(\vartheta,n)=\sum_{k=-j}^{j} k^n|\bar{D}_{mk}{}^j(\vartheta)|^2, \tag{10}$$

where the $\bar{D}_{MK}{}^I$ are the normalized symmetric top eigenfunctions. Their absolute squares are independent of the angles ϕ and ψ, and equal to $|\bar{d}_{mk}{}^j(\vartheta)|^2$, where $\bar{d}_{mk}{}^j$ is real and obeys the equation:

$$\left\{\frac{d^2}{d\vartheta^2}+\cot\vartheta\frac{d}{d\vartheta}-\frac{1}{\sin^2\vartheta}[m^2+k^2-2mk\cos\vartheta]+j(j+1)\right\}d_{mk}{}^j(\vartheta)=0. \tag{11}$$

When multiplied by $[8\pi^2/(2j+1)]^{1/2}$, the functions $\bar{D}_{mk}{}^j$ become identical to the matrix elements $D_{mk}{}^j$ of unitary irreducible representations of the group of rotations in three dimensions. The corresponding $d_{mk}{}^j(\vartheta)$ represent rotations with $\phi=\psi=0$. This circumstance immediately yields the simplest sum of the type (10):

$$S_m{}^j(\vartheta,0)=\sum_k|\bar{d}_{mk}{}^j(\vartheta)|^2 =\frac{(2j+1)}{8\pi^2}\sum_k d_{mk}{}^j(\vartheta)d_{km}{}^j(-\vartheta)=\frac{(2j+1)}{8\pi^2}, \tag{12}$$

in which the m index and angular variable, of which the sum is independent, are sometimes omitted in this report. The symmetries of the $d_{mk}{}^j$ functions[16] also guarantee at once that

$$S_m{}^j(\vartheta,n)=\pm S_{-m}{}^j(\vartheta,n), \tag{13}$$

where the plus sign applies for n even and the minus sign for n odd.

It develops that for small n the sums $S_m{}^j(\vartheta,n)$ are quite simple in their angular dependence, involving at most the nth powers of $\cos\vartheta$. This circumstance makes it practical to evaluate several of these sums explicitly by means of recursion formulas. To simplify the calculation, we factorize[17] Eq. (11). If we define (for $m,\ k\leq j$):

$$F_m{}^{\pm}=[v_m(\vartheta)\mp d/d\vartheta], \tag{14}$$

where

$$v_m(\vartheta)=m\cot\vartheta-k\csc\vartheta, \tag{15}$$

then the equations

$$F_m{}^{\pm}d_{mk}{}^j=[j(j+1)-m(m\pm1)]^{\frac{1}{2}}d_{m\pm1,k}{}^j \tag{16}$$

are equivalent to Eq. (11), as one can verify by operating with $F_{m\pm1}{}^{\mp}$. The factorization thus provides recursive identity for the derivative of the $\bar{d}$ function.

Then consider the expression for the following derivative, given by

$$\begin{aligned}(d/d\vartheta)[S_m{}^j(\vartheta,n)+S_{m+1}{}^j(\vartheta,n)] &=2\cot\vartheta[mS_m{}^j(\vartheta,n)-(m+1)S_{m+1}{}^j(\vartheta,n)]\\ &\quad+2\csc\vartheta[S_{m+1}{}^j(\vartheta,n+1)-S_m{}^j(\vartheta,n+1)],\end{aligned} \tag{17}$$

when evaluated by Eqs. (16) above. Direct evaluation, on the other hand, using the known expression for $S_m{}^j(\vartheta,n)$ yields simply an algebraic function, so that the above equation becomes a recursion relation in m for $S_m{}^j(\vartheta,n+1)$. One need only know one particular $S_m{}^j(\vartheta,n+1)$ to obtain from this recursion formula all the $S_m{}^j(\vartheta,n+1)$. The final step is, therefore, to evaluate

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[14] R. Chaudhry, R. Vandenbosch, and J. Huizenga, Bull. Am. Phys. Soc. **6**, 419 (1961); Phys. Rev. **126**, 220 (1962).

[15] S. Cohen and W. Swiatecki, Aarhus Universitet, January 1961 (unpublished).

[16] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957).

[17] Compare P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), Part I.

$S_j{}^j(\vartheta, n+1)$ from

$$(d/d\vartheta)[S_j{}^j(\vartheta,n)] = 2j\cot\vartheta S_j{}^j(\vartheta,n) - 2\csc\vartheta S_j{}^j(\vartheta, n+1). \quad (18)$$

For $(n+1)$ odd, this final process can be replaced by the simpler observation from Eq. (13) that

$$S_0{}^j(\vartheta, n+1) = 0. \quad (19)$$

This procedure yields the following results for sums required in the text:

$$S_m{}^j(\vartheta,0) = S^j(0) = (2j+1)/8\pi^2, \quad (20a)$$

$$S_m{}^j(\vartheta,1) = m\cos\vartheta\, S^j(0), \quad (20b)$$

$$S_m{}^j(\vartheta,2) = \tfrac{1}{2}\{[j(j+1)-m^2] - [j(j+1)-3m^2]\cos^2\vartheta\}S^j(0), \quad (20c)$$

$$S_m{}^j(\vartheta,3) = \tfrac{1}{2}m\cos\vartheta\{[3j(j+1)-3m^2-m] - [3j(j+1)-5m^2-m]\cos^2\vartheta\}S^j(0), \quad (20d)$$

$$S_m{}^j(\vartheta,4) = \tfrac{1}{8}\{[3j^2(j+1)^2-6m^2j(j+1)+3m^4 - 2j(j+1)+5m^2] - [6j^2(j+1)^2 - 36m^2j(j+1)+30m^4-8j(j+1) + 30m^2]\cos^2\vartheta + [3j^2(j+1)^2-30m^2j(j+1) + 35m^4-6j(j+1)+25m^2]\cos^4\vartheta\}. \quad (20e)$$

2. Clebsch-Gordan Identities

In the evaluation of Eqs. (1) and (3) there occur sums over Clebsch-Gordan coefficients of the form

$$F(L,j) = \sum_{IM} |C_{0MM}{}^{LjI}|^2 f[I(I+1), M]. \quad (21)$$

To evaluate such expressions, consider the equality

$$\Phi_\mu{}^L\chi_m{}^j = \sum_I C_{\mu m M}{}^{LjI}\Psi_M{}^I, \quad (22)$$

where $\mathbf{L}$ and $\mathbf{j}$ are commuting angular momentum operators whose vector sum is $\mathbf{I}$. Then $\Psi_M{}^I$ is an eigenfunction of any function $f[\mathbf{I}\cdot\mathbf{I}, \mathbf{I}_z]$ of the operators $\mathbf{I}\cdot\mathbf{I}$ and $\mathbf{I}_z$. Therefore, evaluation of the diagonal matrix element of the appropriately chosen operator $\mathbf{f}$ leads to the desired result (20). That is,

$$\langle\Phi_\mu{}^L\chi_m{}^j|f[(\mathbf{L}+\mathbf{j})\cdot(\mathbf{L}+\mathbf{j}), (L_z+j_z)]|\Phi_\mu{}^L\chi_m{}^j\rangle = \sum_I |C_{\mu m M}{}^{LjI}|^2 f[I(I+1), M]; \quad (23)$$

summing (22) over m, and setting $\mu=0$, one obtains (20).

The following special choices of the function $f(J,M)$ give the results listed:

$$f=1, \qquad F=(2j+1); \quad (24a)$$

$$f=I(I+1), \qquad F=(2j+1)[L(L+1)+j(j+1)]; \quad (24b)$$

$$f=M^2, \qquad F=\tfrac{1}{3}(2j+1)j(j+1); \quad (24c)$$

$$f=I^2(I+1)^2, \qquad F=(2j+1)[L^2(L+1)^2 + (10/3)L(L+1)j(j+1) + j^2(j+1)^2]; \quad (24d)$$

$$f=M^2I(I+1), \qquad F=\tfrac{1}{3}(2j+1)j(j+1)[L(L+1)+j(j+1)]; \quad (24e)$$

$$f=M^4, \qquad F=(1/15)(2j+1)j(j+1)\times[3j(j+1)-1]. \quad (24f)$$

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Polarization of Protons in $Be^9(d,p)Be^{10}$†

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The polarization of protons from the $Be^9(d,p)Be^{10}$ reaction has been measured at an incident deuteron energy of 10 MeV. The measurement has been carried from 13°(lab) to 80°(lab). The polarization is positive at forward angles but changes sign at about 70°(lab). Elastic scattering of protons from helium and carbon was used as the analyzing reaction. The axis of quantization is taken as $\mathbf{n}=\mathbf{k}_d\times\mathbf{k}_p$.

The experimental study of the deuteron stripping reaction serves two purposes. It can be used to obtain information about the spin, parities, and widths of the nuclear levels studied, and in addition is a sensitive "probe" to gain information about the various interactions involved. A simple Butler-type analysis of the angular distribution of the product nucleons is very often sufficient to determine the angular momentum of the captured nucleon.[1] In such an analysis both the incoming deuteron and the outgoing proton are approximated by plane waves, a simplification which leads to a prediction of zero polarization for the outgoing proton. In more sophisticated treatments either or both the incoming deuteron and outgoing proton waves are distorted by optical model potentials. Although the inclusion of spin-orbit terms in the potential is required

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[1] M. H. Macfarlane and J. B. French, Revs. Modern Phys. **32**, 567 (1960).