Mean Lifetime of the π^0 Meson

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(Received April 10, 1962)

An estimate of the π^0 mean lifetime has been obtained by a method first attempted by Harris, Orear, and Taylor in 1957. The decay distances of monoenergetic π^0 mesons, produced by the $K_{\pi 2}$ decay at rest ($K^+ \to \pi^+ + \pi^0$) and decaying via the Dalitz mode ($\pi^0 \to e^+ + e^- + \gamma$), were measured. In two stacks of nuclear emulsion (Ilford K5), exposed to the 700-MeV/c K^+ beam at the Berkeley Bevatron, 45 events of this type were found. For the π^0 mean lifetime a value of $\tau = (2.3_{-1.0}^{+1.1}) \times 10^{-16}$ sec has been obtained.

1. INTRODUCTION

THE direct determination of the π^0 mean lifetime τ has heretofore been attempted in a number of experiments. However, until 1959 these experiments did not result, practically speaking, in more than the estimate of an upper limit for τ. It was only in 1960 that Glasser *et al.*[1] as well as Blackie *et al.*,[2] and in 1961 Shwe *et al.*[3] succeeded in determining directly the mean lifetime τ by measuring the π^0-decay distances in nuclear emulsion; the π^0 meson was identified by its Dalitz decay. The single Dalitz decay ($\pi^0 \to e^+ + e^- + \gamma$) occurs with a relative frequency of $\approx 1.2 \times 10^{-2}$ and the double Dalitz decay ($\pi^0 \to 2e^+ + 2e^-$) with a relative frequency of $\approx 3.5 \times 10^{-5}$.[4–6] Glasser *et al.*, Blackie *et al.*, and Shwe *et al.* obtained for τ the values $(1.9 \pm 0.5) \times 10^{-16}$ sec, $(3.3_{-1.4}^{+1.5}) \times 10^{-16}$ sec, and $(1.9_{-0.8}^{+1.3}) \times 10^{-16}$ sec, respectively.

In the present experiment a method has been applied, which was first suggested and attempted by Harris *et al.*[7] The $K_{\pi 2}$ decay at rest ($K^+ \to \pi^+ + \pi^0$) is used as a production reaction for monoenergetic π^0 mesons. Each of the two pions is emitted with a momentum of 205.3 MeV/c, corresponding to a kinetic energy of 110.7 MeV for the π^0 meson and of 108.6 MeV for the π^+ meson. In principle, one only needs to measure the decay distance of the π^0 meson in a sufficiently large number of such events; from this one then directly obtains the π^0 mean lifetime. The details of our experiment are described in what follows.

2. EXPOSURE AND SCANNING

Two identical stacks, each consisting of 65 pellicles of Ilford K5 emulsion having the dimensions 6 in. $\times$7 in.$\times$600 μ, were exposed to the 700-MeV/c K^+ beam at the Berkeley Bevatron.

An area scan yielded 18 960 K^+ decays at rest. Among these events there were 61 decays with one Dalitz pair and 1 decay with two Dalitz pairs. From the branching ratios for the K^+ decay[8] and the relative frequencies for the Dalitz decay of the π^0 meson, it follows that among the 18 960 K^+ decays one should have found 71 ± 3 decays with one Dalitz pair and 0.27 ± 0.01 with two Dalitz pairs. That results in a scanning loss of $(14 \pm 12)\%$ for K^+ decays with one Dalitz pair. However, this scanning loss introduces no serious bias, since the probability for overlooking a $K_{\pi 2}$ event cannot be a function of the π^0-decay distance because the mean gap length between two grains of an electron track is about 7 μ whereas the mean π^0 decay length is only ≈ 0.1 μ.

By following the secondary tracks of the 61 K^+ decays with one Dalitz pair, five could be identified as τ' decays, three as K_{e3} decays, and two as $K_{\mu 3}$ decays. Two events were unmeasurable, and for five events it could not be decided which ones of the secondary tracks were the two Dalitz electrons. The remaining 44 events, as well as the one K^+ decay with two Dalitz pairs, were considered as being $K_{\pi 2}$ decays and were measured. From the branching ratios for the K^+ decay, it follows that about 25% of the 45 measured events should be K_{e3} and $K_{\mu 3}$ decays. The effect of this background can be estimated. Later on it will be accounted for by a correction term.

3. MEASURING PROCEDURE

The measurements were carried out with a Leitz-Ortholux microscope, supplied with a special Zeiss eyepiece filar micrometer. The total magnification of the optical system was about 2000. A Cartesian coordinate system was defined with its (x,y) plane parallel to the plane of the emulsion. The x and y coordinates of the grains of the two electron tracks and of the π^+ track were measured, as well as the dip angles of these tracks. For each event, the center of the last grain of the K^+ track was chosen as the origin of the system.

In order to reduce possible systematic errors as a consequence of thermal motion, the coordinate measurements were carried out in the following manner: The coordinate differences Δx_{ij} and Δy_{ij} between each pair of neighboring grains i and j were measured, the cross hair of the eyepiece micrometer being moved from i to j and back to i. The three readings on the micrometer

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screw obtained in this way $(i-j-i)$, give two measured values for each difference. The effect of the thermal motion on these two values should be exactly opposite and equal if the thermal motion is assumed to be uniform, and therefore should cancel out when one takes the average. Each coordinate difference was measured three times in this way. The arithmetic mean value was taken as the final result. The grain coordinates x and y follow by addition from the measured differences.

For the measurement error of a coordinate difference in x as well as in y, a mean value of

$$F=(0.024\pm0.001)\ \mu$$

has been determined. The measurement uncertainty Δ_i in the coordinates of the ith grain of a track is then given by

$$\Delta_i=F\sqrt{i}.$$

(The first grain of a track is always the one lying next to the origin of coordinates.) On the average the coordinates of six grains per track were measured.

In a few events the track of one of the two Dalitz electrons was not measurable, and in a few other events the two electron tracks were measured as being one track since their opening angle was so small that it was impossible to separate them inside the field of view.

4. ANALYSIS OF THE DATA

The projections of the electron and π^+ trajectories into the (x,y) plane were reconstructed by fitting a straight line $y=ax+b$ to the measured points (x,y). The fit was done by the method of least squares, minimizing the expression

$$S=\sum_{i=1}^{N}[w_{xi}(x_i-x)^2+w_{yi}(y_i-y)^2].$$

In this formula x_i, y_i are the measured coordinates, x, y are the corresponding fitted coordinates, w_{xi} and w_{yi} are the weights, and N is the number of measured grains per track. The weights are given by the following equations:

$$(1/w_{xi})=iF^2+D_i^2[a^2/(1+a^2)]+\langle\Delta^2\rangle_{\text{av}}[a^2/(1+a^2)],$$
$$(1/w_{yi})=iF^2+D_i^2[1/(1+a^2)]+\langle\Delta^2\rangle_{\text{av}}[1/(1+a^2)].$$

In either equation the first term is the pure measurement error, the second term is the uncertainty in the measured coordinates, resulting from multiple scattering, and the third term is the uncertainty which is introduced by the deviation of the grain centers from the true particle trajectories. With a mean $p\beta$ of 61.4 MeV/c for the electrons and a $p\beta$ of 169.7 MeV/c for the π^+ mesons, one gets

$$D_i^2=2.920\times10^{-7}[(x_i^2+y_i^2)^{\frac{1}{2}}/\cos\theta]^3\ \mu^2 \quad \text{for } e \text{ tracks,}$$
$$0.4022\times10^{-7}[(x_i^2+y_i^2)^{\frac{1}{2}}/\cos\theta]^3\ \mu^2 \quad \text{for } \pi^+ \text{ tracks.}$$

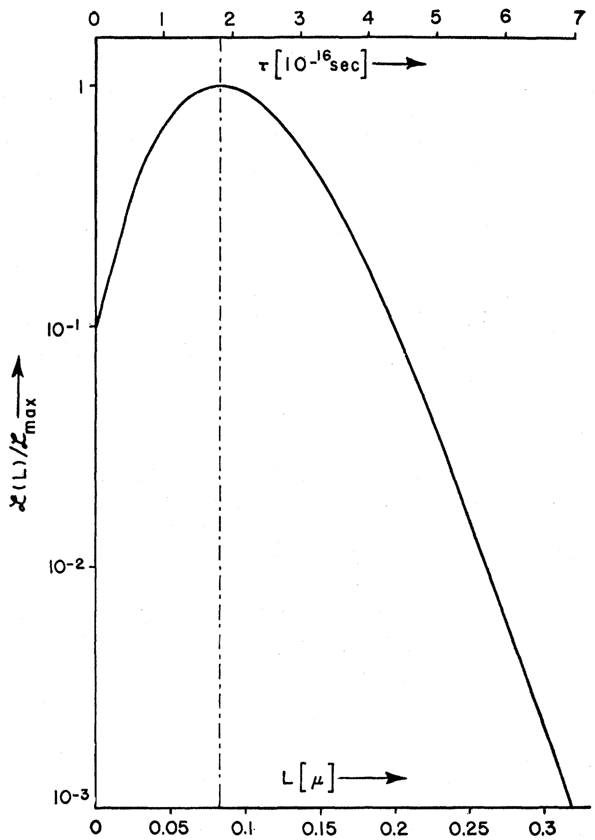

FIG. 1. Likelihood function for our 45 events. The relative likelihood $\mathcal{L}(L)/\mathcal{L}_{\max}$ is plotted logarithmically vs the mean decay length L. The maximum of the function occurs at $L=0.083\ \mu$, corresponding to a mean lifetime τ of 1.8×10^{-16}sec

θ is the dip angle. For $\langle\Delta^2\rangle_{\text{av}}$ the following value was estimated:

$$\langle\Delta^2\rangle_{\text{av}}=0.00125\ \mu^2.$$

This means a rms deviation of the grain centers from the true particle trajectory of about 0.035 μ.

The π^0 starts at the decay point of the K^+ meson. This decay point is defined by the foot of the perpendicular from the center of the last grain of the K^+ track to the π^+ line. The x, y coordinates of the decay point of the π^0 are given by the intersection of the electron line with the π^+ line. The thus determined projection of the π^0-decay length into the horizontal plane [(x,y) plane] has to be divided by $\cos\theta_\pi$ in order to get the true π^0-decay length (θ_π=dip angle of the π^+). For those events of which both electron tracks could be measured, one gets two different π^0 decay lengths this way because of the uncertainties in the measured x, y coordinates. The weighted mean will then give the best estimate.

If the intersection of the electron line with the π^+ line lies on the proper side of the K^+ end, the π^0-decay length is counted positive; otherwise it is counted negative.

TABLE I. Compilation of the π^0-decay distances λ_i and their mean errors δ_i.

Event	λ_i [μ]	δ_i [μ]	Event	λ_i [μ]	δ_i [μ]	Event	λ_i [μ]	δ_i [μ]
M 1	+0.14	0.64	*M* 18	+0.18	0.34	*B* 9	−0.21	0.50
M 2	+0.86	0.59	*M* 19	+0.49	0.33	*B* 10	−0.88	0.60
M 3	−0.32	0.36	*M* 20	−0.06	0.11	*B* 11	−0.33	0.29
M 4	−0.89	1.13	*M* 22	0.00	0.09	*B* 12	+0.48	0.29
M 5	+0.34	0.17	*M* 26	+0.75	0.26	*B* 13	−0.24	0.21
M 6	−0.27	0.10	*M* 27	−1.16	0.94	*B* 14	−0.92	0.94
M 7	−0.83	0.39	*M* 28	−0.14	0.13	*B* 15	+0.28	0.21
M 8	+0.48	0.54	*B* 1	−0.17	0.17	*B* 16	+1.67	0.49
M 9	−0.25	0.19	*B* 2	−0.79	0.58	*B* 17	+0.45	0.61
M 12	+0.38	0.14	*B* 3	−0.09	0.18	*B* 18	+0.34	0.30
M 13	+0.03	0.35	*B* 4	−0.28	0.38	*B* 19	+0.46	1.11
M 14	+0.26	0.29	*B* 5	−1.75	3.31	*B* 21	+0.81	0.30
M 15	−0.07	0.50	*B* 6	−0.12	0.40	*B* 23	−0.33	0.28
M 16	+0.03	0.28	*B* 7	+0.12	0.05	*B* 24	+0.09	0.54
M 17	−0.24	0.25	*B* 8	+0.56	0.60	*B* 25	−0.25	0.39

As an estimate for the accuracy with which the K^+ end of an event could be determined along the π^+ line the distance between the center of the last grain of the K^+ track and the π^+ line has been taken.

The π^0 decay lengths λ_i and their mean errors δ_i are given in Table I.

From the 45 measured π^0 decay distances a mean lifetime τ has been calculated by the maximum-likelihood method. For the ith event the probability density P_i that one measures the decay distance $\lambda_i \pm \delta_i$ is

$$P_i = \frac{1}{L\delta_i(2\pi)^{\frac{1}{2}}} \int_0^\infty \exp\left\{-\left[\frac{(\lambda_i - l)^2}{2\delta_i^2} + \frac{l}{L}\right]\right\} dl.$$

L is the mean decay distance, corresponding to the mean lifetime τ.

$$L = \gamma\beta c\tau.$$

Then the likelihood function is

$$\mathfrak{L}(L) = \prod_{i=1}^{45} P_i.$$

Figure 1 shows a plot of this likelihood function. The 45 measured π^0-decay distances $\lambda_i \pm \delta_i$ give for the mean decay length a most probable value of

$$L^* = (0.083_{-0.043}^{+0.049})\ \mu.$$

This corresponds to a π^0 mean lifetime:

$$\tau^* = (1.8_{-0.9}^{+1.1}) \times 10^{-16}\ \text{sec}.$$

5. CORRECTIONS

The maximum-likelihood result still has to be corrected for systematic errors. Two corrections must be taken into account—a grain correction and a background correction.

(a) *Grain correction.* This correction is necessary because the first grain of the π^+ track as well as that of the two electron tracks can be so close to the last grain of the K^+ track that it is practically impossible to decide if it has been produced by the K^+, π^+, or by one of the two Dalitz electrons, respectively. It is reasonable to assume that for grains with a diameter d this decision cannot be made unless the distance of their centers is $> \frac{1}{2}d\sqrt{3}$, that is, unless the diameter of their circle of intersection is $< \frac{1}{2}d$. The grain diameter has been

$$d = (0.55 \pm 0.12)\ \mu.$$

For the grain densities the following values were determined:

$$g(\pi^+) = (16.5 \pm 0.2)\ \text{grains}/100\ \mu,$$
$$g(e) = (15.5 \pm 0.2)\ \text{grains}/100\ \mu.$$

From this the correction term is

$$K_{\text{grain}} = +(0.009 \pm 0.005)\ \mu.$$

(b) *Background correction.* As mentioned in Sec. 2, the 45 events which were taken as $K_{\pi 2}$ decays and measured, still contain a 25% background of three-body decays, namely, K_{e3} and $K_{\mu 3}$ decays with the ratio 1:1. The maximum kinetic energy of the π^0 from the K_{e3} decay is 130.4 MeV and of the π^0 from the $K_{\mu 3}$ decay 119.1 MeV. The distribution of the angle between the momentum vectors of π^0 and decay lepton e^+ or μ^+, respectively, has to be assumed somewhere between isotropic and peaking at 180°. Then one obtains the following estimate for the correction term:

$$K_{\text{background}} = +(0.014 \pm 0.014)\ \mu.$$

The two electrons from the conversion of a real γ from the π^0 decay may be mistaken for a Dalitz pair if the conversion takes place at a satisfactory small distance from the K^+ end ($\lesssim 5\ \mu$). However, it is not necessary to correct for this effect because these cases are very rare due to the relatively large radiation length of 2.9 cm in nuclear emulsion.

6. RESULTS AND DISCUSSION

For the corrected mean decay length L of the π^0,

$$L = L^* + K_{\text{grain}} + K_{\text{background}},$$

the experimental value

$$L = (0.106_{-0.046}^{+0.051})\ \mu$$

has been obtained. The corresponding corrected mean lifetime of the π^0 meson is

$$\tau = (2.3_{-1.0}^{+1.1}) \times 10^{-16}\ \text{sec}.$$

Within the error limits this value is in good agreement with the results obtained by Glasser *et al.*,[1] Blackie *et al.*,[2] and Shwe *et al.*[3]:

Authors	π^0 mean lifetime
Glasser *et al.*[1]	$(1.9 \pm 0.5) \times 10^{-16}$ sec
Blackie *et al.*[2]	$(3.3_{-1.4}^{+1.5}) \times 10^{-16}$ sec
Shwe *et al.*[3]	$(1.9_{-0.8}^{+1.3}) \times 10^{-16}$ sec

It should be pointed out, that the four experiments are different from each other either by the measuring procedures applied or by the π^0 production reactions. From the good agreement of the results, it may there-

fore be concluded that these experiments do not contain unrecognized systematic errors of a magnitude essentially greater than that of the random errors given by the error limits.

For the weighted mean $\bar{\tau}$ from all four experiments, including the present one, the following value is obtained:

$$\bar{\tau}=(2.1\pm0.4)\times10^{-16} \text{ sec.}$$

ACKNOWLEDGMENTS

The Bologna Emulsion Group contributed the scanning and the measurements of about half of the events used. We are indebted to the members of the group. Furthermore, we are grateful to Dr. I. Derado for useful discussions and to Dr. H. L. de Vries for teaching us the programming for the electronic computer G 3. We also want to thank Dr. N. Schmitz for his valuable help in carrying out the exposure of the two stacks. Our particular thanks are due to the scanners, especially to Mrs. H. Baumbach and Miss B. Frentzel-Beyme. We are thankful to Dr. K. Gottstein for his interest in this work. Last but not least, we want to express our indebtedness to Dr. E. Lofgren and to the Bevatron crew who made the exposure possible.