Superfluid Helium Critical Velocities in a Rotating Annulus*

PHILIP J. BENDT
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received March 12, 1962)

The critical velocity of rotation to bring superfluid helium at 1.3°K into motion was measured in an annulus 2.2 mm wide, 1.8 cm deep, and 8.83 cm mean diameter. Superfluid currents were detected after the annulus was stopped, and after the normal fluid had come to rest, with a modified form of "Rayleigh disk." Well-defined critical velocities were found, and these varied from 0.81 cm/sec (1.75 rpm of the annulus) to 0.55 cm/sec (1.19 rpm) depending on the past history of the liquid. The higher critical velocity applies to liquid in which a superfluid current has not been established previously. The "memory" of the liquid helium for previous superfluid currents lasts between 50 and 75 min. A discussion is given of the evidence that these currents constitute an irrotational velocity field ($v\propto 1/r$) in the annulus, which is free of vortex lines.

I. INTRODUCTION

When the superfluid component of liquid helium moves through a narrow channel faster than the "critical velocity," it loses its apparent property of completely frictionless flow. In explanation, Landau[1] suggested that excitations are created in the superfluid at the walls of the channel. Gorter and Mellink[2] proposed a mutual friction between the superfluid and normal-fluid components, which is proportional to $(v_s-v_n)^3$, and that this interaction is too small to be observed at low velocities. Other authors have suggested that some form of turbulence sets in, as in classical fluids at high Reynolds numbers, and the turbulence has been identified with tangled vortex lines.[3] Dash[4] summarized the measurements of critical velocities with the empirical formula $v_c=0.087(d\times\rho_s/\rho)^{-1/2}$, where v_c is in cm/sec and d is "a characteristic dimension of the liquid geometry" in cm. Atkins[5] summarizes the measurements in channels wider than 0.001 cm with the relation $v_c=0.01/d$, where d is the width of the channel in cm.

The published critical velocities in channels have not included measurements on liquid helium in a rotating container. Experiments on the hydrodynamics of rotating liquid helium have shown a number of strange phenomena,[6] which suggested this would be a useful field of investigation. In a related experiment, Hollis-Hallet[7] has made measurements of the apparent viscosity of liquid helium in his rotating cylinder viscometer. The liquid was contained in a 1 mm wide gap between a stationary inside cylinder of radius 2 cm, and a rotating outside cylinder. While no critical velocity was observed, Hollis-Hallet found that the effective viscosity at 1.3°K varied with the linear velocity of the outside cylinder from 17 μP at the lowest velocity (0.107 cm/sec) to 31 μP at 2.74 cm/sec. Reppy and Lane[8] in a rotating 2.5-cm diam closed cylinder (not an annulus) at 1.2°K found the critical velocity for bringing the superfluid into simulated solid body rotation to be between 0.08 and 0.14 cm/sec. However, if the helium in the bucket was at rest for 24 h previous to the experiment, this critical velocity was about 0.25 cm/sec. We observe a similar dependence on the previous history of the liquid helium.

In the present experiment, liquid helium at the start of each measurement was contained at rest in a narrow circular annulus. The experiment measured how fast the annulus had to be rotated to bring the superfluid into motion. This was determined by observing the presence of a current after the annulus was stopped, and after a delay sufficiently long for the normal fluid, due to its viscosity, to come to rest.

II. METHODS OF DETECTING SUPERFLUID CURRENTS

The problem of building a detector for superfluid currents which will work reliably in an open annulus 2.2 mm wide has not been solved in a satisfactory way. We desired a detector which would leave the annulus clear during rotation, and which could be lowered into the annulus after rotation was stopped. A rectangular "Rayleigh disk" 1.6 mm wide was tried. This was suspended on a torsion fiber and oriented at 45° with the direction of flow. Whenever this was lowered into the annulus, it was pulled by surface tension forces flat against one wall or the other, as soon as it touched the liquid. A Rayleigh disk will work, however, in a wider channel, where the liquid miniscus is flat in the center.

Several detectors depending on drag forces were also tried,[9] as well as a Venturi meter, but these proved not to be sufficiently sensitive to observe a current. Other methods were considered; charged ions apparently are scattered only by the excitations comprising the normal fluid, and particles of solid hydrogen floating on the sur-

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] L. Landau, J. Phys. (U.S.S.R.) **5**, 71 (1941); **6**, 91 (1947).
[2] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).
[3] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 114, 128 (1957); **A242**, 493 (1957); **A243**, 400 (1958).
[4] J. G. Dash, Phys. Rev. **94**, 825, 1091 (1954).
[5] K. R. Atkins, *Liquid Helium* (Cambridge University Press, New York, 1959), p. 200.
[6] J. R. Pellam, Phys. Rev. Letters **5**, 189 (1960); J. D. Reppy, D. Depatie, and C. T. Lane, *ibid.* **5**, 541 (1960); P. P. Craig, *ibid.* **7**, 331 (1961).
[7] A. C. Hollis-Hallet, Proc. Cambridge Phil. Soc. **49**, 717 (1953).
[8] J. D. Reppy and C. T. Lane, *Proceedings of the Seventh International Conference on Low-Temperature Physics* (University of Toronto Press, Toronto, 1960), Paper 19-4.
[9] P. J. Bendt, reference 8, Paper 23-17.

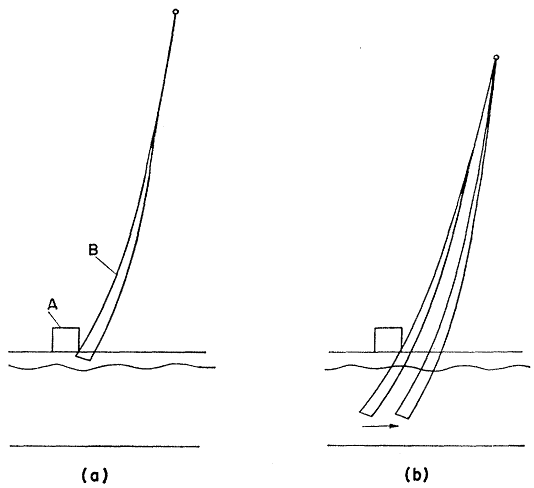

FIG. 1. Side view of detector and annulus. (a) The aluminum foil B in the raised position, touching the bar A, which is soldered across the top of the annulus. (b) When the foil is lowered, it breaks away from the bar and hangs free, as shown by arrow.

face apparently will move only with the normal fluid. Attenuation of second sound will show the presence of vortex lines,[10] but there may be none in the superfluid current.[11] The moment of inertia of the liquid was too small compared with that of the annulus to detect currents by methods which depend on measuring the angular momentum of the system.

The detector with which results were obtained is a variation of the rectangular "Rayleigh disk." It consisted of a strip of aluminum foil (Dural) 0.0002 in. thick, 1.6 mm wide, and 4.5 cm long, curved and twisted as shown by B in Fig. 1(a). To overcome the surface tension problem, it was mounted rigidly at the top, allowing no sideways motion. The support for the foil could be raised and lowered from outside the cryostat. Across the top of the annulus, above the liquid level, a small bar was soldered, shown by A in Fig. 1(a). In the raised position, the aluminum foil touched the bottom edge of this bar. As the support was lowered, the foil continued to adhere to the bar, sliding across the edge. It was apparently held by surface tension of liquid which reached the bar by film flow. When the support had been lowered 1.5 cm (foil immersed 1.3 cm), the foil would break loose from the bar to which it was held, as shown in Fig. 1(b). This was quite reproducible.

The lower end of the foil was twisted so that it entered the liquid at an azimuthal angle of 45° with a tangent to the annulus. A superfluid current in the annulus applied a torque to the foil, as it would to a Rayleigh disk. Twisting the lower end applied a bending moment to the curved portion of the foil. The result was that the foil would break away from the bar when the support was lowered about 0.5 cm (foil immersed 0.3 cm), clearly distinguishable from the height at breakaway in the absence of a current. The force required to detach the foil from the bar is estimated from the surface tension of liquid helium, and independently from the modulus of rigidity of aluminum, to be about 0.02 dyn. However, the detector was so critical in its adjustment, and so difficult to keep working correctly, that much less data was obtained from the experiment than was desired.

III. APPARATUS

The liquid helium was contained in a brass annulus machined to have smooth walls. The annulus was 2.2 mm wide, 18 mm deep, and 8.83 cm mean diam, and was filled to approximately 2 mm below the top edges. The support for the detector was guided by vertical posts mounted on top of the annulus, as shown in Fig. 2. The detector support was raised and lowered by a $\frac{1}{10}$-in.-diam thin-wall stainless steel tube, which slide inside a $\frac{1}{8}$-in.-diam thin-wall tube, and passed through a Wilson seal in the "rotating head" at the top of the Dewar.

The annulus and detector were contained in a closed cylindrical glass vessel 10 cm in diameter. This was sealed to Kovar and rigidly supported by thin-wall tubing 90 cm below the rotating head. The annulus was filled by condensing helium gas admitted to the glass vessel through a capillary. About $\frac{4}{5}$ of the free volume inside the glass vessel was taken up with an empty vacuum-tight brass cylinder, so the volume of liquid helium in the glass vessel and outside the annulus was small. The rotating glass vessel was submerged in a

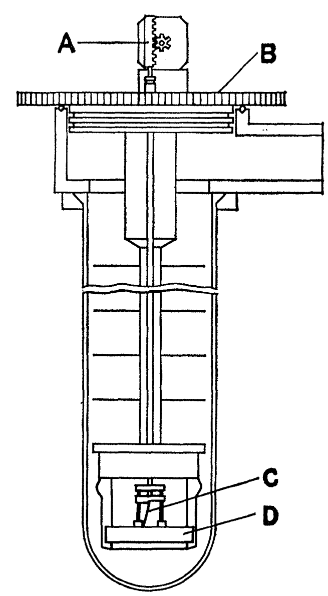

FIG. 2. Schematic diagram of the cryostat, not drawn to scale. A is the rack and pinion used to raise and lower the detector. B is the 9-in. diam gear which forms the perimeter of the rotating head. C is the aluminum foil mounted on the detector support assembly. D is the brass annulus inside the rotating glass vessel.

[10] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. **A238**, 204, 215 (1956).

[11] P. J. Bendt and T. A. Oliphant, Phys. Rev. Letters **6**, 213 (1961).

stationary helium bath which was pumped to about 1.2 mm of Hg pressure. The vapor pressure of the helium bath was measured with an oil manometer.

The rotating head at the top of the Dewar was machined of brass with double O-ring sliding seals. A small motor, gear box, and rack and pinion were mounted on top of the head and rotated with it. This was used to raise and lower the detector support. During the experiment, the position of the rack was measured to determine how far the detector was lowered into the annulus.

The rim of a 9-in.-diam steel gear was shrunk fit over the outside edge of the rotating head, and was driven by a spur gear mounted on the cryostat frame. The mechanical drive was mounted on a separate structure, which was supported on the floor of the room with vibration-damping mounts. A V belt was used to deliver torque to the spur gear. The mechanical drive consisted of an electric motor, speed reducing gear box, and Vickers hydraulic variable-speed drive. The speed of the Vickers drive could be continuously varied from maximum speed in one direction of rotation (about 4 rpm of the annulus) through zero to maximum speed in the opposite direction. The speed control on the Vickers drive was operated by a worm gear and worm turned at 1 rpm by another electric motor. This arrangement provided smooth start-up, and slowing down, of the annulus.

The mechanical drive was started and stopped manually, while the power for the motor which changed the speed of the Vickers drive was handled by an industrial electric timer. Setting the timer for 3 min brought the annulus from rest to a speed of 1.2 rpm. The annulus was allowed to rotate at constant speed for 3 to 10 min. The direction of the speed-control motor was reversed, and starting the timer again would bring the annulus to a very low speed in approximately 3 min. The annulus was stopped manually, while rotating very slowly, in order to position the annulus with the detector facing a slit in the Dewar silvering. The detector was illuminated through heat-absorbing glass by a small lamp, and was observed with a 15-power cathetometer while it was lowered slowly into the annulus.

The similarity of the aluminum foil detector to a gold-leaf electroscope raises the question of whether it was responding to electrostatic charges generated by moving liquid helium, rather than to flow of the liquid past the foil. Electrostatic effects were observed with a nonconducting Mylar foil instead of the aluminum foil, and also when a charge was deposited on the outside of the rotating glass vessel, by rubbing it with a cloth. To minimize electrostatic effects, the aluminum foil was mounted in electrical contact with the all-metal detector support assembly, which was soldered to the top of the brass annulus. In addition, the detector support was grounded at the top of the cryostat by the thin-wall stainless steel tube used to raise and lower the foil.

As will be discussed below, on nearly all runs the foil was raised immediately after breakaway. It was then lowered into the liquid a second time, and on every occasion except one, it did not break away on the second immersion until reaching 1.3 cm. One situation whereby electrostatic repulsion could cause early breakaway would be to accumulate charge on the annulus, detector support, and foil, during rotation. It is not understood how this charge could fail to leak to ground, or how the metal parts could be discharged during the 10- to 20-sec immersion of the foil in the liquid helium. Likewise, the possibility of charges being induced on the foil at the time it is immersed are ruled out by the fact that early breakaway was not repeated when the foil was lowered a second time. However, the possibility of subtle and unsuspected electrostatic effects cannot be entirely ruled out.

IV. RESULTS

All measurements were made at 1.3°K, at which temperature ρ_s/ρ is about 0.95. The velocity relaxation time for the normal fluid, due to its viscosity, is short because of the low density of normal fluid, and because or the large ratio of wall area to volume in the narrow annulus. The relaxation time for laminar flow of normal fluid in the annulus can be calculated by classical hydrodynamics, and is 5 to 10 sec. The motion of the normal fluid is believed to follow the annulus with a time delay of the order of two or three relaxation times. We searched for currents after the annulus was stopped, following a waiting period of not less than 1 min, during which time the normal fluid should effectively come to rest. The "currents" we detected after 1 min were believed to be superfluid currents. The detector responded in the same manner to both directions of rotation. This was expected, since the torque on a Rayleigh disk is proportional to v^2.

The measurements show a well-defined "critical velocity" of rotation of the annulus, above which the superfluid is brought into motion. When the annulus was rotated at constant speed below the critical velocity, for 3 to 10 min, no superfluid currents were detected. The experiment also showed that the critical velocity depends on the previous history of the liquid helium. The measured critical velocities were the same for both directions of rotation.

The experimental results are shown in Fig. 3. The ordinate is the vertical length of detector which was immersed in the liquid at breakaway. A current was presumed to be present when breakaway occurred at about 0.3 cm. The position of the detector when breakaway occured at about 1.3 cm (no current detected) was not measured accurately after the first few times, and probably varied more than shown in Fig. 3.

When liquid helium had been "at rest", the critical velocity was measured to be 0.81 ± 0.03 cm/sec (mean linear velocity of the annulus, equivalent to 1.75 rpm). Here, "at rest" means that the superfluid had not been brought into motion since helium gas was condensed to fill the annulus with liquid, or that a time interval

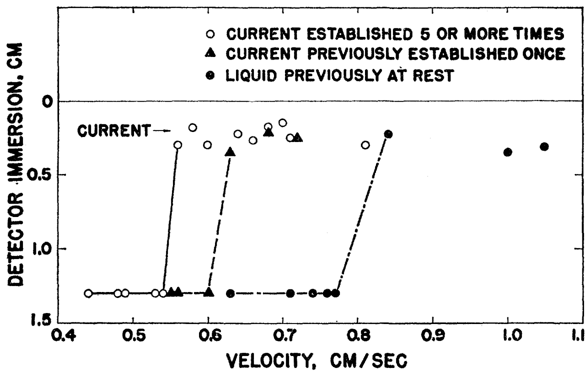

FIG. 3. Critical linear velocities of the annulus for detecting a superfluid current, showing the dependence on the past history of the liquid helium. Ordinate is the vertical length of detector which was immersed in the liquid at break away.

of at least 75 min had lapsed since a current had been detected and destroyed by the detecting foil. If the past history included just one previously established current (detected and destroyed 10 to 50 min previous), the critical velocity was reduced to 0.62±0.02 cm/sec (1.34 rpm). The critical velocity continued to get smaller as more superfluid currents were established and destroyed, until after five currents it was 0.55±0.01 cm/sec (1.19 rpm). This appeared to be the smallest critical velocity which could be obtained by repeatedly generating and destroying superfluid currents.

Referring to the equations in Sec. I summarizing critical velocities in nonrotating channels, Dash's equation predicts, for our channel width and temperature, 0.19 cm/sec, and Atkins' equation predicts 0.05 cm/sec. We have no explanation of the larger values observed in this experiment, except to note (a) they are comparable to critical velocities in narrower channels, and (b) they imply that it is difficult to bring superfluid into rotation. Also, the phenomena to which the term "critical velocity" is applied were different in the experiments which the above equations summarize. For example, the wide channel data included by Atkins were channels 2.4 and 4.0 mm wide used by Vinen[3] to measure the onset of attenuation of second sound in the presence of a heat current.

A further result is concerned with the destruction of the superfluid currents in the annulus. On most of the runs where breakaway occurred at about 0.3-cm immersion, the aluminum foil was withdrawn out of the liquid immediately after breakaway, and then lowered into the liquid a second time. The foil was first immersed 10 to 20 sec, counting the time to lower and raise the detector. The maximum immersion was about 1/5 the liquid depth of 1.6 cm. In every measurement except one, the breakaway on the second immersion occurred at 1.3 cm, indicating the current had been destroyed by the first immersion. Regarding the one exception, the annulus had been rotated at 1.05 cm/sec, and the vane was immersed about 10 sec on each of the first two immersions; on the third immersion breakaway occurred at 1.3 cm.

There are two "lifetimes" of interest in this experiment. The first is the lifetime of the superfluid currents after they are created by rotating the annulus faster than the critical velocity. Unfortunately, this is not known, but on two separate runs currents were detected 10 min after rotation of the annulus had stopped. This is of interest, inasmuch as experiments by Hall and Vinen[10] in a wider annular geometry rotated about ten times as fast indicated that superfluid *turbulence*, as observed by second sound attenuation, died out in 3 min. The second lifetime of interest is the memory in the liquid of past superfluid currents, which causes the critical velocity to decrease from 0.81 to 0.55 cm/sec. This time is of the order of an hour, being greater than 50 and less than 75 min for measurements made in our annulus. This memory may be stored in the liquid helium by a few persistent random lengths of vortex line. These would not constitute a macroscopic current, but when the annulus and normal fluid are rotated again, they might nucleate the process which brings the superfluid component into rotation.

V. EVIDENCE FOR AN IRROTATIONAL VELOCITY FIELD

Feynman[12] predicted that liquid helium in a rotating cylinder would simulate solid body rotation, and the circulation $\oint \mathbf{v}\cdot d\mathbf{l}$ would be quantized, equal to $n(h/m)$, where h is Planck's constant, m is the mass of a He^4 atom, and n is a positive integer including zero. This condition would be satisfied for all paths in the liquid by an array of vortex lines parallel to the axis of rotation. Bendt and Oliphant[11] suggested that a superfluid velocity field with $v\propto 1/r$ (r is the distance from the axis of rotation), called an irrotational velocity field,

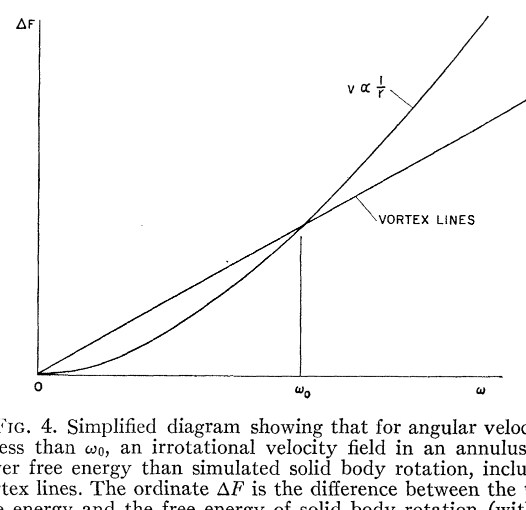

FIG. 4. Simplified diagram showing that for angular velocities ω less than ω_0, an irrotational velocity field in an annulus has lower free energy than simulated solid body rotation, including vortex lines. The ordinate ΔF is the difference between the total free energy and the free energy of solid body rotation (without vortex lines).

[12] R. P. Feynman, *Progress in Low-Temperature Physics* (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, p. 17.

without any vortex lines, would be established at small angular velocities in a rotating annulus. This is because it would have lower free energy than the velocity field described by Feymann. This is shown by the simplified free-energy diagram Fig. 4, where the ordinate ΔF is the excess energy added to the energy of solid body rotation by the above requirement on the circulation. The straight line in Fig. 4 shows that the ΔF added by vortex lines to solid body rotation is proportional to the angular velocity ω. However, for an irrotational velocity field, ΔF is quadratic in ω, and for an annulus there is always some value of ω, designated ω_0, below which the free energy is lower for an irrotational velocity field than for vortex lines. As the ratio of the inside radius to the outside radius of the annulus becomes smaller, ω_0 becomes smaller; ω_0 is identically zero for a simple cylinder.

Bendt and Oliphant[11] offer the hypothesis that in the absence of vortex lines, the irrotational superfluid current may persist without dissipation after the annulus is brought to rest. If the velocity of rotation of the annulus exceeds ω_0, then the persistent current after the annulus has stopped would presumably correspond to ω_0. On the other hand, according to the accepted concept of vortex lines,[13] the current should not persist if there are vortex lines in the rotating helium, because these will attach themselves to the bottom and sides of the annulus, and will generate turbulence when the annulus is stopped. The ratio of inside radius to outside radius of the annulus used in the experiment was 0.953, yielding a predicted value for ω_0 of 0.23 or 0.82 rpm, depending on the boundary conditions which are assumed.[11] This is not a prediction of the critical velocity measured in the experiment, but rather a prediction of the velocity which was seen by the detector after the annulus was stopped.

The evidence that an irrotational velocity field existed in the annulus is as follows:

(1) The response of the detector was the same, regardless of how fast the annulus had been rotated (as long as it was faster than the critical velocity), and independently of how long we waited after the annulus was stopped (within the experimental range of 1 to 10 min). We believe we observed a *reproducible* velocity, as is predicted for an irrotational velocity field.

(2) The ease with which the currents were destroyed by immersing the foil 1/5 of the channel depth at one location for less than 20 sec. This result points to a *coherent* flow pattern around the annulus, which is characteristic of an irrotational velocity field, but not of vortex lines.

(3) The lifetime of the currents, known to be at least 10 min, is long compared with the observed lifetime of detectable superfluid turbulence generated by stopping a wider annulus from high velocities of rotation.[10]

ACKNOWLEDGMENTS

We wish to express our appreciation to Dr. T. A. Oliphant and Dr. E. F. Hammel for their continued interest and support of this experiment.

[13] H. E. Hall, *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1960), Vol. 9, p. 89.