Angular Distribution of γ Radiation Following Direct Nuclear Reaction*

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The γ-ray angular distributions following the direct reaction $C^{12}(p,p'\gamma_{4.43\text{ MeV}})C^{12}$ have been calculated for the cases of single-particle excitation and collective excitation using Glendenning's optical potential and the surface interaction model. In the case of particle excitation, the effective two-body potential has been assumed to have the form $V_d(|\mathbf{r}_p-\mathbf{r}_n|)[a_0+a_1\boldsymbol{\sigma}(p)\cdot\boldsymbol{\sigma}(n)]$. Comparison with experiments at this laboratory for incident proton energies $E_p=6.69$, 8.46, 8.71, and 9.72 MeV shows remarkably good fits for $a_1/a_0=\pm0.67$. For the case of collective excitation, the agreement has been found to be poor. The plane-wave Born approximation for the incoming and outgoing waves does not yield satisfactory results for either mechanism.

I. INTRODUCTION

RECENTLY, some γ-angular distributions following the reaction $C^{12}(p,p'\gamma_{4.43\text{ MeV}})C^{12}$ have been measured at our laboratory[1] for incident proton energies between 5 and 12 MeV. These measurements were carried out at incident proton energies where the excitation curve shows no resonant structure, as well as at energies in the vicinity of resonance peaks. At the former energies, it is expected that the reaction proceeds by a direct process.

For the angular correlation between the γ ray and the inelastically scattered proton from the 4.43-MeV level in C^{12}, Levinson and Banerjee[2] have obtained good theoretical results using the approach of particle direct interaction.

On the other hand, from an analysis of the inelastic electron-scattering cross section for C^{12}, Ferrell and Visscher[3] have reached the conclusion that some collective behavior exists in the 4.43-MeV level in C^{12}. From the strength of the $E2$ transition from this level to the ground state in C^{12}, Devons *et al.*[4] and Kurath[5] also have suggested the same fact.

For the angular distributions of protons and α particles inelastically scattered from the 4.43-MeV state in C^{12}, the analysis has been performed by many authors.[2,6,7] They have shown that the shape of such angular distributions of protons and α particles are insensitive to the type of interaction (particle interaction and collective interaction). However, the shape of the angular distribution of γ rays following the direct reaction, in general, depends on the interaction type. Thus, it is of interest to analyze the angular distributions of 4.43-MeV γ rays following the direct reaction of protons with C^{12} from the point of view of both the particle and collective interactions, since this may give some information on the properties of the 4.43-MeV state of C^{12} and on the interacting two-body force.

Glendenning[6] has shown that if suitable depths of the real and imaginary parts are chosen, the rectangular-well optical potential yields good results for the angular distributions of particles (protons, neutrons, and α particles) inelastically scattered from several levels in a number of nuclei by the direct reaction process. Furthermore, he has shown that the surface interaction model is appropriate for such a direct reaction.

In our calculation, the distorted-wave Born approximation is used for the incoming and outgoing proton waves. For the reason mentioned above and for ease of calculation, the distorted potential is taken to be Glendenning's optical potential in addition to the Coulomb potential, and the surface interaction model is introduced.

II. ANGULAR DISTRIBUTION

If the scattered proton is not observed, the angular distribution function for the γ ray following the direct reaction is given by

$$W(\theta)=\frac{1}{2(2J_i+1)^{\frac{1}{2}}}\sum_{M_iM_f\sigma_1\sigma_2}\int d\Omega\left|\sum_M\langle\phi_f(J_fM_f)|H_\gamma|\phi(JM)\rangle\langle\phi(JM)\psi_f(\Omega,\mathbf{r}_p,\sigma_2)|V|\phi_i(J_iM_i)\psi_i(\mathbf{r}_p,\sigma_1)\rangle\right|^2, \quad (1)$$

where H_γ and V represent the interaction Hamiltonian for the γ-ray emission and inelastic scattering, respectively; $\int d\Omega$ represents the integral over direction Ω of the scattered proton; $\phi(JM)$ is the nuclear wavefunction for the state with spin J and z-component M, the subscripts i, f distinguishing between the initial and final states, while the intermediate (excited) state has no subscript; ψ_i and ψ_f are the wavefunctions of the incident proton and the scattered proton, respectively; σ is the spin coordinate of the particle.

We assume here a pure multipole for the γ radiation. This assumption is correct for our analysis, since the γ radiation from the 4.43-MeV state (2^+) to the ground state (0^+) in C^{12} is known to be $E2$. For unpolarized

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[1] H. S Adams, J. D. Fox, N. P. Heydenburg, and G. M. Temmer, Phys. Rev. **124**, 1899 (1961).
[2] C. A. Levinson and M. K. Banerjee, Ann. Phys. (New York) 2, 471, 499 (1957); **3**, 67 1958).
[3] R. A. Ferrell and W. M. Visscher, Phys. Rev. **104**, 475 (1956).
[4] S. Devons, G. Manning, and J. H. Towle, Proc. Phys. Soc. (London) **A69**, 173 (1956).
[5] D. Kurath, Phys. Rev. **106**, 975 (1957).
[6] N. K. Glendenning, Phys. Rev. **114**, 1297 (1959).
[7] J. S. Blair, Phys. Rev. **115**, 928 (1959); E. Rost and N. Austern, *ibid.* **120**, 1375 (1960) and references given there.

γ radiation, Eq. (1) becomes[8]

$$W(\theta)=\sum_k A_k(J_fJ)B_k(JJ_i;\psi_f\psi_i)P_k(\cos\theta), \tag{2}$$

$$A_k=\sum_L \frac{1}{4\pi}(2J+1)(2L+1)(-1)^{2J+J_f-k-1}(L1L-1|k0)W(JJLL;kJ_f)|\langle J_f\|L\|J\rangle|^2, \tag{2a}$$

$$B_k=\frac{1}{2(2J_i+1)^{\frac{1}{2}}}\sum_{M_iMM'\sigma_1\sigma_2}(-1)^M(JMJ-M'|kM-M')$$
$$\times\int d\Omega\langle\phi(JM')\psi_f|V|\phi_i(J_iM_i)\psi_i\rangle^*\langle\phi(JM)\psi_f|V|\phi_i(J_iM_i)\psi_i\rangle, \tag{2b}$$

where L is the multipole order of the γ radiation; $(a\alpha b\beta|c\gamma)$ the Clebsch-Gordan coefficient; $W(JJLL;kJ_f)$ the Racah coefficient; $\langle J_f\|L\|J\rangle$ the reduced matrix element for the γ emission, as defined by Devons and Goldfarb[8]; k is an even integer.

ψ_i and ψ_f are written in partial wave expansion as follows:

$$\psi_i=(4\pi)^{\frac{1}{2}}\sum_{L_1} i^{L_1}\exp(i\delta_{L_1})(2L_1+1)^{\frac{1}{2}} \times f_{L_1}(k_1,r_p)Y_{L_1}{}^0\chi_s{}^{\sigma_1}, \tag{3a}$$

$$\psi_f=4\pi\sum_{L_2M_2} i^{L_2}\exp(-i\delta_{L_2})Y_{L_2}{}^{M_2*}(\Omega) \times f_{L_2}(k_2,r_p)Y_{L_2}{}^{M_2}\chi_s{}^{\sigma_2}, \tag{3b}$$

where the $Y_L{}^M$ are spherical harmonics, and the direction of the incident beam is taken as the polar axis; $\chi_s{}^\sigma$ is the spin wavefunction; δ_L is the phase shift for the Coulomb potential. The radial wavefunction $f_L(k,r)$ satisfies the equation

$$\left[\frac{1}{r}\frac{d^2}{dr^2}r-\frac{L(L+1)}{r^2}-\frac{2M}{\hbar^2}U-\frac{2kn}{r}+k^2\right]f_L(k,r)=0, \tag{4}$$

where U is the distorting optical potential assumed to have a rectangular well shape, following Glendenning; M is the reduced mass of the target nucleus and the projectile. The Coulomb parameter is

$$n=ZZ'Me^2/\hbar^2k,$$

where Z and Z' are the atomic numbers of the target nucleus and the projectile, respectively.

A. Particle Excitation

We assume that one of the particles bound in the last shell of the target nucleus jumps to an upper level (with possibility of an exchange process) by the direct interaction with the incoming particle. The interaction Hamiltonian is given by

$$V=\sum_{n=1}^{q} V_{pn}, \tag{5}$$

where V_{pn} is the two-body potential between the incident particle (denoted by p) and the nth bound particle. The summation is taken over all the bound particles (q in number) in the last shell.

For V_{pn} we assume a surface interaction form for the reason mentioned in Sec. I. Since Glendenning's argument[6] for the exchange contribution, advanced for the angular distribution of the inelastically scattered particle, is applicable in our case, the space exchange character is excluded from the two-body potential. In addition, the wavefunction describing the system consisting of the projectile and the bound particles is not antisymmetrized with respect to interchange of the projectile with one of the bound particles. The tensor force is neglected. Thus, the two-body potential is assumed to be composed of a spin-flip part plus a non-spin-flip part,

$$V_{pn}=\delta(r_p-R_I)V_d(|\mathbf{r}_p-\mathbf{r}_n|)[a_0+a_1\boldsymbol{\sigma}(p)\cdot\boldsymbol{\sigma}(n)], \tag{6}$$

where R_I is the interaction radius.

Introducing the notation

$$\begin{aligned}\sigma_0{}^0&=1, &\sigma_{-1}{}^1&=\frac{-1}{\sqrt{2}}(\sigma_x-i\sigma_y),\\ \sigma_0{}^1&=\sigma_z, &\sigma_1{}^1&=\frac{1}{\sqrt{2}}(\sigma_x+i\sigma_y),\end{aligned} \tag{7}$$

and expanding V_d in a reduced tensor operator $C_\mu{}^\lambda$, following Racah's treatment,[9]

$$C_\mu{}^\lambda=\left(\frac{4\pi}{2\lambda+1}\right)^{\frac{1}{2}}Y_\lambda{}^\mu(\theta,\varphi),$$

the expansion coefficient being $v_\lambda(r_pr_n)$, Eq. (6) becomes

$$V_{pn}=\delta(r_p-R_I)\sum_{\beta\gamma\lambda\mu}a_\beta v_\lambda(r_pr_n)(-1)^{\mu+\gamma} \times\sigma_\gamma{}^\beta(p)C_\mu{}^\lambda(p)\sigma_{-\gamma}{}^\beta(n)C_{-\mu}{}^\lambda(n). \tag{8}$$

For the nuclear state, we use the j-j coupling shell-model wavefunction.[10] In fact, Levinson and Banerjee[2]

[8] S. Devons and L. J. B. Goldfarb, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, Part III.

[9] G. Racah, Phys. Rev. **62**, 438 (1942).

[10] From analysis of electron scattering from C^{12}, Pal and Nagarajan [Phys. Rev. **108**, 1577 (1957)] have concluded that the L-S limit comes closest to the observed value, this being in contradiction to Levinson and Banerjee's conclusion.

have reported that this model yields a remarkably good fit to the $p'-\gamma$ angular correlation from the 4.43-MeV level in C^{12} as well as to the angular distribution of inelastically scattered protons. For the sake of brevity, we assume $J_i=M_i=0$. This assumption is sufficient for our purpose, since an even-even nucleus is considered here. Let us assume that one particle in the j shell is excited to the j' shell by the collision. Thus, the wavefunctions of the initial and excited states are expressed as follows:

$$\phi_i(J_iM_i)=|j^qJ_i=M_i=0\rangle,$$
$$\phi(JM)=|j^{q-1}(j)j'JM\rangle. \qquad (9)$$

Using the Racah technique[9,11] for tensor operators, and using (3), (8), and (9), we find

$$\begin{aligned}\langle\phi(JM)\psi_f|V|\phi_i(J_iM_i)\psi_i\rangle &= \langle j^{q-1}(j)j'JM;\psi_f|V|j^qJ_i=M_i=0;\psi_i\rangle\\ &= q^{\frac{1}{2}}(j^{q-1}(j)jJ_i=0]j^qJ_i=0)\langle jj_n'JM;\psi_f|V_{pn}|jj_nJ_i=M_i=0;\psi_i\rangle\\ &= q^{\frac{1}{2}}(4\pi)^{\frac{3}{2}}\sum_{L_1L_2M_2} i^{L_1-L_2}\exp[i(\delta_{L_1}+\delta_{L_2})](2L_1+1)^{\frac{1}{2}}Y_{L_2}^{M_2}(\Omega)\\ &\qquad\times\sum_{\lambda\beta} a_\beta f_{\lambda\beta}(L_1L_2M_2;JM)F_\lambda(L_1L_2;ll'), \end{aligned}\qquad (10)$$

$$f_{\lambda\beta}=\sum_{\gamma\mu}(-1)^\lambda(2\lambda+1)(2\beta+1)\left(\frac{2(2j'+1)(2l'+1)(2L_2+1)}{2L_1+1}\right)^{\frac{1}{2}}(\lambda\mu\beta\gamma|JM)$$
$$\times(l'0\lambda0|l0)(L_20\lambda0|L_10)(s-\sigma_2\beta-\gamma|s-\sigma_1)(L_2-M_2\lambda-\mu|L_10)\begin{Bmatrix}\beta & \lambda & J\\ s & l & j\\ sl & ' & j'\end{Bmatrix}, \qquad (10a)$$

$$F_\lambda(L_1L_2;ll')=\frac{1}{2\lambda+1}f_{L_2}^*(R_I)f_{L_1}(R_I)R_I^2\int R_{n'l'}(r_n)v_\lambda(R_Ir_n)R_{nl}(r_n)r_n^2dr_n, \qquad (10b)$$

where $(j^{q-1}(j)jJ_i=0]j^qJ_i=0)$ is the coefficient of fractional parentage; $\begin{Bmatrix}\beta & \lambda & J\\ s & l & j\\ s & l' & j'\end{Bmatrix}$ is the Wigner 9-j symbol; R_{nl} is the radial wavefunction of the bound nucleon in the state with quantum numbers n and l.

Inserting (10) into (2b) and using the algebra for angular momentum, B_k becomes, neglecting irrelevant constant factors,

$$B_k\propto\sum_{\beta\lambda\lambda'}\sum_{L_1L_1'L_2}|a_\beta|^2(-1)^{\beta+L_2}(2\beta+1)(2J+1)(2j'+1)(2l'+1)(L_10L_1'0|k0)W(J\lambda J\lambda';\beta k)W(\lambda L_1\lambda'L_1';L_2k)$$
$$\times G_\beta^*(L_1'L_2;\lambda')G_\beta(L_1L_2;\lambda), \qquad (11)$$

$$G_\beta(L_1L_2;\lambda)=i^{L_1-L_2}\exp[i(\delta_{L_1}+\delta_{L_2})](2\lambda+1)[(2L_1+1)(2L_2+1)]^{\frac{1}{2}}$$
$$\times(L_20\lambda0|L_10)(l'0\lambda0|l0)\begin{Bmatrix}\beta & \lambda & J\\ s & l & j\\ s & l' & j'\end{Bmatrix}F_\lambda(L_1L_2;ll'). \qquad (11a)$$

In the expression (11), the cross term of $\beta=0$ and 1 vanishes. Remembering that k is even, we find from Eq. (11) that $L_1+L_1'=$even. If $l+l'$ and J are even, which is true for the reaction considered here, we have $\lambda=\lambda'=J$, and L_1+L_2 and $L_1'+L_2$ are even numbers. In this case, since the integral in F_λ contributes only a constant factor to B_k, the shape of the γ-angular distribution $W(\theta)$ is independent of the shape of the space portion $V_d(|\mathbf{r}_p-\mathbf{r}_n|)$ of the two-body potential, and independent of the form of the radial wavefunction of the bound nucleon.

B. Collective Excitation

For collective excitation, the interaction Hamiltonian V must be the deformed part of the optical potential. We restrict ourselves to quadrupole deformation. Hence the effective nuclear radius is

$$R=R_0[1+\sum_\mu\alpha_{2\mu}Y_2^\mu(\theta,\varphi)]. \qquad (12)$$

For small shape oscillations of a spherical nucleus, $\alpha_{2\mu}$ is

$$\alpha_{2\mu}=\left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}}[b_\mu+(-1)^\mu b_{-\mu}^*], \qquad (13)$$

where $\hbar\omega$ is the energy of one surface phonon; C the surface tension; b_μ^* and b_μ are the creation and annihilation operators for one phonon in the state $|2\mu\rangle$, respectively. For the rotation of a slightly deformed, axially symmetric nucleus, $\alpha_{2\mu}$ is

$$\alpha_{2\mu}=\left(\frac{4\pi}{5}\right)^{\frac{1}{2}}\beta Y_2^{\mu*}(\Theta), \qquad (14)$$

where β is the usual deformation parameter; Θ represents the orientation of the symmetry axis.

The interaction Hamiltonian V_{coll}, to first order in

[11] G. Racah, Phys. Rev. **63**, 367 (1943); N. Noya, A. Arima, and H. Horie, Suppl. Progr. Theoret. Phys. (Kyoto) **8**, 33 (1958).

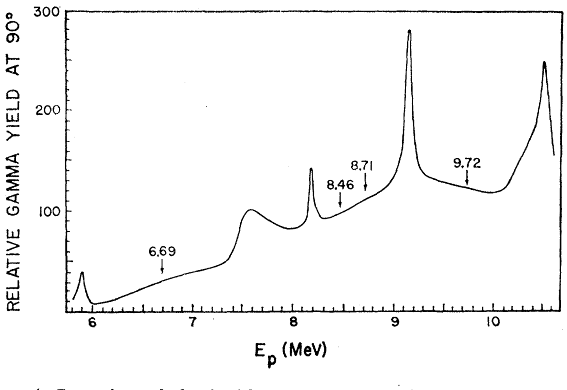

FIG. 1. Location of the incident proton energies at which calculations were carried out. Excitation curve from reference 1.

the deformation, is given by

$$V_{\text{coll}}=R_0(\partial U/\partial r)\sum_\mu \alpha_{2\mu}Y_2{}^\mu(\theta\varphi). \tag{15}$$

Since the optical potential is assumed to have the rectangular well shape, $\partial U/\partial r$ has a δ-function form

$$\partial U/\partial r\propto\delta(r-R). \tag{16}$$

Since $(-1)^\mu b_{-\mu}{}^*$ in Eq. (13) has the same property as $Y_2{}^{\mu*}$, for both shape oscillation and rotation, the matrix element of V_{coll} has the following form

$$\langle V_{\text{coll}}\rangle\propto\langle\phi_{\text{coll}}(JM)|Y_2{}^{\mu*}(\text{collective coordinate})|\times\phi_{i\text{ coll}}(J_i=M_i=0)\rangle\langle\psi_f|\delta(r_p-R_I)Y_2{}^\mu|\psi_i\rangle, \tag{17}$$

where ϕ_{coll} is the wavefunction for the collective nuclear state; R is replaced by R_I. On the other hand, according to the shell model, for the last shell in C^{12}, we have $l=l'=1$ and $J=2$. Therefore, for the reasons mentioned in the last part of Sec. A, only the term with $\lambda=2$ in the expression (8) contributes to $W(\theta)$. Thus, for $a_1=0$, the matrix element of V_{pn} has the form, aside from an irrelevant constant factor,

$$\langle V_{pn}\rangle\propto\langle j^{q-1}(j)j_n'JM|v_2(R_Ir_n)Y_2{}^{\mu*}(\theta_n\varphi_n)|\times j^{q-1}(j)j_nJ_i=M_i=0\rangle\langle\psi_f|\delta(r_p-R_I)Y_2{}^\mu|\psi_i\rangle. \tag{18}$$

The first matrix element in both the expressions (17) and (18) reduces to the form (constant)$\times\delta_{\mu M}$. Therefore, expressions (17) and (18) differ only by a constant factor. Thus, the shape of the angular distribution $W(\theta)$ for collective excitation is the same as that for particle excitation in the case of the non-spin-flip two-body potential.

III. RESULTS OF CALCULATION

According to the j-j coupling shell model, the excitation to the first excited state ($J=2$) of C^{12} corresponds to the single particle transition $1P_{3/2}\to 1P_{1/2}$. Thus, we have $j=\frac{3}{2}$, $j'=\frac{1}{2}$ and $l=l'=1$, resulting in $\lambda=\lambda'=2$ as already mentioned in Sec. II B.

For the distorted optical potential U we use, following Glendenning,

$$U=V_0+iW_0 \quad \text{for} \quad r<R$$
$$=0 \quad \text{for} \quad r>R, \tag{19}$$

$V_0=35$ MeV, $W_0=10$ MeV, $R=3.2\times10^{-13}$ cm, $R_I=3.45\times10^{-13}$ cm.

For the incoming and outgoing waves (3), partial waves with angular momentum up to 4 are taken into the calculation. For the incident energies ($\lesssim 10$ MeV) considered here, the partial waves with angular momentum greater than four have a negligible effect on $W(\theta)$.

The calculations have been carried out for the incident proton energies $E_p=6.69$, 8.46, 8.71, and 9.72 MeV. The location of these incident energies in the excitation curve[1] are shown by arrows in Fig. 1. The results of calculation are shown in Fig. 2. As seen from Fig. 2, the curves for the spin-flip potential ($a_0=0$) and the non-spin-flip potential ($a_1=0$) (shown by full curves I and II, respectively) are quite different.

For the same two-body potential as (6), the angular distribution of the inelastically scattered proton is, aside from irrelevant constant factors, given by

$$\sigma(\theta)\propto\sum_{\lambda\beta}\sum_{L_1L_1'L_2L_2'}|a_\beta|^2(-1)^\lambda\frac{2J+1}{2\lambda+1}(2j'+1)(2l'+1)(2\beta+1)[(2L_2+1)(2L_2'+1)]^{\frac{1}{2}}G_\beta{}^*(L_1'L_2';\lambda)G_\beta(L_1L_2;\lambda)\times\sum_k(L_10L_1'0|k0)(L_20L_2'0|k0)W(L_1L_2L_1'L_2';\lambda k)P_k(\cos\theta). \tag{20}$$

In our case, λ in Eq. (20) is regarded as a constant parameter and the summation over λ is removed. G_β has the form $f(\beta,\lambda)\times g(L_1L_2,\lambda)$. Therefore, the term depending on β contributes to expression (20) only as a constant factor. Thus, the shapes of the angular distributions of the inelastically scattered proton are the same for both the spin-flip and non-spin-flip two-body potential. This conclusion is independent of the choice of distorted potential, and independent of the surface interaction assumption. This conclusion should be contrasted with the result for the angular distribution of γ radiation.

For all four incident proton energies, good fits to the experiments are found for $a_1/a_0=\pm0.67$. The curves obtained for this mixing ratio are shown by the full curves III in Fig. 2, being compared with the experimental points. This value of a_1/a_0 might, in general, change by changing the distorted potential and the surface interaction assumption. However, suppose that we were to change the distorted potential or the type

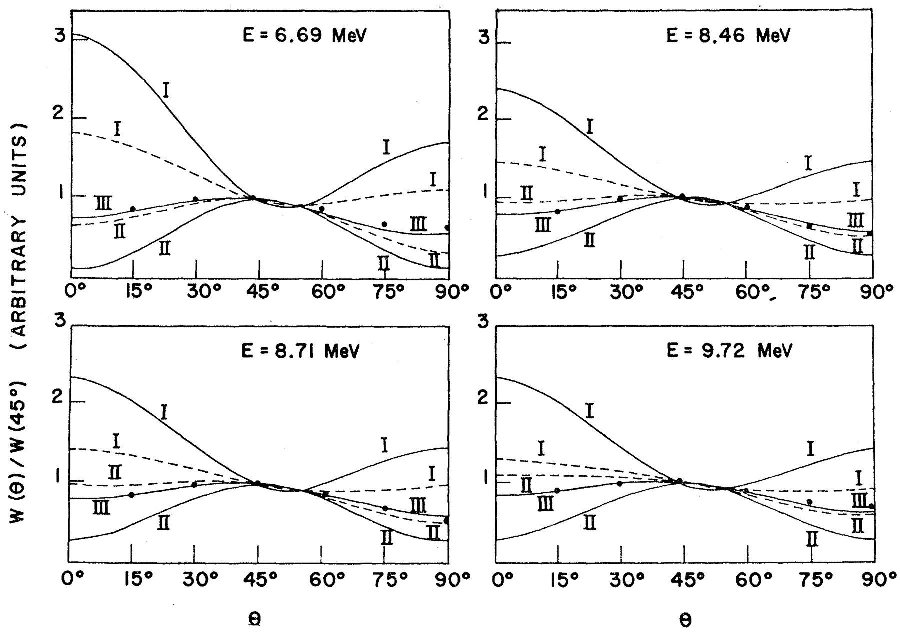

FIG. 2. γ angular distributions from the 4.43-MeV level in C^{12}. The points are the experimental results of reference 1. Solid curves I are for distorted wave and particle excitation due to spin-flip potential ($a_0=0$, $a_1=1$); solid curves II for distorted wave and non-spin-flip potential ($a_0=1$, $a_1=0$), or for distorted wave and collective excitation; solid curves III for distorted wave and $a_1/a_0=\pm 0.67$. Dashed curves I are for plane wave and spin-flip potential; dashed curves II for plane wave and non-spin-flip potential, or for plane wave and collective excitation.

of effective two body interaction (e.g., surface or volume) between the projectile and the bound particle. If the angular distributions of the inelastically scattered protons from the first excited state of C^{12} can still be explained under the new conditions, one can show that the value of the ratio a_1/a_0 will remain essentially unaltered.[12]

[12] This is found to be approximately true when the incident proton energy is low so that the partial waves of the incoming protons with $L_1>2$ and the partial waves of the outgoing protons with $L_2>1$ give small contribution to the cross section. If we neglect the effect of such partial waves, and if only $\lambda=2$ is allowed, the angular distributions of the inelastically scattered protons and of the γ radiation have the following forms:

$$\sigma(\theta_p)\propto \sum_{L_2(L_1)} a_{L_1L_2}\sum_M F_{L_1L_2,JM,\beta}(\theta_p),$$

$$W_\beta(\theta_\gamma)\propto \sum_{L_2(L_1)} a_{L_1L_2}\sum_M \int d\Omega F_{L_1L_2,JM,\beta}(\theta_p)W_{JM}(\theta_\gamma)$$

where $\sum_{L_2(L_1)}$ represents the sum over two terms with ($L_2=0$, $L_1=2$) and ($L_2=L_1=1$); $a_{L_1L_2}$ is independent of M and β; $W_{JM}(\theta_\gamma)$ represents the angular distribution function of γ radiation following the transition ($JM\rightarrow J_f=M_f=0$). The effects of the distorted potential and the surface interaction assumption are contained only in $a_{L_1L_2}$ and affect the shapes of $\sigma(\theta_p)$ and $W_\beta(\theta_\gamma)$ through the ratio between the two $a_{L_1L_2}$. Therefore, if the shape of $\sigma(\theta_p)$ is unaffected by a change in the distorted potential or the surface-interaction assumption, since this implies no change in the ratio of the $a_{L_1L_2}$, the shape of $W_\beta(\theta_\gamma)$ and the ratio a_1/a_0 remain unchanged. At the lowest incident proton energy $E_p=6.69$ MeV considered in our analysis, the approximation mentioned above is well justified. Nevertheless, good fits for all four incident energies including $E_p=6.69$ MeV are obtained for the same value of a_1/a_0.

The plane-wave Born approximation for the incoming and outgoing waves does not yield satisfactory results by any choice of the mixing ratio of the two types of the two-body potential.

The theoretical curves for collective excitation are identical to those for particle excitation for $a_1=0$, as mentioned previously, and hence are not in agreement with the experiments. Thus, our analysis shows that the 4.43-MeV level in C^{12} is probably a particle-excited state. However, the possibility of a small admixture of the collective mode in this level remains. For a small admixture of the collective mode, the ratio a_1/a_0 would change to a slightly larger (absolute) value.

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