$S_j{}^j(\vartheta, n+1)$ from

$$(d/d\vartheta)[S_j{}^j(\vartheta,n)]=2j\cot\vartheta S_j{}^j(\vartheta,n) -2\csc\vartheta S_j{}^j(\vartheta, n+1). \quad (18)$$

For $(n+1)$ odd, this final process can be replaced by the simpler observation from Eq. (13) that

$$S_0{}^j(\vartheta, n+1)=0. \quad (19)$$

This procedure yields the following results for sums required in the text:

$$S_m{}^j(\vartheta,0)=S^j(0)=(2j+1)/8\pi^2, \quad (20a)$$

$$S_m{}^j(\vartheta,1)=m\cos\vartheta\, S^j(0), \quad (20b)$$

$$S_m{}^j(\vartheta,2)=\tfrac{1}{2}\{[j(j+1)-m^2] -[j(j+1)-3m^2]\cos^2\vartheta\}S^j(0), \quad (20c)$$

$$S_m{}^j(\vartheta,3)=\tfrac{1}{2}m\cos\vartheta\{[3j(j+1)-3m^2-m] -[3j(j+1)-5m^2-m]\cos^2\vartheta\}S^j(0), \quad (20d)$$

$$S_m{}^j(\vartheta,4)=\tfrac{1}{8}\{[3j^2(j+1)^2-6m^2j(j+1)+3m^4 -2j(j+1)+5m^2]-[6j^2(j+1)^2 -36m^2j(j+1)+30m^4-8j(j+1) +30m^2]\cos^2\vartheta+[3j^2(j+1)^2-30m^2j(j+1) +35m^4-6j(j+1)+25m^2]\cos^4\vartheta\}. \quad (20e)$$

2. Clebsch-Gordan Identities

In the evaluation of Eqs. (1) and (3) there occur sums over Clebsch-Gordan coefficients of the form

$$F(L,j)=\textstyle\sum_{IM}|C_{0MM}{}^{LjI}|^2 f[I(I+1), M]. \quad (21)$$

To evaluate such expressions, consider the equality

$$\Phi_\mu{}^L\chi_m{}^j=\textstyle\sum_I C_{\mu m M}{}^{LjI}\Psi_M{}^I, \quad (22)$$

where $\mathbf{L}$ and $\mathbf{j}$ are commuting angular momentum operators whose vector sum is $\mathbf{I}$. Then $\Psi_M{}^I$ is an eigenfunction of any function $f[\mathbf{I}\cdot\mathbf{I}, \mathbf{I}_z]$ of the operators $\mathbf{I}\cdot\mathbf{I}$ and $\mathbf{I}_z$. Therefore, evaluation of the diagonal matrix element of the appropriately chosen operator $\mathbf{f}$ leads to the desired result (20). That is,

$$\langle\Phi_\mu{}^L\chi_m{}^j|f[(\mathbf{L}+\mathbf{j})\cdot(\mathbf{L}+\mathbf{j}), (L_z+j_z)]|\Phi_\mu{}^L\chi_m{}^j\rangle =\textstyle\sum_I|C_{\mu m M}{}^{LjI}|^2 f[I(I+1), M]; \quad (23)$$

summing (22) over m, and setting $\mu=0$, one obtains (20).

The following special choices of the function $f(J,M)$ give the results listed:

$$f=1, \qquad F=(2j+1); \quad (24a)$$

$$f=I(I+1), \qquad F=(2j+1)[L(L+1)+j(j+1)]; \quad (24b)$$

$$f=M^2, \qquad F=\tfrac{1}{3}(2j+1)j(j+1); \quad (24c)$$

$$f=I^2(I+1)^2, \qquad F=(2j+1)[L^2(L+1)^2 +(10/3)L(L+1)j(j+1) +j^2(j+1)^2]; \quad (24d)$$

$$f=M^2I(I+1), \qquad F=\tfrac{1}{3}(2j+1)j(j+1)[L(L+1)+j(j+1)]; \quad (24e)$$

$$f=M^4, \qquad F=(1/15)(2j+1)j(j+1)\times[3j(j+1)-1]. \quad (24f)$$

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Polarization of Protons in $Be^9(d,p)Be^{10}$†

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The polarization of protons from the $Be^9(d,p)Be^{10}$ reaction has been measured at an incident deuteron energy of 10 MeV. The measurement has been carried from 13°(lab) to 80°(lab). The polarization is positive at forward angles but changes sign at about 70°(lab). Elastic scattering of protons from helium and carbon was used as the analyzing reaction. The axis of quantization is taken as $\mathbf{n}=\mathbf{k}_d\times\mathbf{k}_p$.

THE experimental study of the deuteron stripping reaction serves two purposes. It can be used to obtain information about the spin, parities, and widths of the nuclear levels studied, and in addition is a sensitive "probe" to gain information about the various interactions involved. A simple Butler-type analysis of the angular distribution of the product nucleons is very often sufficient to determine the angular momentum of the captured nucleon.[1] In such an analysis both the incoming deuteron and the outgoing proton are approximated by plane waves, a simplification which leads to a prediction of zero polarization for the outgoing proton. In more sophisticated treatments either or both the incoming deuteron and outgoing proton waves are distorted by optical model potentials. Although the inclusion of spin-orbit terms in the potential is required

† This research was supported in part by the Air Force Office of Scientific Research.
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to produce a polarization in elastic scattering, optical model distortion can produce polarization in stripping reactions even in the absence of any spin-orbit potential. The measurement of polarization is one of the most direct methods of determining the importance of such distortions in stripping reactions.

Since distorted wave calculations require long and tedious computer calculations, only general predictions will be discussed here. Huby *et al.*[2] have discussed the nature of the distorted wave theory on the assumption that spin-orbit potentials are negligible. They show that if (as is usual) the nucleon is captured into a definite state of orbital angular momentum l, the polarization of the outgoing nucleon is

$$P_l(\mathbf{k}_d,\mathbf{k}_p)=\frac{1}{3}\left(\frac{\theta_{l+1/2}{}^2}{l+1}-\frac{\theta_{l-1/2}{}^2}{l}\right)\frac{\langle m\rangle}{\theta_{l+1/2}{}^2+\theta_{l-1/2}{}^2}. \quad (1)$$

Here the $\theta_{j=l\pm\frac{1}{2}}{}^2$ are the reduced widths corresponding to the capture of the nucleon into states of total angular momentum $j=l\pm\frac{1}{2}$, while $\langle m\rangle$ is the expectation value of the orbital angular momentum along the quantization axis (normal to the reaction plane). Equation (1) states that the polarization is essentially a measurement of $\langle m\rangle$, weighted by the two widths which enter with opposite signs. This is a result of assuming that the proton spin enters the reaction only through its coupling in the deuteron to the spin of the neutron; the neutron spin is in turn coupled to the orbital momentum through $\mathbf{j}=\mathbf{l}+\mathbf{s}$. It can be noted from Eq. (1) that, since $\langle m\rangle\leqslant l$, the theory without spin-orbit potentials limits the maximum polarization to $\frac{1}{3}$ if the nucleon is captured with $j=l-\frac{1}{2}$ and $\frac{1}{3}l/(l+1)$ if it is captured with $j=l+\frac{1}{2}$. On the other hand the inclusion of spin-orbit coupling between the proton or deuteron and the nucleus can lead to complete polarization.[3] Thus the magnitude of the polarization gives some indication of the importance of these terms. Also note that for two similar reactions where the $\langle m\rangle$ can be expected to be roughly the same (e.g., stripping to neighboring levels which are of the same l in the same or adjacent nuclei), the polarizations are related by

$$P_{l+1/2}/P_{l-1/2}=-l/(l+1). \quad (2)$$

There is no indication from Eq. (1) what the sign of the polarization is until the sign of $\langle m\rangle$ is determined from a numerical calculation. Newns[4] and Tobocman[5] have used semi-classical arguments to show how distortion of the proton or deuteron waves can lead to polarization, and find that the sign of the polarization depends on whether the deuteron or proton distortion predominates. They also show that the sign of the polarization depends on the total angular momentum j of the captured nucleon, and is opposite for $j=l\pm\frac{1}{2}$ capture. A predominant deuteron interaction will give a positive polarization for $j=l+\frac{1}{2}$. Here we use the Basel convention: P is positive when it is parallel to $\mathbf{k}_d\times\mathbf{k}_p$. An empirical rule, based on available polarization data, seems to indicate that for $l=1$ stripping in low-Z elements the polarization in the stripping peak is positive or negative depending on whether j is $l+\frac{1}{2}$ or $l-\frac{1}{2}$. This rule is also obeyed in the $C^{12}(d,n)N^{13}$ reaction.[6] There are some exceptions to this rule, one being $B^{10}(d,p)B^{11}$ to the first-excited state.[7] However, independent evidence indicates that this reaction proceeds by spin-flip stripping.[8] One really surprising exception to this rule seems to be the $B^{10}(d,p)B^{11}$ ground state reaction, in which the polarization changes sign as the energy increases. Hensel[7] at $E_d=7.8$ MeV and Hird[9] at 8.9 MeV find that the sign rule is obeyed but Takeda[10] at 11.4 MeV finds the sign opposite to Hensel and Hird and so opposite to the "sign rule." Incomplete results of an experiment done by one of us (RWB) at 10 MeV gives an appreciable polarization of the same sign as at lower energies so that there is no change of sign up to $E_d=10$ MeV. The rapid change of sign between 10 and 11.4 MeV is extremely interesting in view of the small variation with energy (measurements from $E_d=7$ to 15 MeV) of the polarization in the reaction $C^{12}(d,p)C^{13}$ to the ground state.[11–13] It would be of great interest to have some additional measurements of the polarization in the reaction $B^{10}(d,p)B^{11}$ in the interval of $E_d=10$ to 12 MeV to see how it varies with energy.

At angles past the stripping peak, a simple argument by Newns and Refai[14] predicts a change of sign of the polarization. The argument separates the stripping amplitude into a predominant Butler term and (hopefully) much smaller "extra" terms, attributing the polarization to interference between the Butler and extra terms. This means that the polarization will change sign at the theoretical stripping minimum where the Butler term changes sign. This picture will be incorrect if the "extra" terms are of significant im-

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[13] A. Isoya, S. Micheletti, M. J. Marrone, and L. H. Reber, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Burkes (Academic Press Inc., New York, 1961), p. 595.
[14] H. C. Newns and M. Y. Refai, Proc. Phys. Soc. (London) **71**, 627 (1958).

portance. Cross terms between the "extra" amplitudes will be most noticeable near the stripping minimum where the Butler amplitude is small.

In order to examine the validity of these arguments, we have been measuring the angular dependence of polarizations produced by stripping on light nuclei. The reaction $Be^9(d,p)Be^{10}$ has been selected to complement the measurements which have been performed on carbon.[11,12] The two reactions should have roughly the same optical wave functions since the l values are the same, the nuclear radii are similar, and the Q values differ by less than 2 MeV. They differ primarily in the total angular momentum transfer; $C^{12}(d,p)C^{13}$ has $j=l-\frac{1}{2}$ while $Be^9(d,p)Be^{10}$ has $j=l+\frac{1}{2}$. Thus polarization of opposite signs is expected.

EXPERIMENTAL PROCEDURE

The polarization of the protons was measured by scattering them from a second target of a known analyzing power P_2, where P_2 is identical to the polarization which would have been induced in the scattered protons, had the initial beam been unpolarized. The polarization of the protons produced in the stripping is then found from the right-left asymmetry of the second scattering by

$$P_1P_2=A. \tag{3}$$

This asymmetry A is defined in terms of the number of protons scattered to the left N_L and the right N_R by

$$A=(N_R-N_L)/(N_R+N_L). \tag{4}$$

The two principal components of the apparatus were a small scattering chamber with a beryllium foil target at its center and a polarization analyzer which could be rotated around the target to determine the angular dependence of the polarization. Two different types of analyzers were used. In the first, which has been described elsewhere,[11] protons from the beryllium target were scattered left or right in the analyzer by helium gas at high pressure; the scattering angle on either side was restricted by a fixed "Venetian blind" arrangement, and the protons were detected in nuclear emulsions. In the second, described in more detail below, the protons were scattered left and right by a carbon foil and detected by telescopes consisting of a thin proportional counter and a scintillation detector operated in coincidence.

The beryllium targets were made from 1-mil commercially available beryllium foil. In the stripping peak a single thickness was sufficient, while at larger angles a double thickness was used in order to compensate for the decrease in cross section with angle. Since the first-excited level in beryllium is at 3.37 MeV, it was still possible to resolve the ground state completely from the excited state.

The target was placed inside a small scattering chamber whose window arrangement was such as to allow an essentially continuous variation of the angle of observation from 15° to 160° (lab). For angles less than 20° a special scattering chamber was constructed which allowed observation down to about 7°. The 10-MeV deuteron beam of the Washington University cyclotron was brought into the chamber through a 5/16-in. circular tantalum baffle, and after passing through the target was stopped by 10-mil tantalum sheet. During the course of this work, improvements in the stability and focusing of the cyclotron beam permitted us to replace the circular baffle by a 0.15-in. slit. The large-angle data taken with the helium polarimeter and all of the data from the carbon polarimeter were taken with the smaller baffle. The protons left the target chamber through a thin (1-mil) Dural window and then entered the polarimeter through a similar window after a brief passage in air.

CARBON POLARIMETER

Since polarization measurements are double "scattering" experiments intensity considerations make inevitable the use of thick targets in both the polarizing and analyzing reactions. This severely reduces the energy resolution so that only well-separated levels can be measured. The resolution is further reduced when helium is used as an analyzer since helium has a very low mass and because gas targets have inherently poor geometry. In an effort to improve the energy resolution over that obtainable with helium, a polarimeter has been built using the elastic scattering of protons by carbon as an analyzing reaction. The polarization induced in protons by elastic scattering by carbon has been measured in the last few years at several laboratories. In particular, it has been shown[15–18] that the polarization is both large and smoothly varying in the forward direction at energies above 12 MeV. Yamabe *et al.*[17] and Brockman[15] have shown that the polarization is about 50% at 45° lab while Sanada[18] has shown that the polarization at 50° lab is approximately 70%. We have set the scattering angle in our polarimeter at 50° in order to be able to use the larger value of P_2.

The polarimeter consists of a carbon target flanked by two identical counter telescopes. Several target foils of self supporting ground graphite were used, ranging in thickness from 20 to 30 mg/cm². They were located 10.25 in. from the first target and subtended a half-angle of 1.75°. The entire polarimeter could be rotated about the axis of the proton beam; this made it possible to average out any slight mechanical asymmetries in the polarimeter, nonuniformities in the carbon target, or differences in the counter efficiency. Each of the counter telescopes consisted of an argon (4% CO_2)

[15] K. W. Brockman, Phys. Rev. **110**, 163 (1958).
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[18] J. Sanada, Suppl. Helv. Phys. Acta **6**, 249 (1961).

filled proportional counter in front of a 3/8 in. by 5/8 in. NaI scintillation counter whose mean acceptance angle was 49.4° with a half angular spread of 4.0°. The outputs of these counters, after amplification and pulse height discrimination, were fed into slow coincidence circuits, $2\tau=0.4\,\mu$sec. The coincident pulses were used to gate the Huchinson-Scarrott multichannel analyzer which recorded the pulse spectra of the scintillators. In order to detect any malfunction of the analyzer, the pulses were also recorded by scalers. The memory of the analyzer was split so that the spectrum from one counter was stored in the even numbered channels and from the other in the odd numbered channels.

With careful selection of targets, scintillators, etc., the polarimeter could resolve proton groups which were separated by more than 1 MeV.

Background, always a problem in double scattering experiments because of the low counting rates, was effectively eliminated by the coincidence system and energy discrimination. The accidentals rate of the coincidence system was negligible. It was determined by measuring the coincidences between the left proportional counter and the right scintillator or vice versa. There was some background which was attributed to neutron-induced reactions in the walls of the chamber. Real coincidences were produced when the charged reaction products passed through both counters.[19] This background disappeared when the beryllium target, a favorite source of high energy neutrons, was removed. The background was, in general, less than 5% of the real counts and in test cases changed the asymmetry by less than 0.01. Therefore, the background was ignored for the rest of the angles.

It should be noted that there are some complications in using carbon as an analyzer for stripping reactions due to contamination of the proton beam with scattered deuterons. Unless the stripping reaction has a high Q or unless the proton beam is moderated before striking the carbon, protons produced by stripping on the carbon will obscure those to be measured. Also when the scattered deuteron beam is intense, say at small angles or at high primary beam currents, additional moderation may have to be placed before or after the carbon to prevent the numerous low-energy reaction products coming from the carbon from saturating the counters. These problems were fairly serious at this laboratory since the 10-MeV energy of the deuteron beam did not allow the use of much moderator if the proton energy at the second reaction was to be kept above 12 MeV. Below this energy the polarization constant P_2 for carbon changes very rapidly with energy. This energy restriction also prevented measurements above about 40°.

[19] It is interesting to note that the use of methane, which contains hydrogen, as a moderator in the counter gas doubled the background. If a thin plastic scintillator had been used for this purpose the peak would have been badly obscured.

We found it necessary to be careful in using thin proportional counters in double scattering experiments. The high ionization level, due both to events occurring in the second target and to the high room background, leads to the loss of up to 10% of the real events because of dead time which in turn could lead to errors up to 0.10 in the asymmetry. This effect was checked by measuring the asymmetry twice at 19° at beam currents varying by a factor of 3. The two values were equal within statistics (± 0.02) showing that the counters have equal dead time. In addition there was no change in the counting rate of the proportional counters when the polarimeter was inverted, so that any asymmetry should be eliminated by the experimental procedure.

In order to eliminate any systematic error arising from differences of the right and left counters, the asymmetry A was measured twice for each stripping angle; the two measurements were under identical conditions except that the roles of the counters were reversed by rotating the counter 180° about the axis of the proton beam. This procedure will average out any small errors due to asymmetries in the polarimeter such as different dead times of the proportional counters, nonuniform second targets, etc. The error between the two values of A was always within statistics and was not systematic, indicating that the two counters were identical. At several angles further checks were made by measuring the asymmetry for both plus and minus stripping angles. Again no errors were found outside of statistics.

N_L and N_R were found by the usual method of adding up the number of counts in a peak between two cutoff limits. One advantage of performing each measurement twice with the counters in opposite locations is that it makes the evaluation of A insensitive to the exact location of the cutoff points. If an extra channel is included for one of the counters, it has opposite effects in the two determinations and so cancels.

When the second target subtends a finite solid angle, the variation of cross section with angle for the first reaction introduces a spurious asymmetry A_σ. In this event, the product of the polarizations is not given by Eq. (4) but by[20]

$$P_1P_2=(A-A_\sigma)/(1-AA_\sigma).$$

If A and/or A_σ are small, then this is just

$$P_1P_2=A-A_\sigma. \tag{5}$$

For our polarimeter this spurious asymmetry is given by

$$A_\sigma=7.64\times10^{-4}(\sigma_1'/\sigma_1)(\sigma_2'/\sigma_2-1.70).$$

Here, σ_1 and σ_2 are the differential cross sections for $Be^9(d,p)Be^{10}$ and $C^{12}(p,p)C^{12}$, respectively, while σ_1' and σ_2' are their derivatives with respect to angle. Values for σ_1 and σ_1' were determined by careful measurement of the cross section, while σ_2 and σ_2' were determined

[20] R. E. Warner and W. P. Alford, Phys. Rev. **114**, 1338 (1959).

TABLE I. Polarization of protons from $Be^9(d,p)Be^{10}$ at $E_d=10$ MeV as measured with the helium polarimeter.

θ (lab)	θ (c.m.)	L	R	$A=P_1P_2$	P_1(%)
15.5°	17.3°	720	913	−0.118	18.5± 3.8
19.5°	21.8°	1674	1676	−0.001	0.1± 2.7
24.5°	27.4°	1106	1200	−0.041	6.3± 3.2
29.0°	32.4°	1274	1458	−0.0673	10.4± 3.0
32.5°	36.2°	273	304	−0.0537	8.3± 6.4
37.8°	42.0°	332	416	−0.112	17.3± 5.6
42.5°	47.2°	98	143	−0.187	28.5± 9.7
46.0°	50.8°	184	234	−0.120	18.2± 7.4
52.0°	57.3°	53	83	−0.221	33.4±12.7
56.0°	61.5°	44	69	−0.221	33.0±13.7
61.5°	67.2°	74	83	−0.0573	8.5±11.8
64.5°	70.7°	85	67	+0.118	−17.5±12.0
70.0°	76.4°	91	71	+0.123	−18.2±11.8
76.0°	82.5°	57	43	+0.140	−20.6±14.6
80.0°	86.8°	55	38	+0.183	−26.9±15.0

from the data of Peelle[21] and Nagahara.[22] Unfortunately, these measurements were taken at 10-deg intervals so that the determination of σ_2' at 50° is accurate only to about ±10%. However A_σ is small, less than 0.05, so that its error should be less than 0.005. This uncertainty, along with the uncertainty of A due to wandering of the beam in the baffles, has been accounted for by adding ±0.01 to the statistical error. The validity of Eq. (5) as well as the alignment was checked by substituting a gold foil for the carbon second target. The scattering from gold at these energies is near Coulombic and has been shown to have a small polarization.[17] The asymmetry reduced, as expected, to A_σ.

DISCUSSION

The polarizations are given in Tables I and II and in addition are displayed in Fig. 1, where the cross section[23] and data at other energies[9,24] (see also Table III) are shown for comparison. Since the polarization parameter P_2 used in calculation of the polarization, was taken from the literature, it is difficult to determine its accuracy. Thus the standard error given in the tables has been evaluated on the basis of the statistics of A alone.

TABLE II. Polarization of protons from $Be^9(d,p)$ at $E_d=10$ MeV as measured with the carbon polarimeter.

θ_{lab}	$\theta_{\text{c.m.}}$	A	A_σ	P(%)
13°	14.5°	−0.068±0.023	+0.018	+12.3±3.0
15.5°	17.3°	−0.063±0.020	+0.014	+11.0±2.9
19°	21.3°	−0.061±0.015	+0.000	+8.7±2.1
22°	24.5°	−0.077±0.014	−0.016	+8.6±2.0
25°	27.9°	−0.076±0.018	−0.025	+7.3±2.6
28°	31.2°	−0.118±0.020	−0.036	+11.7±2.9
31°	34.6°	−0.127±0.022	−0.051	+10.9±3.1
34°	37.8°	−0.147±0.031	−0.051	+13.7±4.4
37°	41.1°	−0.139±0.033	−0.047	+13.2±4.7
40°	44.4°	−0.192±0.027	−0.033	+22.7±3.9

[21] R. W. Peelle, Phys. Rev. **105**, 1311 (1957).
[22] Y. Nagahara, J. Phys. Soc. Japan **16**, 133 (1961).
[23] B. Zeidman and J. M. Fowler, Phys. Rev. **112**, 2020 (1958).
[24] C. R. Lubitz, University of Michigan Technical Report UMRI-2842 (unpublished).

TABLE III. Polarization of protons from $Be^9(d,p)$.

E_d (MeV)	θ	P(%)	References
7.8	20° (c.m.)	12.5±2.5	Green and Parkinson
	42° (c.m.)	21.0±3.0	(See reference 24)
8.9	15° (lab)	18.3±7.0	Hird
	20° (lab)	8.2±6.0	(See reference 9)

The data show several salient features. First, the polarization is found to be positive up to moderately large angles. This is in agreement with the Tobocman sign rule, indicating that the deuteron interaction is predominant. There is a maximum at about 60° followed by a sharp drop, with a change of sign of the polarization near 70° (c.m.). This is to be compared with the first minimum in the Butler stripping curve at around 58° (its exact position depends on the choice of stripping radius used to obtain a "best" fit). This, along with the maxima appearing where Newns' and Refai's[14] argument would instead predict a change of sign, indicates that cross terms among the "extra" interaction amplitudes make a substantial contribution at these angles.

At every angle, the polarization is smaller in magnitude and opposite in sign to that for the carbon reaction[11,12] at the same bombarding energy (except in the region of sign change). The experimental errors are such that it is difficult to fix a definite ratio of the polarizations in carbon and beryllium, but $|P_{\text{Be}}/P_{\text{C}}|$ seems to be greater than the value 1/2 which is expected from Eq. (2) if one assumes equal values $\langle m\rangle$ in the two reactions. The polarization also rises above the limit of 1/6 given by Eq. (1), indicating that spin-orbit potentials are present.

It is interesting to compare our polarizations with left-right asymmetries in the stripping cross section of the same reaction obtained by Hird and Strzalkowski[25] using partially polarized 6-MeV deuterons. Satchler[26] has derived a relationship between the polarization produced in a stripping reaction and the asymmetry produced in the cross section through the use of polarized beams:

$$[\sigma(\theta)]_{\text{pol}}=[\sigma(\theta)]_{\text{unpol}}[1+3\mathbf{P}_d\cdot\mathbf{P}_p]. \quad (6)$$

Here $\mathbf{P}_p$ is the polarization of the resulting protons if unpolarized deuterons are incident, $\mathbf{P}_d$ is the vector polarization of the incident deuterons, and the $\sigma(\theta)$ are the differential cross sections measured, respectively, with polarized and unpolarized deuterons. This relation is derived assuming negligible spin-orbit or other spin coupled interactions. If it is assumed that the polarization changes very slowly with energy in this reaction, as is the case in $C^{12}(d,p)C^{13}$, then Hird's asymmetries can be predicted from our polarizations. The great

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[26] G. R. Satchler, Nuclear Phys. **6**, 543 (1958).

similarity[27] (cf. Fig. 1) between the two sets of data indicates that Eq. (6) is valid. This is surprising since the agreement is at angles where non-negligible spin-orbit potentials are indicated by the large value of P. No definite conclusions can be drawn since there is unfortunately a great difference in incident energy and they did not know the polarization of their beam (their very rough estimate for P_d is 60%). If our data at smaller angles (35° to 55°) are compared with their asymmetries, Eq. (6) would indicate that their deuteron beam had a polarization P_d of about 40%. At larger angles the polarizations are too small and the errors too large for any reliable estimates of P_d.

The importance of spin-orbit distorting potentials is difficult to estimate. At angles approaching the first stripping minimum and beyond, their presence (or that of some spin-dependent potential) is required to explain the high polarizations found both in this experiment and in $C^{12}(d,p)C^{13}$. However, in the region of the stripping peak, the importance of the spin-orbit terms is not so clear. In neither this, nor in the early carbon data[11,12] does the polarization exceed the limits placed on it by the central potential theory. Indeed, distorted-wave calculations on $C^{12}(d,p)C^{13}$ by Gibbs,[28] using spin-orbit potential, show considerably polarization at high angles but comparatively little in the forward direction. The same is true of a calculation by Robson.[29] All this would lead one to hope that the spin-orbit terms could be neglected in the forward direction, but there have recently been found very large polarizations[13] in the extreme forward direction in $C^{12}(d,p)C^{13}$. The question arises whether this is a phenomenon of the particular reaction or if it is true of all the $l=1$, low-Z stripping reactions. In $Be^9(d,p)$ the polarizations which have been found do not exceed the theoretical limit in the region of the stripping peak. A calculation which has been made by Lubitz[24] for 8-MeV deuterons, is shown in Fig. 1. As can be seen, the curve has roughly the same shape as the data but the calculated polarizations are too small and the cross-over is at too small an angle. It would be interesting to see if a believable potential could be found which would give the correct magnitude for the polarization or if here too one must resort to spin-dependent terms.

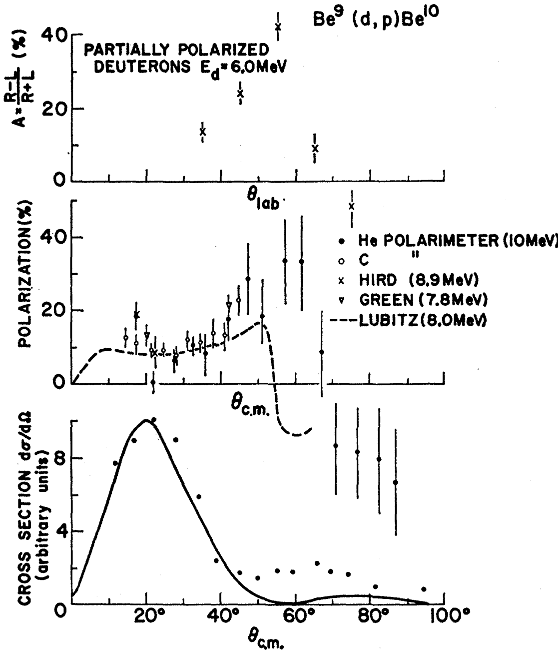

FIG. 1. Proton polarization in $Be^9(d,p)Be^{10}$. (*Upper*) Asymmetry measurement using partially polarized deuterons. $E_d=6.0$ MeV. Hird *et al.* (reference 25). (*Center*) Proton polarization as measured with a helium and carbon polarimeter. (See Tables I, II, and III.) Dashed line is a calculation of Lubitz for $E_d=8$ MeV. (See reference 24.) (*Lower*) Cross section vs angle. Solid line is a Butler-type calculation. Data points are by Zeidman and Fowler (reference 23).

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[27] Since the polarization of Hird's beam is incomplete, the fact that their asymmetries have the same magnitude as our polarizations can only be considered fortuitous; what is important is that the shapes are the same.

[28] W. R. Gibbs and W. Tobocman (private communication).

[29] D. Robson, Nuclear Phys. 22, 34 (1961).