PHYSICAL REVIEW VOLUME 127, NUMBER 5 SEPTEMBER 1, 1962

Effect of Hole-Hole Scattering on the Mobility of p-Type Germanium

JOACHIM APPEL AND RALPH BRAY*
John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California

(Received April 20, 1962)

A quantitative comparison between theory and experiment is presented for the effect of hole-hole scattering on the mobility in both uncompensated and heavily compensated p-type germanium. In the former case, excellent agreement is obtained for impurity concentrations $N_I \leq 10^{16}$ cm^{-3}. An appreciable discrepancy is noted for compensated samples which is attributed to the use of the Born-approximation scattering cross section which gives equal weight to scattering of holes by ionized donors and acceptors.

FOR p-type germanium, the pertinent parameters for the lattice scattering mobility of light and heavy holes have been determined, and it has been found that the appropriate combination of lattice and impurity scattering gave quantitative agreement with the experimental mobility data for impurity concentrations below about 10^{15} cm^{-3}; however, for higher concentrations, the experimental data fell below the theoretical curve.[1] The main purpose of this paper is to discuss the influence of hole-hole scattering on the mobility and to show that to a large extent the above stated discrepancy can be explained by hole-hole scattering. The latter has been incorporated in a theory of electronic transport phenomena which is based on a mathematical procedure described elsewhere.[2] This procedure is obtained by a systematic generalization of Kohler's variation principle to a multiband conductor. For p-type germanium the band structure is taken to be parabolic, with spherical energy surfaces, and twofold degeneracy at the center of the Brillouin zone. The dynamical interaction between charge carriers is described by a shielded Coulomb potential. The hole mobility is computed with acoustical and optical phonon scattering, ionized impurity scattering, and interband and intraband hole-hole scattering. The parameters for the lattice scattering mobility are those obtained by Brown and Bray.[1] Intraband scattering induced by acoustical and optical phonons and by ionized impurities is taken into account via appropriate scattering operators.[2] Interband scattering due to phonens is not explicitly incorporated in the theoretical consideration; however, it is taken into account approximately by assuming the lattice scattering mobility of light holes to be larger than the corresponding mobility of heavy holes by a mass ratio $m_h/m_l = 8.15$. Interband impurity scattering is neglected.

In Fig. 1, experimental and theoretical results are presented. The dashed curve is calculated without considering hole-hole scattering. It can be seen that this curve fits the experimental data well up to impurity concentrations of about 10^{15} cm^{-3}.[3] The computation of the full curve includes hole-hole scattering and obviously this curve extends the good agreement up to impurity concentrations of about 10^{16} cm^{-3}. The inclusion of hole-hole scattering reduces the mobility not only because the drift momentum is randomized over the different energy groups when carriers of the same mass or of different masses are scattered by each other,[4] but also because in scattering events involving two carriers of different masses (i.e., interband hole-hole scattering), the requirement of momentum conservation need not imply velocity or current conservation.

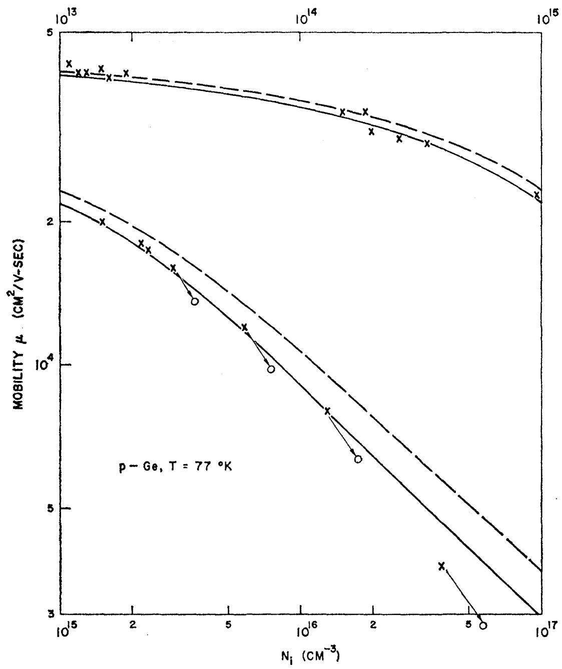

FIG. 1. The dashed curve represents the calculated mobility without hole-hole scattering; the solid curve includes hole-hole scattering. The experimental data represented by (×,○) were measured by Brown and Bray (reference 1). The crosses represent Hall mobilities at $H = 3550$ G. The arrows, connecting some of the crosses to circles, represent approximate corrections for the transformation from Hall to conductivity mobility, as described in reference 1.

* Permanent address: Department of Physics, Purdue University, West Lafayette, Indiana.

[1] D. M. Brown and R. Bray, preceding paper [Phys. Rev. 127, 1593 (1962)].

[2] J. Appel, Phys. Rev. 122, 1760 (1961); 125, 1815 (1962).

[3] It should be noted that the calculation based on the variational principle is in very good agreement with the simpler one based on combining lattice and impurity scattering relaxation times, as described by Brown and Bray, (reference 1).

[4] P. P. Debye and E. Conwell, Phys. Rev. 93, 697 (1954). See reference 2 for further discussion and other references.

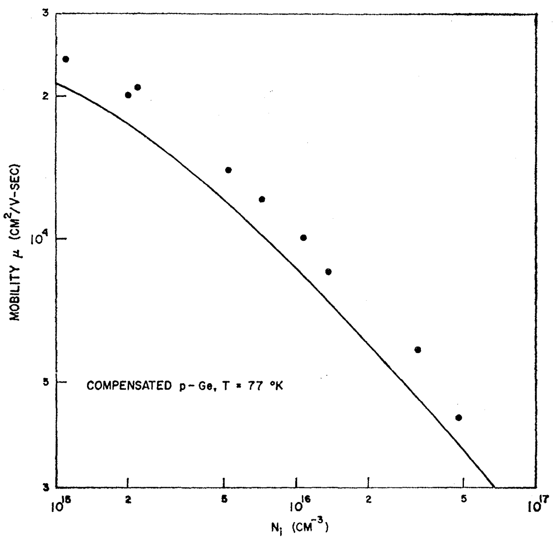

FIG. 2. The experimental data from Fritzsche and Cuevas,[5] represent Hall mobility measurements at $H=7000$ G. Conversion to conductivity mobility may bring the data somewhat closer to the theoretical curve, but the main effect is simply to translate the data along the existing curve, as may be seen from Fig. 1.

For small impurity concentrations ($\lesssim 10^{15}$ cm^{-3}), interband hole-hole scattering is relatively more important; however, intraband scattering accounts for more than half of the mobility reduction at larger concentrations ($\sim 10^{16}$ cm^{-3}), when impurity scattering predominates.

For larger impurity concentrations ($N_I > 10^{16}$ cm^{-3}) the reduction of the mobility due to intercarrier scattering accounts for only part of the difference between the experimental values and the dashed curve. A small part of the residual difference is due to the neglect of freeze out in the theoretical calculations. A larger part is believed to be due to the neglect of interband impurity scattering (and the eventual inadequacy of the use of the Born approximation) which is expected to become important at high impurity concentration.[1]

The theory including hole-hole scattering has also been applied to the mobility at 77°K in highly compensated p-type germanium, for which experimental results have been presented by Fritzsche and Cuevas.[5] Comparison of the theory with the experimental data is shown in Fig. 2. The compensation, produced by transmutation by slow neutrons, reduced the ratio of free carriers to total ionized impurity centers p/N_I to a value of 0.4 in the absence of freeze-out. This reduction in the ratio p/N_I from that in uncompensated samples should have two opposing influences on the mobility at a given value of N_I: (1) The relative influence of carrier-carrier scattering should diminish because $p < N_I$, thereby raising the mobility. (2) The screening of the Coulomb potential for impurity scattering should be reduced, thereby lowering the mobility. The latter effect is dominant and the full curve in Fig. 2, calculated with both these factors included, is lower than the corresponding curve in Fig. 1 for the uncompensated material. It can be seen that, in contrast with the situation in uncompensated material, there is an appreciable discrepancy here between the theory[6] and the experimental data. In the latter case, the theory apparently *overestimates* the impurity scattering, giving mobility values that are *too low* at all values of $N_I = N_A + N_D$. This discrepancy may be due to the fact that attractive acceptors and repulsive donors are not distinguished with the Born approximation scattering cross section on which the present calculation was based. Blatt[7] has shown with the partial wave method, that for an attractive potential, the correct total scattering cross section of charge carriers is in fact larger—and nearer to the Born-approximation scattering cross section—than for a repulsive potential. Hence, a smaller than equal weight should have been given to the ionized donor scattering in p-type germanium.

[5] H. Fritzsche and M. Cuevas, Phys. Rev. **119**, 1238 (1960).

[6] In the theoretical calculations, the value $p/N_I = 0.4$ was assumed to apply at all impurity concentrations. This will obviously be wrong for the more impure samples, where some freeze out must occur at 77°K, in which case $p/N_I < 0.4$. However, this should not affect the main conclusions.

[7] F. J. Blatt, J. Phys. Chem. Solids **1**, 262 (1957).