Pairing Energies of Heavy Nuclei

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An investigation is made into the small-scale fitting of the nuclear mass surface by means of a semi-empirical mass formula. This formula is then used in the determination of pairing energies.

In a previously published paper,[1] Thieberger and de-Shalit derived the following binding energy formula:

$$E=nA+\tfrac{1}{2}n(n-1)a+y(n)D,$$

which they showed could be very accurately fitted to parts of the nuclear mass surface. In this formula E is the relative binding energy of any nucleus in a set of isotopes (or isotones) measured with respect to the binding energy of a given member of the set; n is the number of neutrons (protons) which must be added to the reference nucleus to form the nucleus under consideration; and $y(n)$ is the function: $\tfrac{1}{2}[1-(-1)^n]$. (In the following, whatever is said about sets of isotopes is true also for sets of isotones if the words "protons" and "neutrons," and also the letters "N" and "Z", are interchanged.) A, a, and D are parameters which are obtained by fitting the formula to experimental data. According to certain definitions[2,3] the parameter D represents the negative of the pairing energy.

In the present work similar calculations were

TABLE I. Neutron pairing energies (in MeV).

	Z	N	$-D$		Z	N	$-D$		Z	N	$-D$		Z	N	$-D$
Ni	28	31	1.51	Nb	41	51	0.22	Po	84	127	0.77	U	92	137	0.77
		33	1.17			53	0.63			129	0.83			139	0.82
		35	1.75			55	0.79			131	0.91			141	0.51
		37	1.45	Mo	42	51	0.84			133	0.68			143	0.64
Cu	29	33	1.17			53	0.87	At	85	129	0.45			145	0.52
		35	1.15			55	0.81			131	0.72			147	0.65
		37	1.13	Sn	50	65	0.80			133	0.49	Np	93	137	0.33
Zn	30	33	1.72			67	1.16	Em	86	129	0.91			139	0.51
		35	1.81			69	1.32			131	1.02			141	0.40
		37	1.97			71	1.36			133	1.02			143	0.56
		39	1.76			73	1.38			135	0.80			145	0.48
Ga	31	33	1.33			75	1.30	Fr	87	131	0.48			147	0.66
		35	1.24	Te	52	71	1.02			133	0.56	Pu	94	139	0.64
		37	1.20			73	0.98			135	0.60			141	0.71
		39	0.82			75	1.13			137	0.76			143	0.61
As	33	41	1.28			77	1.04	Ra	88	131	0.90			145	0.59
		43	1.33			79	0.89			133	0.69	Am	95	141	0.47
		45	0.99	I	53	69	1.25			135	0.64			143	0.34
Se	34	41	1.70			71	1.43			137	0.82			147	0.53
		43	1.60			73	1.35			139	0.97			149	0.78
		45	1.61			75	1.30			141	0.68			151	0.62
		47	1.35			77	1.08	Ac	89	135	0.53	Cm	96	141	0.68
Br	35	41	0.80			79	1.24			137	0.62			143	0.68
		43	1.32			81	0.33			139	0.76			145	0.52
		45	1.38	Xe	54	73	1.41	Th	90	133	0.90			147	0.62
		47	0.92			75	0.99			135	0.64			149	0.46
		49	1.06			77	1.26			137	1.00			153	0.46
Kr	36	41	1.91			79	0.89			139	0.78	Bk	97	149	0.39
		43	1.99	Tl	81	123	0.71			141	0.70			151	0.43
		45	1.88			125	0.22			143	0.62			153	0.62
		47	1.69	Pb	82	127	0.69			145	0.65	Cf	98	147	0.66
		49	1.60			129	0.72	Pa	91	135	0.27			149	0.62
Zr	40	51	0.84			131	0.50			137	0.60			151	0.53
		53	0.58			133	1.13			139	0.51	E	99	147	0.39
		55	0.80	Bi	83	127	0.24			141	0.47			151	0.37
		57	1.08			129	0.44			143	0.52			153	0.51
						131	0.50			145	0.57			155	0.57
										147	0.66				
										149	0.62				

* This work was carried out at the University of Cape Town in 1959.
[1] R. Thieberger and A. de-Shalit, Phys. Rev. **108**, 378 (1957).
[2] E.g., R. D. Evans, *The Atomic Nucleus* (McGraw-Hill Book Company, New York, 1955), p. 304.
[3] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939). Note, however, that these definitions differ from that in M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955), p. 24.

performed (on a desk calculator) with the difference that the first two parameters only were fitted (by considering only even-N isotopes), and the formula was then used to calculate the pairing energy $-D$ of the remaining odd isotopes of the set.

The binding energy data used for the calculations were those published by Wapstra[4] (for nuclei with $Z<80$) and by Forman and Seaborg[5] (for nuclei with $Z>80$). The latter contained many interpolated binding energies and the pairing energies corresponding to these interpolated values are therefore omitted. Tables I and II show the results obtained for neutron and proton pairing energies, respectively. The rms deviations σ between the calculated and experimental binding energies were calculated using the formula:

$$\sigma=\left[\sum_{i=1}^{L}\Delta_i^2/(L-k)\right]^{1/2},$$

where L is the number of data and k is the number of parameters. ($k=3$ in this case, including the binding energy of the reference nucleus.) A summary of the values of σ obtained is given in Table III. The accuracy, in general, increased with mass number, although this was partly due to the fact that the number of data also increased with mass number.

The results for pairing energies show an average trend in keeping with several empirical formulas (e.g., $H=12A^{-1/2}$, where H is the pairing energy in milli-mass-units[6]), but there are large local variations. In particular, the pairing energies seem to decrease markedly near magic numbers.

TABLE II. Proton pairing energies (in MeV).

N	Z	$-D$	N	Z	$-D$	N	Z	$-D$	N	Z	$-D$
35	29	0.73	58	45	1.45	83	57	1.32	138	89	0.83
	31	0.86		47	2.42		59	1.20		91	0.85
37	29	0.60		49	2.16	84	57	0.72		93	0.94
	31	0.80	59	45	1.47		59	1.15	139	89	0.82
39	31	0.42		47	0.65	86	61	0.71		91	0.60
	33	0.87	60	45	1.86	88	61	1.08		93	0.70
40	31	1.74		47	1.87	116	77	2.03	140	89	0.94
	33	1.97		49	1.03	120	79	0.74		91	0.96
	35	1.11	61	45	0.62	126	83	0.85		93	0.63
41	31	1.03		47	1.11		85	0.82	141	91	0.63
	33	1.23		49	1.42	127	83	0.35		93	0.47
	35	0.98	62	45	1.33		85	0.68		95	0.63
42	33	1.67		47	1.46	128	83	0.97	142	91	0.86
	35	1.91		49	1.37		85	1.13		93	0.84
43	33	1.33	64	47	1.85	129	83	0.60		95	0.53
	35	1.38		49	1.48		85	0.69	143	91	0.81
44	33	1.65	65	47	1.07	130	83	1.03		93	0.70
	35	1.91		49	1.64		85	1.11		95	0.49
	37	1.62	66	47	1.85		87	1.26	144	91	1.03
45	33	1.01		49	1.48	131	83	0.86		93	0.77
	35	1.38	70	51	1.38		85	0.80		95	0.76
	37	0.73	72	51	1.21		87	0.67	145	91	0.96
46	35	1.65		53	1.15	132	83	1.18		93	0.73
47	35	0.97		55	1.46		85	1.06	146	93	0.92
	37	1.00	73	51	1.02		87	1.11		95	0.74
	39	0.35		53	1.03	133	85	1.12		97	0.90
48	35	1.35		55	1.36		87	1.02	147	93	0.75
	37	1.80	74	51	1.01		89	0.54		95	0.55
	39	1.24		53	0.93	134	85	1.13		99	0.35
49	37	0.82		55	0.98		87	1.04	148	93	0.73
	39	1.27	75	53	1.19		89	0.93		95	0.81
50	37	1.23		55	1.60		91	1.14		97	0.65
	39	1.15	78	53	1.01	135	87	0.87		99	0.60
51	37	1.24		55	1.07		89	0.72	149	95	0.94
	39	0.75	80	53	0.85		91	0.80		97	0.62
54	41	1.62	81	57	0.64	136	87	1.01	150	95	0.90
55	41	1.36		59	1.13		89	0.76		97	0.76
56	41	1.12	82	55	1.73		91	0.88		99	0.65
	43	2.14		57	0.92	137	87	1.03	152	97	0.83
				59	1.04		89	0.81		99	0.70
							91	0.83			
							93	0.52			

TABLE III. Rms deviations, σ, obtained during fits on isotopes and isotones.

Values of σ (MeV)	Number of times σ occurred within given range: For isotopes	For isotones
$\sigma\leqslant 0.03$	15	21
$0.03<\sigma\leqslant 0.10$	13	7
$0.10<\sigma\leqslant 0.30$	10	5
$\sigma>0.30$	2	3

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[4] A. H. Wapstra, Physica **21**, 385 (1955).
[5] B. M. Forman, Jr., and G. T. Seaborg, J. Inorg. Nuclear Chem. **7**, 305 (1958).
[6] A. E. S. Green and D. F. Edwards, Phys. Rev. **91**, 46 (1953).